# THE ENERGY AND ECONOMIC PARAMETERS OF *Bi-Te* BASED THERMOELECTRIC GENERATOR MODULES FOR WASTE HEAT RECOVERY

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• *The results of computer simulation of thermoelectric generator module for waste heat recovery are presented. The effect of module construction on its energy and economic parameters is analyzed. Possibilities for the reduction of specific cost of modules are considered.*

## Introduction

A relevant application of thermoelectric power generation is waste heat recovery from vehicular exhaust gas [1-3], steel industry [4], the turbines of gas pumping stations [5], etc. Alongside with thermoelectric method, there are other waste heat recovery techniques, for instance, with the use of a vapour cycle [6]. Although such type of recovery compares favourably in efficiency, at low temperatures (below 350 to 400 °C) it is generally useless. At the same time, there are many heat sources where the temperature level is 150 to 200 °C. Where waste heat recovery is concerned, the factor of module efficiency is less vital than its specific cost. However, at the present time the cost of commercially available thermoelectric generator modules, such as Hi-Z or Komatsu [7-8], is rather high – about 10 to 20 $/W, which is the main obstacle on the way to mass use of generator modules for heat recovery.

The purpose of the work is to analyze the possibility for improvement of economic parameters of modules at possibly high efficiency values. Computer simulation methods [9-11] were employed for the analysis with regard to a three-dimensional distribution of the temperature and electric fields and the temperature dependences of module structural unit parameters.

## 1. Simulation of thermoelectric generator module

### 1.1. Physical model

A physical model represented in Fig. 1 was considered for the calculation of the energy parameters of thermoelectric generator module.

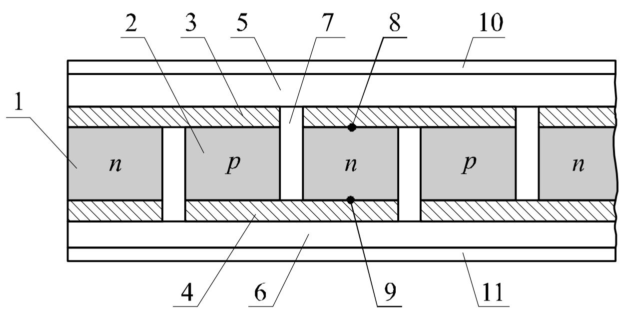


*Fig. 1. Physical model of thermoelectric generator module. 1 – n-type leg, 2 – p-type leg, 3, 4 – electric interconnects, 5, 6 – ceramic plates, 7 – gas, 8, 9 – electric contacts between the legs and connecting plates, 10 – thermal contact between ceramic plate and hot thermostat, 11 – thermal contact between ceramic plate and cold thermostat.*

A thermoelectric generator module comprises *n*-type legs 1 and *p*-type legs 2 connected into a series electric circuit by interconnects 3 on the hot side and interconnects 4 on the cold side of module. Electric contacts 8 and 9 between the legs and connecting plates are characterized by the electric contact resistances different for the hot and cold module sides. Ceramic plates 5 and 6 on the hot and cold module sides, respectively, serve as the reinforcing base for thermoelectric module. The space between module legs is filled with gas 7. Module thermal contacts 10 and 11 between ceramic plates and heat supply/heat sink are characterized by thermal contact resistances.

**1.2. Mathematical description of a physical model**

The basic equations for finding the temperature, potential and current distributions in a thermally and electrically conducting medium with regard to thermoelectric effects will be obtained from the general laws of energy and electric charge conservation. The law of energy conservation

$$\operatorname{div}\vec{w} = 0, \tag{1}$$

$$\vec{w} = \vec{q} + U\vec{j}\,. \tag{2}$$

In (1, 2) $\vec{w}$ is energy flux density, $\vec{q}$ is heat flux density, $U$ is electrochemical potential, $\vec{j}$ is electric current density.

$$\vec{q} = -\kappa\nabla T + \Pi\vec{j}\,, \tag{3}$$

where $\Pi$ is the Peltier coefficient, $\kappa$ is thermal conductivity.

$$\Pi = \alpha T, \tag{4}$$

where $\alpha$ is the Seebeck coefficient, $T$ is temperature.

