T.A. Ismailov[1], O.V. Yevdulov[1], M.A. Khazamova[1], R.A.-M. Magomadov[2]

[1]Federal State Budget Educational Institution of Higher Professional Education "Dagestan State Technical University", 70, Ave. Imam Shamil, Makhachkala, 367015, Russia;
[2]Federal State Budget Educational Institution of Higher Professional Education "Grozny State Oil Technical University", 100, Ordzhonikidze Sq., Grozny, 364051, Russia

MATHEMATICAL MODEL OF A THERMOELECTRIC SYSTEM FOR LOCAL THERMAL EFFECT ON HUMAN HAND

The paper is concerned with a mathematical model of a thermoelectric system for thermal effect on human hand. The model is based on the numerical solution of a system of differential thermal conductivity equations for irregular-shaped bodies. Two-dimensional and one-dimensional plots of temperature variation in different zones of affected object are presented.

Key words: human hand, physiotherapy, thermal effect, temperature field, thermal model, mathematical model.

Introduction

Methods of local thermal effect are widely used in medical practice in the therapy and prophylaxis for stimulation of physiological processes. Thermal effect exerts significant influence on the energy balance of human organism. The reaction of vessels to thermal procedure propagates over the entire body surface, but it is most extensively expressed at a point of immediate application of heat. Under a local temperature effect, skin whose receptors sense pain (mechanical, thermal), temperature (cold, heat) and tactile stimulation is in the intimate contact with the temperature irritant. In so doing, blood flow in skin vessels varies by a factor of 100 to 180, and heat transfer regulation takes place mostly due to a change in blood flow, especially in the tissues of hands and feet [1].

This serves the basis for a number of methods employed in physiotherapeutic practice to which we can refer baths with gradually increased temperature (the Hauffe baths), contrast baths, partial (hand) baths, etc., as well as using various media, such as paraffin, ozokerite and others [2].

Partial or local baths include hydropathic procedures affecting certain part of the body. Wide application is enjoyed by hand baths with corresponding vessels made of galvanized iron [3]. In the process, a forearm, one or both hands, as well as the entire arm are immersed in water. The administered baths are of different temperature and duration, namely warm baths at water temperature 37 – 38° C of duration up to 20 – 30 minutes, hot baths at water temperature 40 – 44° C of duration from 10 to 20 minutes and cold baths at water temperature 8 – 14° C of duration from 5 to 12 minutes.

To intensify the irritant action, baths of contrast temperatures can be administered [4]. In so doing, two baths are used, one of which is filled with hot (40 – 45° C), and the other – with cold water (8 – 10° C). A patient alternatively immerses his hand first into hot water for 1 – 2 minutes, and then to cold water for 10 – 15 seconds, repeating this procedure several times. Cold hand baths are generally recommended at acute inflammatory processes on the hand, contrast baths – at hyperhidrosis, acrocyanosis, etc., hot baths – for infiltrate resolution, etc.

The disadvantages of the above methods of physiotherapeutic procedures include their low efficiency and discomfort, complexity and inconvenience of realization, insufficient accuracy of thermal effect dosing.

The method of thermal effect discussed above can be realized through use of thermoelectric power converters [5]. In this case the above disadvantages are largely eliminated.

The purpose of this study is mathematical simulation, as well as theoretical studies of a thermoelectric system (TES) for thermal effect on human hand with a view to perform efficient physiotherapeutic and rehabilitation procedures.

Zone model

The object of study is a device in the form of a construction comprising a flexible elastic base with embedded thermoelectric modules that have flexible metal heat leveling plates on their junctions (Fig. 1). The device is brought into intimate thermal contact with the affected object (human hand zone).

Fig. 1. The appearance of TES for a local thermal effect on human hand.

A thermal model of TES realizing physiotherapeutic procedures on the middle third of the arm is given in Fig. 2. In conformity with the specific operation of TES realizing thermal effect so that thermal flux from the device is directed normal to the surface of hand, the thermal model of affected zone on which basis the system is calculated, has the appearance shown in Fig. 2. Here, the following parts should be pointed out: 1 – skin cover, 2 – muscular tissue and 3 – bone tissue, distinguished by thermophysical parameters and the level of internal heat release.

