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DYNAMICS OF TRANSIENT PROCESSES IN MULTI-STAGE THERMOELECTRIC MODULES

The dynamics of reaching the operating mode by thermoelectric cooler (TEC) is an important characteristic for any device where it is employed, and the methods for computational and experimental determination of its parameters are rather relevant. In paper [1], the expressions for time relaxation of a unit leg and a single-stage TEC were derived and generalized for the case of a two-stage TEC under different thermal load and operating conditions. These results were experimentally verified in [2]. The purpose of this paper is to get a method for estimating the relaxation time of transient processes in TEC with any number of stages, to study theoretically and experimentally the dynamic curves of reaching the steady-state mode by TEC and to compare the measured and calculated results.

Key words: multi-stage thermoelectric module, time constant, relaxation time.

Introduction

The time necessary to reach the operating mode (relaxation time) of a multi-stage thermoelectric module is of significant applied relevance. We understand the module relaxation time as a time during which the temperature deviation from the equilibrium value of the module is reduced by the factor *e*.

Numerous attempts have been made to estimate the relaxation time of multi-stage modules [1, 2, 3], however, they are rather a matter of judgment, disregarding important design details of module. In paper [4] the method of calculation of non-stationary mode for multistage thermoelectric modules is offered. In this paper we also solve the one-dimensional time-dependent heat conduction equation for a multi-stage TEC as accurately as possible, give the method of calculation of a multistage module relaxation time and compare the results with experimental data.

Problem solution method

Let us consider a multi-stage module (Fig. 1).

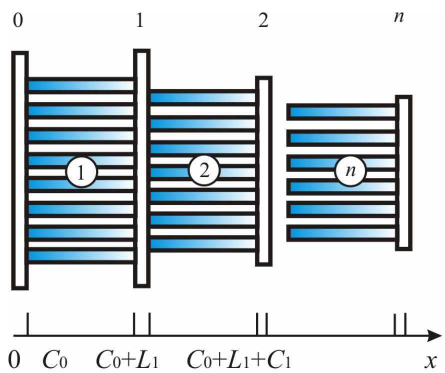

Fig. 1. Schematic of a multi-stage module.

In the figure, module stages are numbered, starting from the "hottest" stage. Stage numbers are indicated in circles. The hot side temperature of *i-th* stage will be denoted as T_i, the cold side temperature – T_{0i}. The first stage is connected to heat sink that has the temperature T_a. Heat spreader numbers start from 0 and are given at the top of the figure. The thickness of *i*-th heat spreader is C_i, the length of leg of *i*-th stage is L_i.

Let temperature distribution along the module be described by function $T(t, x)$, where t is time, x is coordinate. If (·) x is in the region of heat spreader, then $T(t, x)$ meets the equation

$$\frac{\partial T_{i,c}\left(t,x\right)}{\partial t}=a_c^2\frac{\partial^2 T_{i,c}\left(t,x\right)}{\partial x^2}, \tag{1}$$

where symbol i indicates ceramics number, and a_c – heat spreader thermal diffusivity:

$$a_c=\sqrt{\frac{\kappa_c}{c_c d_c}}, \tag{2}$$

where κ_c is thermal conductivity of heat spreader material, c_c is its heat capacity, and d_c is density.

If (·) x is in the region of legs, then

$$\frac{\partial T_{i,ty}\left(t,x\right)}{\partial t}=a_{i,ty}^2\frac{\partial^2 T_{i,ty}\left(t,x\right)}{\partial x^2}+\frac{J^2}{c_{i,ty}d_{i,ty}\sigma_{i,ty}}, \tag{3}$$

where i is stage number, symbol ty is used to designate the type of leg conductivity $ty = n, p$, respectively, $a_{i,ty}=\sqrt{\frac{\kappa_{i,ty}}{c_{i,ty}d_{i,ty}}}$ is temperature conductivity of the leg of ty type, and $\kappa_{i,ty}$, $c_{i,ty}$, $d_{i,ty}$ is thermal conductivity, heat capacity and density of the leg of ty type in *i-th* stage. For simplicity, all the legs will be assumed to be connected in series, so current density J depends neither on conductivity type ty, nor on stage number, $\sigma_{i,ty}$ is electric conductivity of the leg of *i*-th stage and ty type.