The electric current density is found from the equation

$$\vec{j} = -\sigma\nabla U - \sigma\alpha\nabla T\,, \tag{5}$$

where $\sigma$ is electric conductivity.

Substituting (2), (3) in (1) yields

$$-\nabla(\kappa\nabla T) + (\nabla\Pi + \nabla U)\vec{j} = 0\,. \tag{6}$$

From expression (6), using (4) and (5), we obtain an equation for finding the temperature distribution

$$-\nabla\left(\left(\sigma\alpha^2 T + \kappa\right)\nabla T\right) - \nabla\left(\sigma\alpha T\nabla U\right) = \sigma\left(\left(\nabla U\right)^2 + \alpha\nabla T\nabla U\right). \tag{7}$$

For finding the electric potential distribution we will use the law of electric charge conservation

$$\operatorname{div}\vec{j} = 0\,. \tag{8}$$

Substituting (5) in (8) yields the following equation:

$$-\nabla\left(\sigma\alpha\nabla T\right) - \nabla\left(\sigma\nabla U\right) = 0\,. \tag{9}$$

System (7), (9) is a system of differential equations with second-order variable coefficients in partial derivatives describing the temperature and potential distribution in the inhomogeneous thermoelectric medium. A peculiarity of the system of equations (7) and (9) is that parameters $\alpha$, $\sigma$, $\kappa$

depend on space coordinates $x$, $y$, $z$ both explicitly and implicitly through temperature $T(x, y, z)$. As a result, the use of numerical computer methods for solving such kind of equations becomes inevitable.

### 1.3. Computer description of the model

In a computer model the thermoelectric field is represented by a two-element matrix in a functional space of twice differentiable functions including the temperature and electric fields:

$$M = \begin{pmatrix} T(x,y,z) \\ U(x,y,z) \end{pmatrix}. \tag{10}$$

Matrix $M$ satisfies one matrix differential equation

$$-\nabla(-c\nabla M) = f\ , \tag{11}$$

whose components are equations (7) and (9), if matrix nonlinear coefficients of equation (11) are of the form

$$c = \begin{vmatrix} \sigma\alpha^2 T + \kappa & \sigma\alpha T \\ \alpha\sigma & \sigma \end{vmatrix},\ f = \begin{vmatrix} \sigma\left(\left(\nabla U\right)^2 + \alpha\nabla T\nabla U\right) \\ 0 \end{vmatrix}. \tag{12}$$

Such representation of a thermoelectric medium allows solving equation (11) in Comsol Multiphysics simulation medium. The results of solving equation (11) are three-dimensional temperature and electric fields in given geometry of thermoelectric module. Knowing these values, one can easily calculate the basic energy characteristics of the module.

## 2. Research on the effect of module geometry on the energy and economic parameters

### 2.1. Effect of leg height

Leg height is one of important geometric parameters of a thermoelectric module. With a reduction of leg height, the module thermal resistance decreases, heat flux through the module and its electric power increase, accordingly. Moreover, consumption of thermoelectric material, the most expensive component of a thermoelectric module, decreases in proportion to leg height. The competing factors preventing from a continuous reduction of leg height are contact thermal and electric resistances. With a reduction of leg height, the losses of temperature differences in the thermal contacts between heat sinks and ceramic plates increase considerably, and the negative effect of the Joule heat released in the electric contact junctions between thermoelectric legs and interconnects becomes much more pronounced.

Computer simulation was used to study the effect of module leg height on its energy characteristics. Let us consider this situation with specific reference.