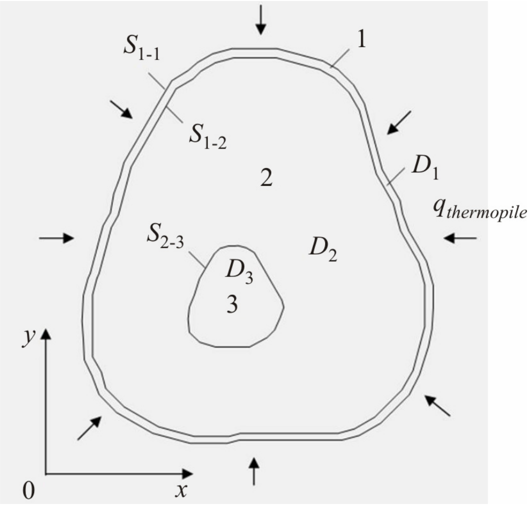

Fig. 2. Thermal model of TES realizing physiotherapeutic procedures on the middle third of the arm.

Let the area restricted by skin cover be denoted as D_1, by muscular tissue – as D_2, by bone tissue – as D_3, the surface of skin cover contacting to TES – as S_{1-1}, to muscular tissue – as S_{1-2}, the surface of muscular tissue contacting to bone tissue – as S_{2-3}, S'_{2-3}. Then a description of thermophysical processes occurring in human arm thermally affected by TES in the generalized mathematical form can be represented as follows:

– with effect on the middle third of the arm

$$a_1\frac{\partial^2 T_1}{\partial x^2}+a_1\frac{\partial^2 T_1}{\partial y^2}+\frac{Q_{vn1}}{c_1\rho_1}=\frac{\partial T_1}{\partial \tau}\ \text{at}\ x,y\in D_1;$$

$$a_2\frac{\partial^2 T_2}{\partial x^2}+a_2\frac{\partial^2 T_2}{\partial y^2}+\frac{Q_{vn2}}{c_2\rho_2}=\frac{\partial T_2}{\partial \tau}\ \text{at}\ x,y\in D_2; \qquad (1)$$

$$a_3\frac{\partial^2 T_3}{\partial x^2}+a_3\frac{\partial^2 T_3}{\partial y^2}+\frac{Q_{vn3}}{c_3\rho_3}=\frac{\partial T_3}{\partial \tau}\ \text{at}\ x,y\in D_3;$$

$$T_1,T_2,T_3=309.6\ \text{K}\ \text{at}\ \tau=0;$$

$\lambda_1\frac{\partial T_1}{\partial n_1}=\alpha\left(T_1-T_{thermopile}\right)$ at $x,y\in S_{1-1}$ – with a non-perfect contact between skin cover and the operating surface of TES;

$\lambda_1\frac{\partial T_1}{\partial n_1}=q_{thermopile}$ at $x,y\in S_{1-1}$ – with a perfect contact between skin cover and the operating surface of TES;

$$\lambda_1\frac{\partial T_1}{\partial n_1}=\lambda_2\frac{\partial T_2}{\partial n_2}\ \text{at}\ x,y\in S_{1-2};$$

$$\lambda_2\frac{\partial T_2}{\partial n_2}=\lambda_3\frac{\partial T_3}{\partial n_3}\ \text{at}\ x,y\in S_{2-3};$$

where T_1, T_2, T_3, T'_3 are the temperatures of skin cover, muscular and bone tissue; a_1, a_2, a_3 are thermal diffusivity coefficients of skin cover, muscular and bone tissue; λ_1, λ_2, λ_3 are thermal conductivity coefficients of skin cover, muscular and bone tissue; c_1, c_2, c_3 is specific heat of skin cover, muscular and bone tissue; ρ_1, ρ_2, ρ_3 is the density of skin cover, muscular and bone tissue; Q_{vn1}, Q_{vn2}, Q_{vn3} is the specific amount of heat released in skin cover, muscular and bone tissue; α is coefficient of heat exchange between skin cover and the operating surface of TES; $T_{thermopile}$ is the temperature of the operating surface of thermopile; $q_{thermopile}$ is the density of heat flux on the operating surface of thermopile; n_1, n_2, n_3 are normals to surfaces S_{1-1}, S_{1-2}, S_{2-3}, respectively; $n_i=(xh_i+yz_i)$; h, z are unit vectors; $i=1\ldots3$.