From the condition of continuity of temperatures it follows that at the boundary of ceramics and legs the following relations are fulfilled

$$T_{0,c}\left(t,C_0\right)=T_{1,ty}\left(t,C_0\right), \tag{4}$$

$$T_{k,c}\left(t,\left[C_0+\sum_{i=1}^{i=k}\left(C_i+L_i\right)\right]\right)=T_{k,ty}\left(t,\left[C_0+\sum_{i=1}^{i=k}\left(C_i+L_i\right)\right]\right),\quad k=1,..,n, \tag{5}$$

$$T_{k-1,c}\left(t,\left[\sum_{i=1}^{i=k}\left(C_{i-1}+L_i\right)\right]\right)=T_{k,ty}\left(t,\left[\sum_{i=1}^{i=k}\left(C_{i-1}+L_i\right)\right]\right),\quad k=1,..,n. \tag{6}$$

We will assume heat flow through the legs of thermoelectric module to be uniformly distributed along the heat spreader section. In this case the one-dimensional approximation remains valid. For the zero heat spreader or for the cold ends of legs of any one of stages such assumption seems quite natural, since the legs are distributed along the ceramic surface uniformly enough. As regards the hot ends of legs for stages starting from the second one, the legs there are distributed along the area which is appreciably smaller than the total area of the heat spreader and such an assumption for them looks somewhat non-natural. Condition for continuity of heat flows will yield on the heat spreader-leg contact with regard to (5), (6)

$$-S_{i-1}k_c\frac{\partial T_{i,c}\left(t,x\right)}{\partial x}+N_i s\sum_{ty=n,p}\left(k_{ty}\frac{\partial T_{i,ty}\left(t,x\right)}{\partial x}+\alpha J T_c\left(t,x\right)\right)\Bigg|_{x=C_0,C_0+\sum_{i=1}^{i=k}\left(C_i+L_i\right)}=0,\quad ty=n,p,\quad k=1,..,n, \tag{7}$$

where S_{i-1} is the area of i-1-th heat spreader, N_i is the number of thermoelement pairs, s is cross-section of thermoelement legs (equal for both conductivity types). Introducing a designation for filling factor of k-th ceramics of i-th stage

$$K_{i,k} = \frac{2N_i s_i}{S_k}, \quad k = i, i-1, \tag{8}$$

we can write (7) in the form

$$\left. \left(k_n \frac{\partial T_{i,n}(t,x)}{\partial x} + k_p \frac{\partial T_{i,p}(t,x)}{\partial x} \right) - \frac{2k_c}{K_{i,i-1}} \frac{\partial T_{i-1,c}(t,x)}{\partial x} + \bar{\alpha}_i J T_{i,c}(t,x) \right|_{x=C_0,\, C_0+\sum_{i=1}^{i=k}(C_i+L_i)} = 0, \tag{9}$$

where $\bar{\alpha}_i = (\alpha_{in} + \alpha_{ip})/2$.

Similarly, on the leg-heat spreader contact with regard to (4) we have

$$\left. k_c \frac{\partial T_c(t,x)}{\partial x} - \frac{K_{i,i}}{2} \left(k_n \frac{\partial T_n(t,x)}{\partial x} + k_p \frac{\partial T_p(t,x)}{\partial x} \right) - K_{i,i} \bar{\alpha} J T_{ty}(t,x) \right|_{\sum_{i=1}^{i=k}(C_{i-1}+L_i)} = 0, \quad k = 1,..,n. \tag{10}$$

The boundary conditions on the cold heat spreader of module:

$$\kappa_c \frac{\partial T_{n,c}\left(t, C_0 + \sum_{i=1}^{i=n} (C_i + L_i) \right)}{\partial x} = q_0, \tag{11}$$

where q_0 is the density of heat flow to the cold heat spreader of module.