Input parameters of a model: $T_h = 200$ °C is heat supply temperature, $T_c = 40$ °C is heat sink temperature, $a \times b = 2.4 \times 2.4$ mm is cross-section of leg, $d_a = 1.4$ mm is the distance between legs, $N = 49$ is the number of thermoelectric couples in a module, $\alpha_n(T)$, $\alpha_p(T)$, $\sigma_n(T)$, $\sigma_p(T)$, $\kappa_n(T)$, $\kappa_p(T)$ are the Seebeck coefficient, electric conductivity and thermal conductivity of *n*- and *p*-type thermoelectric material, respectively (standard for *Bi-Te* based materials); $R_{t1} = 4$ K/W is thermal contact resistance between heat supplies and ceramic plates, $r_{c1} = 10^{-5}$ $\Omega\cdot\text{cm}^2$ is contact electric resistance in the hot part of module, $r_{c2} = 5\cdot10^{-6}$ $\Omega\cdot\text{cm}^2$ is contact electric resistance in the cold part of module.

Calculation of module manufacturing cost was done with account of material cost, salary (based on the average worker's salary in China) and profit (~ 20%).

Figs. 2 and 3 show the leg height dependences, calculated in simulation, of the efficiency and specific cost of thermoelectric module.

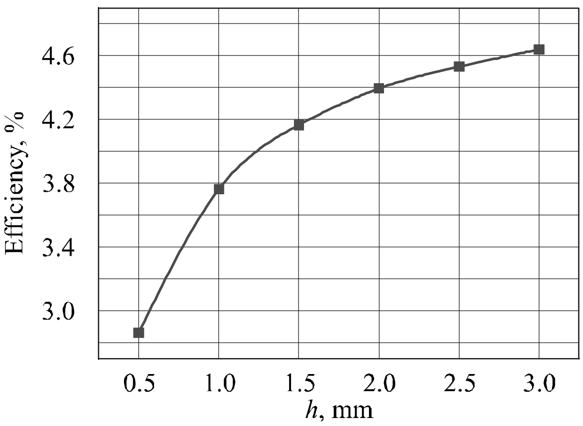


*Fig. 2. Module efficiency versus leg height.*

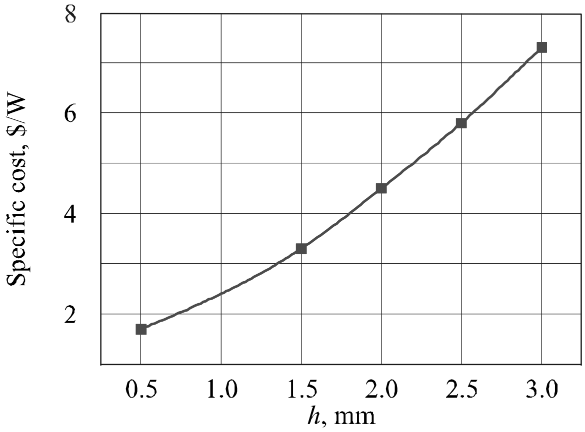


*Fig. 3. Module specific cost versus leg height.*

### 2.2. Effect of contact electric resistances of anti-diffusion layers

The electric power, efficiency and specific cost of module were calculated as a function of contact electric resistance for different leg heights. Contact electric resistance varied in the range from 0 to $10^{-5}$ $\Omega\cdot cm^2$. The results of calculations are given in Figs. 4 and 5. As can be seen from Figs. 4 and 5, with a reduction of leg height, the negative effect of contact electric resistances on the electric power and efficiency of module is particularly pronounced.

In general, the necessity to reach possibly lower contact resistance values is evident. At the present time, the values of contact resistances of *Ni* anti-diffusion layers reach $3 \div 5\cdot10^{-6}$ $\Omega\cdot cm^2$. Hence it follows that to achieve acceptable efficiency values, the height of legs should lie within 1 to 1.5 mm.