The system of equations (1) was solved by finite-element numerical method in conformity with procedure set forth in [6] and implemented in the Elcut applied software package. The results obtained make it possible to determine temperature variation at different points of the biological object, namely human arm, as well as to follow up its variation versus the value of heat flux from TES (cooling capacity and heating capacity of thermopiles) and the ambient conditions.

Simulation results

The numerical experiment was performed in conformity with the required conditions of physiotherapeutic procedures, namely the range of temperatures achieved by the biological object from 277 to 317 K, the duration of effect – from 10 to 30 minutes, possible alternation of cooling and heating the respective zone of a biological object.

The initial data was assumed as follows: $\lambda_1 = 0.407$ W/(m·K), $\lambda_2 = 0.439$ W/(m·K), $\lambda_3 = 0.34$ W/(m·K); $\rho_1 = 1036$ kg/m^3, $\rho_2 = 1050$ kg/m^3, $\rho_3 = 1036$ kg/m^3; $c_1 = 3458$ J/(kg·K), $c_2 = 4020$ J/(kg·K), $c_3 = 3127$ J/(kg·K); $Q_{vn1} = 0$, $Q_{vn2} = 30$ J/(kg·s), $Q_{vn3} = 0$. The basic geometric dimensions of affected zones are shown in Fig. 3. The data is given in mm.

The calculated results are represented in Figs. 4 – 13.

Fig. 4 shows a two-dimensional temperature field of the middle third of the arm section exposed to cooling by TES under study with the value of thermal flux on the lateral surface of the biological object equal to 2500 W/m^2.

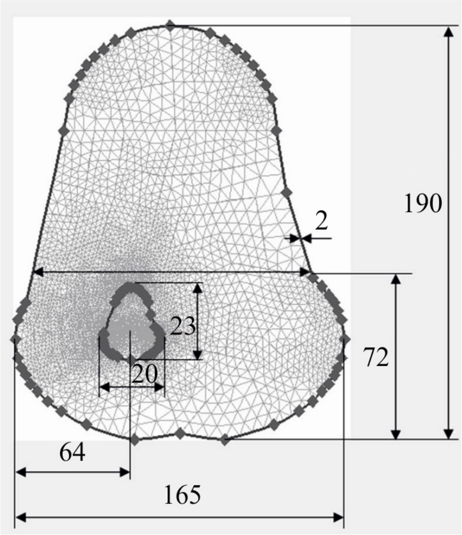

Fig. 3. A model of section of the middle third of the arm assumed in the calculation with the finite-element network and the basic dimensions.

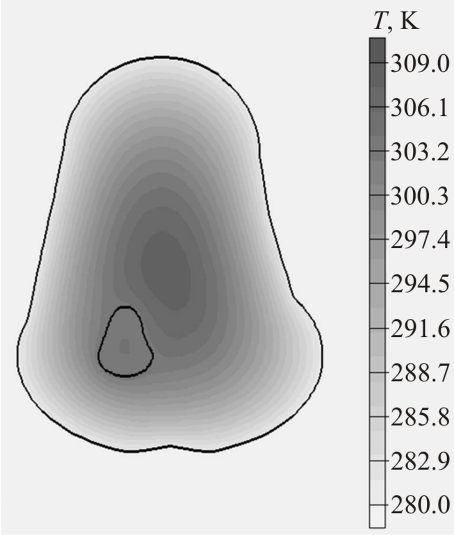

Fig. 4. Two-dimensional temperature section field of the middle third of the arm in steady-state mode under cooling effect for $q_{thermopile} = 2500$ W/m^2.