The initial conditions:

$$T_c(0,x) = T_a. \tag{12}$$

We will sought for the solution of (1) in the form

$$T(t,x) = \tau(t)\varphi(x) + \varphi_{st}(x), \tag{13}$$

where $\varphi_{st}(x)$ is a solution of the steady-state thermal conductivity equation with the same conditions (4) – (10) as for the non-steady solution of thermal conductivity. By virtue of definition (13), function $\varphi(x)$ satisfies Eq.(1) and Eq. (3), where the last term is equal to 0. Functions $\varphi(x)$ on the heat spreader and legs are sewn by means of Eqs. (4) to (10). On the cold heat spreader for the last stage $\varphi(x)$ satisfies Eq.(11) where $q_0 = 0$. The initial conditions for function $\varphi(x)$:

$$\varphi_0(x) = T_a - \varphi_{st}(x). \tag{14}$$

Characteristic numbers λ are found by the method of dividing variables from equation

$$\frac{\partial \tau(t)}{\tau(t)\partial t} = a^2 \frac{d^2\varphi(x)}{\varphi(x)dx^2} = -\lambda_j^2. \tag{15}$$

The solution of Eq.(15) in the region of i-th heat spreader is of the form:

$$\varphi_{i,c}(x) = A_{i,c} \sin\frac{\lambda_j x}{a_c} + B_{i,c} \cos\frac{\lambda_j x}{a_c}, \quad i = 1,..,n, \tag{16}$$

where $A_{i,c}$ and $B_{i,c}$ are integration constants.

In the region of thermoelectric material

$$\varphi_{i,ty}(x) = A_{i,ty} \sin\frac{\lambda_j x}{a_{ty}} + B_{i,ty} \cos\frac{\lambda_j x}{a_{ty}}, \quad i = 1,..,n, \quad ty = n, p, \tag{17}$$

where $A_{i,ty}$, $B_{i,ty}$ are integration constants.

If the solution in the region of heat spreader is known, the solution in the region of legs can be found from (4) – (10) by solving a simple linear system of equations

$$\begin{cases} A_{i,n}\sin\frac{\lambda_j L}{a_n}+B_{i,n}\cos\frac{\lambda_j L}{a_n}=T_{cer} \\ A_{i,p}\sin\frac{\lambda_j L}{a_p}+B_{i,p}\cos\frac{\lambda_j L}{a_p}=T_{cer} \\ A_{i,n}\sin\frac{\lambda_j\left(L+L_i\right)}{a_n}+B_{i,n}\cos\frac{\lambda_j\left(L+L_i\right)}{a_n}-A_{i,p}\sin\frac{\lambda_j\left(L+L_i\right)}{a_p}-B_{i,p}\cos\frac{\lambda_j\left(L+L_i\right)}{a_p}=0 \\ A_{i,n}\frac{\kappa_n\lambda_j}{a_n}\cos\frac{\lambda_j L}{a_n}-B_{i,n}\frac{\kappa_n\lambda_j}{a_n}\sin\frac{\lambda_j L}{a_n}+A_{i,p}\frac{\kappa_p\lambda_j}{a_p}\cos\frac{\lambda_j L}{a_p}-B_{i,p}\frac{\kappa_p\lambda_j}{a_p}\sin\frac{\lambda_j L}{a_p}=\frac{\kappa_c}{K_{i-1,i}}D_{cer} \end{cases} \quad (18)$$

Here, the following designations are introduced:

$$L=\sum_{m=0}^{m=i}\left(C_m+L_m\right) \qquad i=0,1,..n;\ L_0=0,$$

$$Tcer=\varphi_{i-1,c}\left(L\right) \quad D_{cer}=A_{i-1,c}\frac{\lambda_j}{a_c}\cos\frac{\lambda_j L}{a_c}-B_{i-1,c}\frac{\lambda_j}{a_c}\sin\frac{\lambda_j L}{a_c}. \quad (19)$$