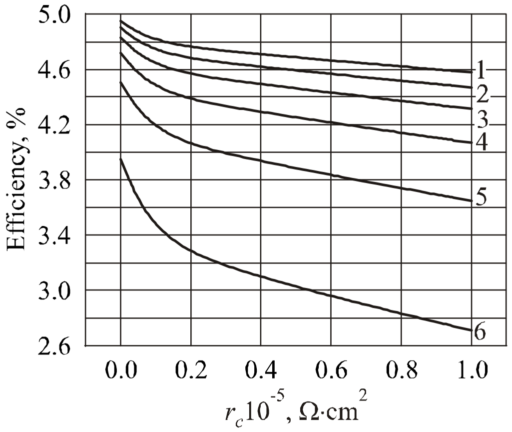


*Fig. 4. Dependences of module efficiency on the contact resistance of anti-diffusion layers for different leg heights h. (1 – 3 mm; 2 – 2.5 mm; 3 – 2 mm; 4 – 1.5 mm; 5 – 1 mm; 6 – 0.5 mm)*

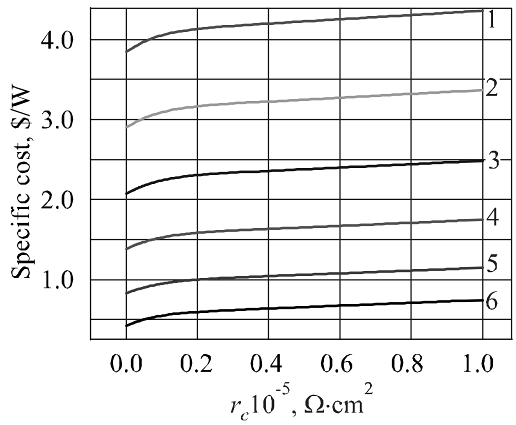


*Fig. 5. Dependences of module specific cost on contact resistance of anti-diffusion layers for different leg heights h. (1 – 3 mm; 2 – 2.5 mm; 3 – 2 mm; 4 – 1.5 mm; 5 – 1 mm; 6 – 0.5 mm)*

### 2.3. Effect of connecting plate thickness

Presented here are the results of simulation of a dependence of the energy and economic parameters of a thermoelectric module on the thickness of copper connecting plates. The thickness varied in the range of 0.1 – 1 mm.

The simulation was done for various leg heights. The electric contact resistance of anti-diffusion layers was assumed equal to $5\cdot10^{-6}$ $\Omega\cdot cm^2$.

The results of simulation are shown in Figs. 6 and 7. As can be seen from the figures, with connecting plate thickness 0.4 to 0.5 mm, the values of module electric power and efficiency reach saturation. In so doing, the smaller is the height of a leg, the more appreciable is the effect of connecting plate thickness on the module efficiency.

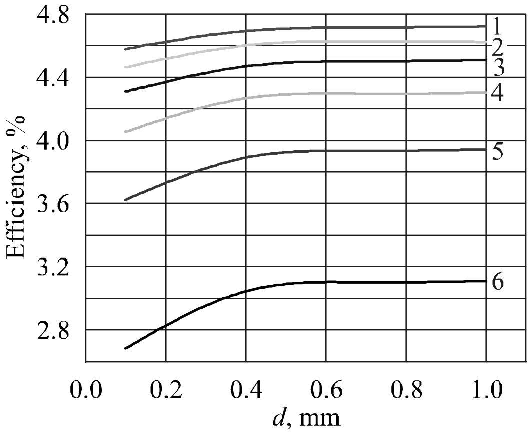


*Fig. 6. Dependences of module efficiency on connecting plate thickness for different leg heights h. (1 – 3 mm; 2 – 2.5 mm; 3 – 2 mm; 4 – 1.5 mm; 5 – 1 mm; 6 – 0.5 mm)*

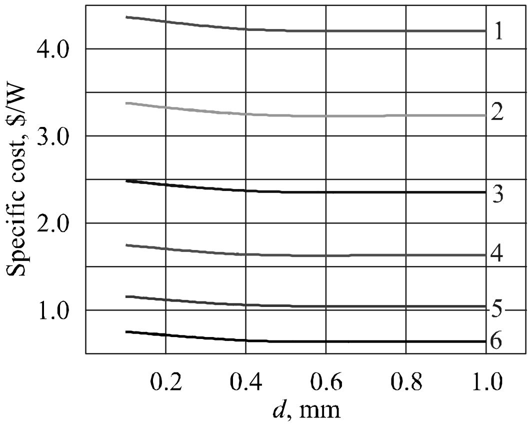


*Fig. 7. Dependences of module specific cost on connecting plate thickness for different leg heights h. (1 – 3 mm; 2 – 2.5 mm; 3 – 2 mm; 4 – 1.5 mm; 5 – 1 mm; 6 – 0.5 mm)*

### 2.4. Effect of heat losses in ceramic plates

Ceramic plates are one of important structural units of thermoelectric module. They serve both as reinforcing module elements and electric insulation between connecting plates and heat supplies. The heat conducting properties of ceramic plates strongly influence the resulting temperature drop on the legs of thermoelectric module.