Figs. 5 and 6 show the respective one-dimensional plots of temperature distribution along the lateral axis on the section of the middle third of the arm, as well as along the lateral axis passing through the bone axis, in the steady-state mode. In so doing, for the case corresponding to Fig. 5 the values are considered for different heat flux values on the lateral surface of affected object equal to $q_{thermopile} = 2500$ W/m^2, $q_{thermopile} = 2250$ W/m^2, $q_{thermopile} = 2000$ W/m^2. According to the data obtained, the temperature of the biological object is reduced with increase in $q_{thermopile}$. In this case maximum temperature reduction is approximately the same and is observed on skin cover layer at $q_{thermopile} = 2500$ W/m^2, making 280 K. Accordingly, the temperature in the centre of the biological object varies only slightly by about 0.5 K which is due to the presence of internal heat release in thermally affected object, as well as due to its low thermal conductivity and relatively high heat capacity.

According to the above plots, the temperature distribution in the affected object depends in a certain way on the presence of bone tissue in its bulk. Under the assumed conditions that determine the absence of internal heat release in the bone tissue, such temperature variation is about 1 – 2 K with respect to the case when this object is absent in the bulk of the tissue.

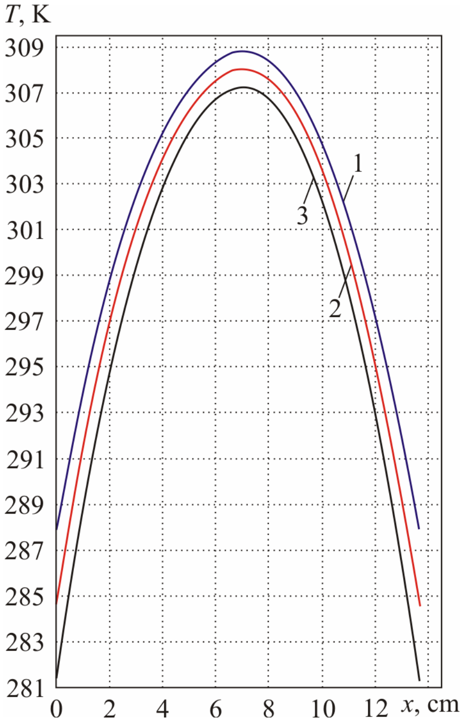

Fig. 5. Temperature distributions along the lateral axis of the middle third of the arm under cooling effect for different values of $q_{thermopile}$
1 – $q_{therrmopile}$ = 2500 W/m^2, 2 – $q_{thermopile}$ = 2250 W/m^2, 3 – $q_{thermopile}$ = 2000 W/m^2.

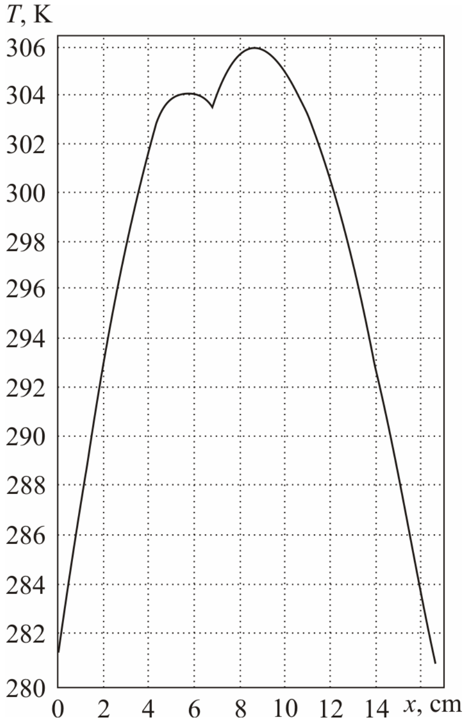

Fig. 6. Temperature distribution along the lateral axis of the bone tissue of the middle third of the arm under cooling effect for $q_{thermopile}$ = 2500 W/m^2.