If the solution in the region of legs is known, in the region of heat spreader the solution will be found from expressions (4) – (10):

$$B_{i,c}=T_{mater}\cos\frac{\lambda_j L}{a_c}-q_{mater}\sin\frac{\lambda_j L}{a_c},\ A_{i,c}=T_{mater}\sin\frac{\lambda_j L}{a_c}+q_{mater}\cos\frac{\lambda_j L}{a_c},\ i=1,..n \quad (20)$$

where the following designations are used:

$$L=\sum_{m=1}^{i}\left(C_{m-1}+L_m\right),\ T_{mater}=A_{i,ty}\sin\frac{\lambda_j L}{a_{ty}}+B_{i,ty}\cos\frac{\lambda_j L}{a_{ty}},\ q_{mater}=\frac{a_c K_{i,i}\left(\kappa_{ni}D_{ni}+\kappa_{pi}D_{pi}+\bar{\alpha}_i JT_{mater}\right)}{\kappa_c\lambda_i},$$

$$D_{ty,i}=A_{i,ty}\frac{\lambda_j}{a_{ty}}\cos\frac{\lambda_j L}{a_{ty}}-B_{i,ty}\frac{\lambda_j}{a_{ty}}\sin\frac{\lambda_j L}{a_{ty}} \qquad ty=n,p.\ (21)$$

The solution for the 0 ceramics is given by:

$$\varphi_{0,c}\left(x\right)=\sin\frac{\lambda_j x}{a_c},x\in[0,D_0], \quad (22)$$

where coefficient $A_{0,c}$ =1, which does not restrict the generality of solution, since the equation for finding λ_j includes the ratio of coefficients.

Solving consecutively the equations (18) and (20) for all the stages, we can obtain the equation for finding characteristic numbers

$$tg\left[\frac{\lambda_i\left(C_0+\sum_{i=1}^{n}\left(C_i+L_i\right)\right)}{a_c}\right]-\frac{A_{n,c}}{B_{n,c}}=0 \quad (23)$$

Relaxation times are related to characteristic numbers in conformity with (15):

$$\tau_i=\left(\frac{1}{\lambda_i}\right)^2. \quad (24)$$

The resulting relaxation time of module is selected as the largest of the set of times (24).

In the above equations the temperature dependences of thermoelectric parameters are disregarded. To take into account the temperature dependences, an approximate compromise decision was taken. The values of thermoelectric parameters for each stage were assumed to be constant, but their values were taken for *i*-th stage at the temperature of legs on the "hot" heat spreader T_i.

Experimental results

The calculated relaxation time was compared to the experimentally obtained data, as well as to the data of previous works. In Table 1 are listed the values obtained for two-stage modules 2MC06-023-12 and 2MC06-043-05. Here I is measuring current, τ is the value of relaxation time calculated by the foregoing method, τ_{exp} is the measured relaxation time, τ_{appr} is the approximate value of relaxation time calculated as the sum of maximum stage-by-stage times at zero current [1, 2]. All the values were obtained in vacuum at the temperature of the hot heat spreader 50 °C.

Table 1

Experimental and calculated values of relaxation times

TE module	I, mA	τ_{exp}, s	τ, s	τ_{appr}, s
2MC06-023-12	300	9.94	9.50	12.7
2MC06-043-05	700	3.19	3.68	3.22

The following parameters were used in the calculations: for thermoelectric material the heat capacity was 0.13 J/g, the density – 7.5 g/cm^3, for ceramics – 0.8 J/g and 3.5 g/cm^3, for solder – 0.17 J/g and 9.3 g/cm^3, respectively, ceramics thickness was 0.5 mm.

The measurements were performed as follows. At given current, the temperature on thermoelectric module was measured as a function of time and maximum temperature difference $\Delta T_{\max}$ was found. After that we constructed the dependence $\ln\left(1-\frac{\Delta T}{\Delta T_{\max}}\right)$, and τ_{exp} was determined by the inverse value of linear approximation slope ratio.

Figs. 2 and 3 explain experimental determination of the relaxation time.