Figs. 8 and 9 show the results of module simulation with a different thickness of ceramic plates, as well as with the use of different materials of ceramic plates: $Al_2O_3$ and *AlN*. The *AlN* material compares favourably in its thermal conductivity and allows reducing to minimum the heat losses in ceramic plates. Its weak point is high cost. It is more expensive that $Al_2O_3$ by a factor of about 10.

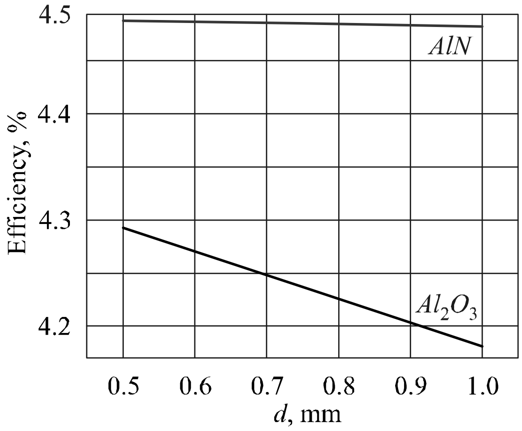


*Fig. 8. Dependences of module efficiency on the thickness of ceramic plates.*

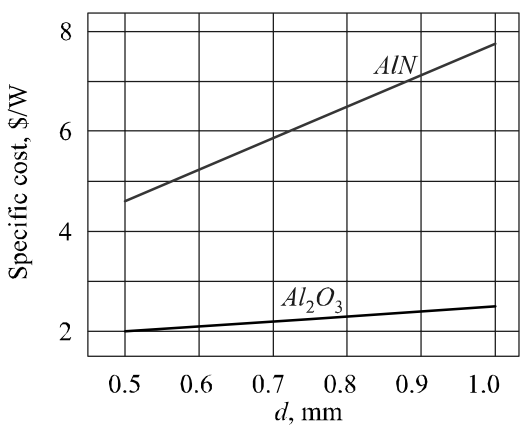


*Fig. 9. Dependences of module specific cost on the thickness of ceramic plates.*

Simulation was done for different thicknesses of ceramic plates in the range of 0.5 to 1 mm with the use of both $Al_2O_3$ and *AlN* for leg height 1.5 mm.

As is obvious from Figs. 8 and 9, the use of ceramic plates based on *AlN* allows increasing marginally the electric power and efficiency of module. But if we compare the specific cost of modules employing $Al_2O_3$ and *AlN* ceramic plates, the economic inefficiency of using ceramic plates based on *AlN* becomes evident. As is seen from Fig. 9, the most efficient way is to use ceramic plates of thickness 0.50 to 0.65 mm.

### 2.5. Effect of heat losses in connecting plates and anti-diffusion layers

Presented here are the results of simulation of the effect of heat losses in module structural units, including connecting plates and anti-diffusion layers.

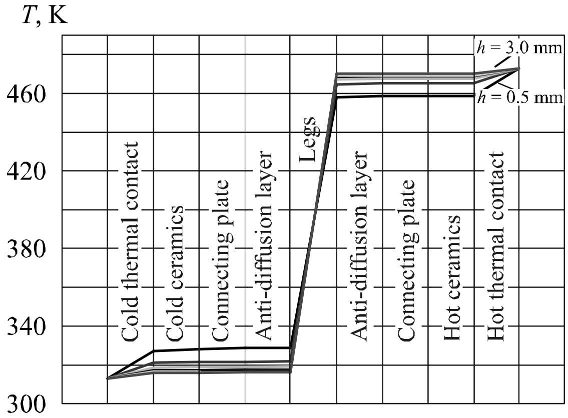


*Fig. 10. Temperature distribution along module structural units h is leg height.*