Fig. 7 depicts a two-dimensional temperature field on the section of the middle third of the arm thermally affected by TES under study with the heat flux value on the lateral surface of the biological object equal to 700 W/m^2. The respective one-dimensional plots are given in Fig. 8. These dependences are mainly similar to those given for the case of TES operation in cooling mode. The difference lies in temperature rise of the affected object with increase in $q_{thermopile}$ value, as well as in the lower values of thermopile power requirement for keeping temperature mode of the biological object corresponding to performance of physio-therapeutic procedures. Under the conditions discussed above the maximum temperature rise on skin cover of the middle third of the arm was obtained at $q_{thermopile}$ = 700 W/m^2, making 315 K. For the case of TES operation in heating mode the availability of bone tissue in the bulk of the biological object also introduces relevant changes into the structure of its temperature field. The above influence is identical to the case of TES operation in cooling mode and amounts to 1 – 2 K.

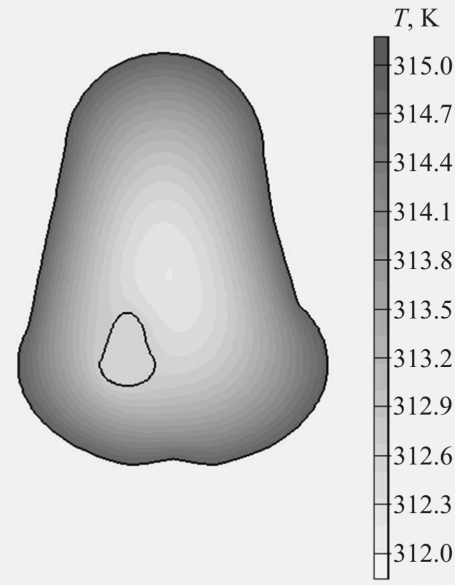

Fig. 7. Two-dimensional temperature field on the section of the middle third of the arm in steady-state mode under heating effect for $q_{thermopile}$ = 700 W/m^2.

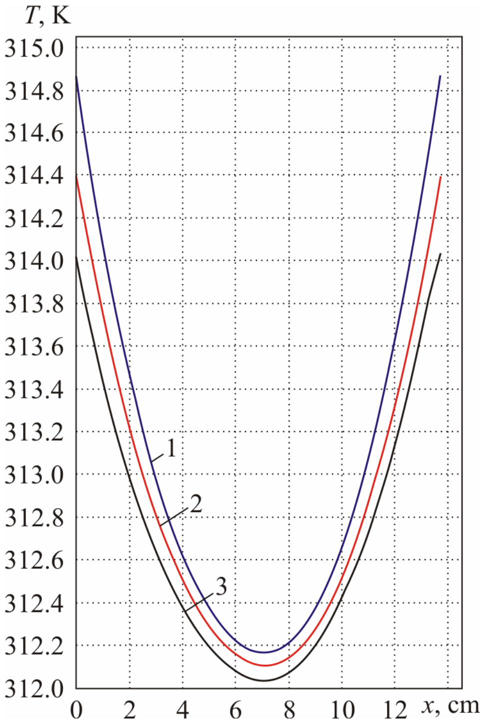

Fig. 8. Temperature distribution along the lateral axis of the middle third of the arm under heating effect for different $q_{thermopile}$ values 1 – $q_{thermopile}$ = 700 W/m^2, 2 – $q_{thermopile}$ = 650 W/m^2, $q_{thermopile}$ = 600 W/m^2.

Fig. 9 depicts a two-dimensional steady-state temperature field on the section of the middle third of the arm for the case corresponding to contrast thermal effect, when heat flux $q_{thermopile.c}$ = –3500 W/m^2 is directed to the right surface of the biological object, and $q_{thermopile.h}$ = 1000 W/m^2 – to the left surface. The respective one-dimensional plots showing temperature distribution on the above hand zone along the lateral axis for different values of $q_{thermopile.c.}$ and $q_{thermopile.h}$, are represented in Fig. 10. In conformity with these dependences, the presence on the opposite lateral surfaces of unlike heat fluxes changes considerably the thermal field of the object with respect to the case when heat flux of the same sign is distributed along the lateral surface. The temperature value decreases from the lateral surface corresponding to heat flux $q_{thermopile\ h}$ to the surface affected by heat flux $q_{thermopile.c}$. In so doing, a change in $q_{thermopile.c}$ from the value of 3500 W/m^2 to 2500 W/m^2 at a constant value of $q_{thermopile.h}$ = 1000 W/m^2 increases the temperature along the lateral axis by about 4 K. Thus, when performing contrast thermal procedures, under conditions corresponding to these dependences, to achieve the required temperature of skin surface which is a direct object of physiotherapeutic effect, one should take into account the mutual influence of $q_{thermopile.h}$ and $q_{thermopile.c}$ values. Selection of $q_{thermopile.h}$