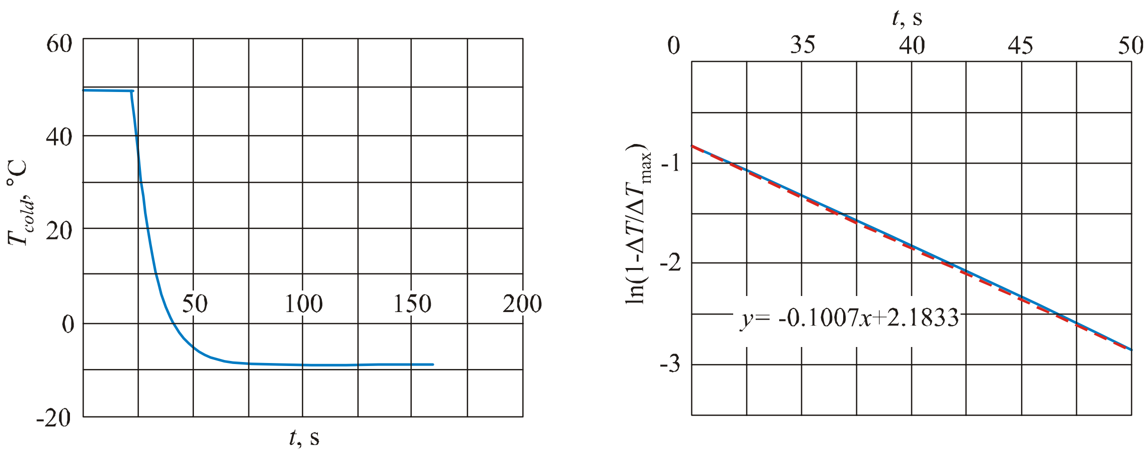

Fig. 2. Finding the relaxation time τ_{exp} of TE module 2MC06-023-12.

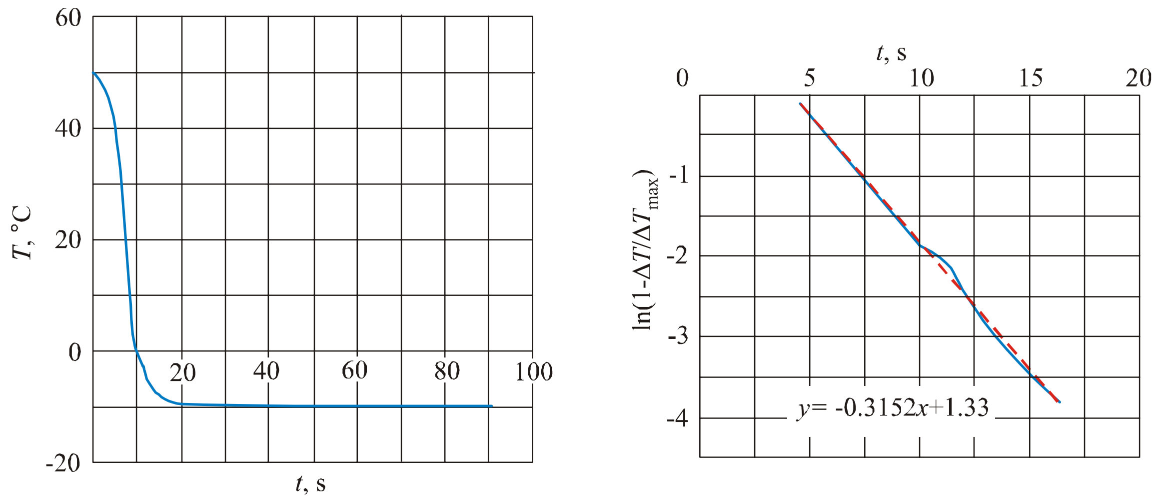

Fig. 3. Finding the relaxation time τ_{exp} of TE module 2MC06-043-05.

It is important to note the specificity of measuring the relaxation time of multi-stage modules. For this purpose the method of measurement on setting the steady-state voltage [2] is, generally speaking, not legitimate, though for a single-stage module where thermoEMF voltage is proportional to temperature difference, it certainly works. On setting the steady-state voltage value, the multi-directional variation of leg temperatures still takes place for some time, so the relaxation time measured by the thermoEMF voltage proves to be underrated.

Conclusion

The elaborated technique is a general method for calculation of the relaxation time of thermoelectric module with any number of stages for arbitrary current.

The calculated results were compared to the relaxation time values measured for two-stage modules. The calculated values are in satisfactory agreement with theoretical ones. Note, however, that it is necessary to supplement the experimental data for modules with more than two stages and perform further comparative analysis.

An important point is that we succeeded in showing that the approximate value of relaxation time calculated as the sum of maximum stage-by-stage times at zero current [1, 2] is rather good approximation. This approximate method can be easily used for solving urgent practical problems of estimating the dynamics of transient processes in thermoelectric modules.

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