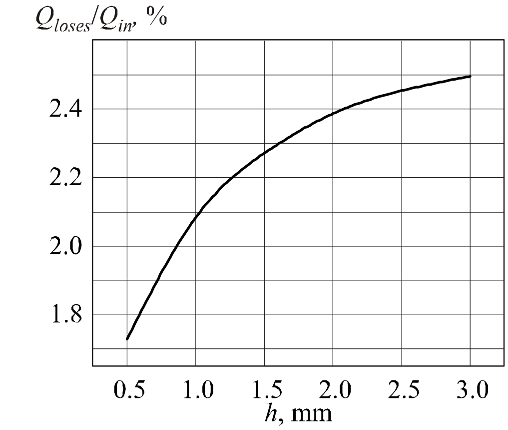


*Fig. 11. Dependence of the amount of heat losses in the space between legs on leg height h.*

Simulation was done for the leg height in the range of 0.5 to 3 mm. The thickness of ceramic plates is 0.63 mm, the thickness of connecting plates is 0.8 mm, the thickness of anti-diffusion layers is 150 µm. The results of simulation are shown in Fig. 10. As is obvious, the most considerable are the losses in temperature difference in thermal contacts between the heat supply and the hot ceramics, as well as between the heat sink and the cold ceramics. In so doing, the smaller is the leg height, the more appreciable are these losses.

Heat losses in connecting plates and anti-diffusion layers are negligibly small even for their considerably larger thicknesses and in the future they can be ignored.

### 2.6. The effect of heat losses in the space between the legs

The space between module legs is filled with gas. Therefore, part of the heat supplied to module is transferred directly from the hot ceramic plate to the cold one through the gas gap. This factor should reduce module efficiency. Computer simulation was used to analyze heat losses in the space between module legs for the case when it is filled with air. The results are given in Fig. 11, where dependence of the ratio of the amount of heat that passed through the air gap to the amount of heat supplied to module is represented.

As can be seen from Fig. 11, the amount of heat losses in the air gap increases with increasing the leg height and can make up to 2.5% for selected geometric dimensions of module.

## Conclusions

The obtained results of the effect of various geometric factors on the efficiency and specific cost of module bring us to the following conclusions.

1. The most important in module geometry is the size of legs. With a reduction of their height

from 3 to 0.5 mm, the efficiency decreases from 4.6% to 2.8%. In so doing, the specific cost decreases from 4.5 to 0.7 \$/W.

2. Another important issue is the quality of anti-diffusion layers. There is a monotonous efficiency decrease and specific cost increase with increase in the contact resistance of anti-diffusion layers. For the leg height 1.5 mm the best contact resistance value ($10^{-6}$ $\Omega\cdot cm^2$) yields the efficiency value 4.5% (in the absence of contact resistance – 4.7%). However, the influence of contact resistances continuously grows with a reduction in the leg height. Thus, with the leg height 0.5 mm, the efficiency of module decreases to 3.4%.
3. The thickness of ceramic plates has a considerable impact on the properties of modules with a reduction of the leg height. Thus, for the legs of height 0.5 mm and ceramic plate thickness 0.5 mm the efficiency decreases by 9%, and the specific cost increases from 0.7 \$/W to 2 \$/W.
4. The use of *AlN* ceramic plates allows minimizing heat losses in ceramics. However, in this case the cost of ceramic plates is considerably higher. Eventually, their use leads to efficiency increase from 4.3% to 4.5%, but the specific cost in this case increases to 5 \$/W.
5. It is established that heat losses in the anti-diffusion layers and connecting plates are marginal and these factors can be ignored.
6. The effect of heat transfer by air between module legs is also marginal. Heat losses are within 1.7 to 2.5% of total heat flux through the module. Accordingly, in these limits the efficiency will decrease and the specific cost of modules will increase. Therefore, the use of low thermal conductivity inert gases instead of air is not efficient.

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