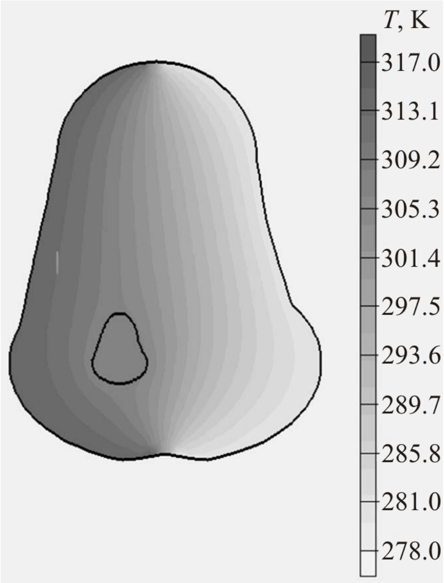

Fig. 9. Two-dimensional temperature field on the section of the middle third of the arm in the steady-state mode under contrast thermal effect for $q_{thermopile.c}$ = 3000 W/m^2, $q_{thermopile.h}$ = 1000 W/m^2.

and $q_{thermopile.c}$ values with the respective analysis of the temperature field of the object according to the model proposed will make it possible to optimize the energy characteristics of TES under study.

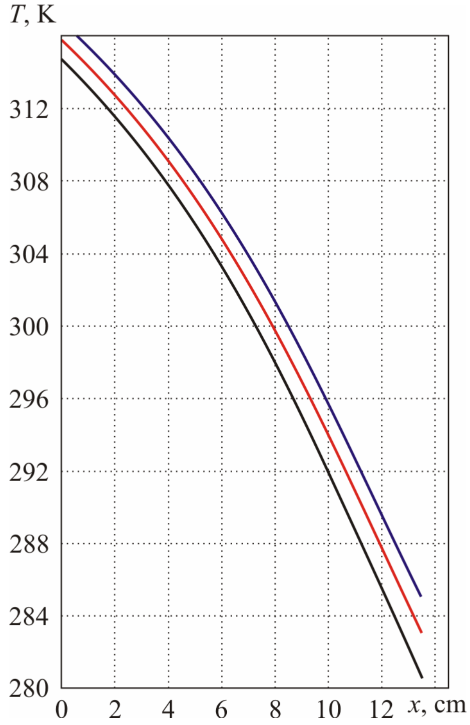

Fig. 10. Temperature distributions along the lateral axis of the middle third of the arm under contrast thermal effect for different values of $q_{thermopile}$. 1 – $q_{thermopile.c}$ = 3500 W/m^2, $q_{thermopile.h}$ = 1000 W/m^2; 2 – $q_{thermopile.c}$ = 3000 W/m^2, $q_{thermopile.h}$ = 1000 W/m^2; 3 – $q_{thermopile.c}$ = 2500 W/m^2, $q_{thermopile.h}$ = 1000 W/m^2.

For the analysis of the dynamic characteristics of TES a change in temperature at different points of a biological object under cooling and heating effect of TES was investigated. Fig. 11 represents temperature variation in time at different points of the middle third of the arm on cooling, and Fig. 12 – on heating for $q_{thermopile.c} = –2500$ W/m^2 and $q_{thermopile.h} = 800$ W/m^2, respectively. A variation in time of skin cover, muscular and bone tissue of biological object has been studied. According to calculated results, the time required to bring TES to the steady-state operating mode lies within relatively narrow limits. For the case corresponding to Figs. 11 – 12, the time required for stabilization of biological object temperature is about 1200 s (20 min.). The above factor should be taken into account when performing physiotherapeutic procedures. It is worthwhile to switch the device on prior to thermal exposure with a view to bring it to operating mode.

It is interesting to study the operation of TES in the mode of contrast thermal effect related to alternative cooling and heating of a biological object. Fig. 13 shows temperature variation on the skin cover of the middle third of the arm in time for such procedure mode. One cycle of contrast effect has been considered, namely object temperature reduction and then its rise and vice versa. Depending on the method of performing physiotherapeutic procedures, the cycle can be repeated several times. In conformity with the data obtained, through use of TES the treatment procedure of contrast thermal effect can be implemented to the full extent. The duration of change from one mode of exposure to another is relatively short. On the dependences shown in Fig. 13 it is of the order of 8 to 9 minutes. It should be noted that to accelerate a change in the modes of exposure, one can use short-time forced operation mode of TES, i.e. increase in thermopile supply current and, accordingly, the value of $q_{thermopile}$ in transient operating mode of device.

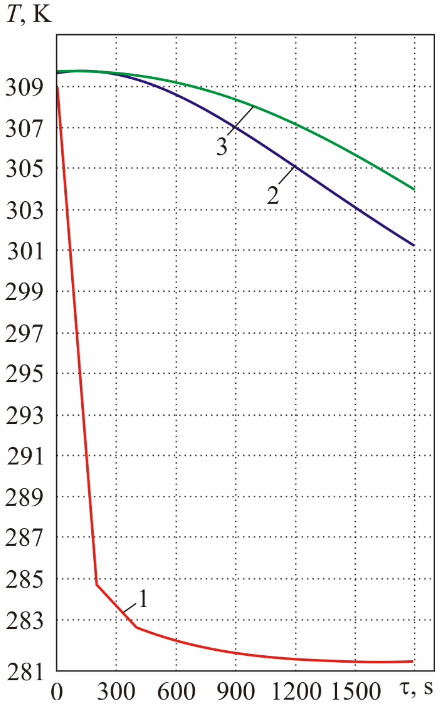

Fig. 11. Temperature variation at different points on the section of the middle third of the arm in time on cooling for $q_{thermopile.c}$ = 2500W/m^2. 1 – skin cover, 2 – bone tissue, 3 – muscular tissue.

Based on the analysis of the obtained values of heat flux on the lateral surface of biological object, equivalent to cooling power and calorific power of thermopile, the latter is subject to calculation. The sought-for values in this case are the geometrical dimensions of thermoelements (TE) forming part of thermopile, the value of supply electrical current, electrical energy consumption. In the majority of cases, ass a thermopile in thermoelectric system (TES) one can use thermoelectric modules (TEM) of standard type that can be selected with the use of special applied software packages.

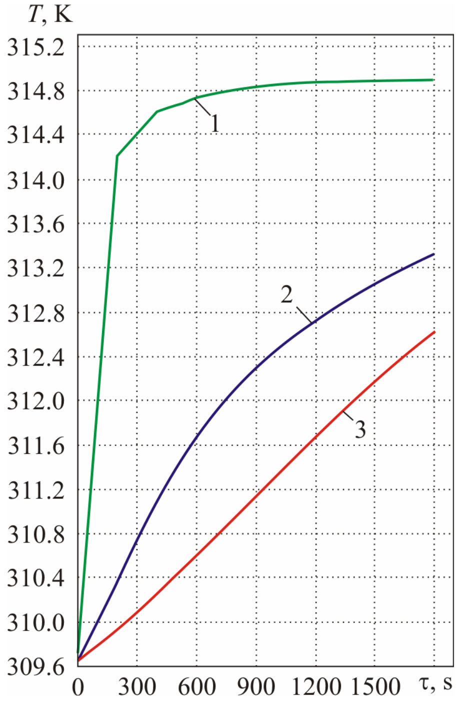

Fig. 12. Temperature variation at different points on the section of the middle third of the arm on heating for $q_{thermopile.h}$ = 800 W/m^2. 1 – skin cover, 2 – bone tissue, 3 – muscular tissue.

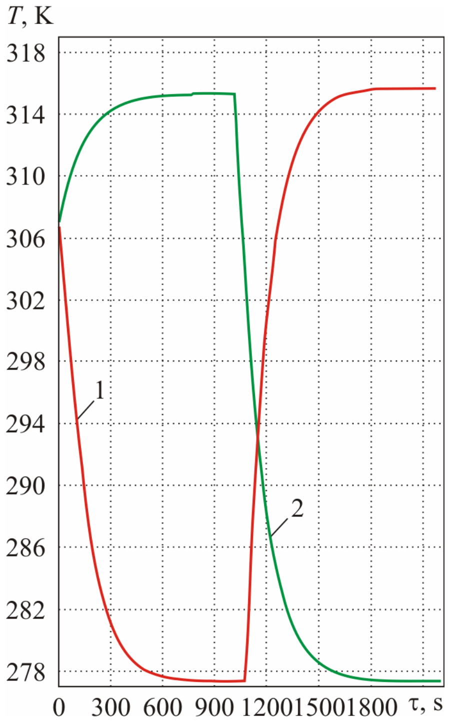

Fig. 13. Time variation of biological object under contrast procedures for $q_{thermopile.c}$ *= 4000 W/m*^{2}*,* $q_{thermopile.\,h}$ *= 1200 W/m*^{2}*. 1 – cooling-heating, 2 – heating-cooling.*

For TES variant under study one can use four standard TEMs of the type TB-127-1.0-1.5 produced by Engineering and Production Firm Kryotherm (Saint-Petersburg), realizing in full measure the required conditions of therapeutic procedures. For their selection a package of software programs “Thermoelectric system calculation” was used [7]. The specifications of this type of TEM are given ibidem.

Conclusions

1. Local thermal effect is widely used in medical practice in the therapy and prophylaxis of various diseases.
2. The above treatment method can be realized through use of TES distinguished by environmental safety, high reliability, efficiency and small overall dimensions.
3. The thermoelectric device studied in the paper is designed as a construction comprising a flexible elastic base with embedded thermoelectric modules that have flexible metal heat leveling plates on their junctions.
4. A thermal model of TES for performing thermal physiotherapeutical procedures on the middle third of the arm is a three-layered structure (skin cover, muscular and bone tissue) of complicated configuration. A thermal flux from thermopiles forming part of the system is directed to the external surface of the skin cover.
5. A mathematical model of TES is system of differential equations in partial derivatives with the boundary conditions of the second, third and fourth kind that has been solved with the aid of a numerical finite-element method.
6. In the numerical experiment, two-dimensional and one-dimensional plots of temperature variation at different points of device-object system have been obtained with different thermopile powers, as well as device operation conditions.
7. It has been established that with increase in thermopile power when operated in cooling mode, the

temperature of a biological object is reduced, and in heating mode – increased. In so doing, in the investigated range of thermopile powers the temperature of skin cover varies most strongly, whereas the temperature of muscular tissue in the centre of a biological object varies only slightly.

8. According to the plots, the temperature distribution in the target object is in a certain manner affected by the presence of bone tissue in its volume. Under the accepted conditions, such temperature variation is about 1 to 2 K as compared to the case when given object is absent throughout the tissue.
9. The presence on different areas of the lateral surface of unlike thermal fluxes changes considerably the thermal field of the object as compared to the case when a thermal flux of the same sign is distributed along the lateral surface. In so doing, there is practically monotonous temperature increase from cooling to heating zone.
10. The plots of temperature variation of different points of human hand with time have been obtained with a local thermal effect through use of TES. In accordance with the data obtained, the time necessary for temperature stabilization on cooling and heating of a biological object is about 20 minutes, and the duration of transient mode with contrast procedures is 8 to 9 minutes.
11. To realize the required conditions of performing thermal procedures, it is sufficient to use standard TEM commercially produced by manufacturing companies.

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