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CRITERIA FOR PERFORMANCE EVALUATION OF THERMOELECTRIC ENERGY CONVERTERS

Performance analysis problems for thermoelectric energy converters are considered on the base of system approach. Methods to construct the limiting opportunities sets for thermoelectric generators and heat pumps under various restrictions are outlined.
Key words: thermoelectric energy converter, system analysis.

Introduction

The literature on the methods of performance evaluation and comparison of thermoelectric energy converters (TEC) is quite extensive [1-8]. The absolute majority of the existing methods are based on the analysis of extreme modes of TEC under various assumptions. However, such analysis gives no sufficient information for making decisions on the choice of real device parameters, since it does not take into account a relationship between technical and economic characteristics in an explicit form. Moreover, technical criteria of TEC efficiency are often competing, i.e. improvement of one of them with a change in the vector of controlling parameters results in the degradation of another. The uncertainty of optimization purposes is also due to the fact that by virtue of specific features of energy conversion system, any real technical task can be implemented in the infinitely large set of variants differing in specific characteristics and project cost.

The task of optimal design is to assure a choice of this set of variants leading to the achievement of the target goal with minimum expenditures. For solving this task a TEC should be considered as a system consisting of thermoelectric circuit, heat source and sink. Having mathematical models of these units and their interaction conditions, one can formulate target function defining the efficiency of device as a whole and find an optimal solution. In some cases for these purposes one employs the empirical dependences which describe the defining parameters of units with a subsequent numerical solution of optimization problem [9]. Understandably, such approach is acceptable only for solving a narrow class of problems, since it is related to obtaining a large volume of experimental data.

The methods of system analysis are successfully used for solving similar problems in a general form [10, 11]. The essence of the method lies in constructing a set of possible solutions (a set of limiting opportunities) of the problem in a space of defining criteria. Analysis of such sets gives an idea of the basic system properties, the character of interaction between units, allows revealing in this space a set of efficient solutions (Pareto set) of the problem restricting the space of optimal solutions of multi-criteria problem. Problem solutions obtained in the generalized variables are universal, since they cover all possible combinations of primary independent variables and allow evaluating possible performance of TEC at early stages of design.

Generalized variables and TEC efficiency criteria

A mathematical model of TEC in the general form has rather high dimensionality, since it comprises considerable amount of independent variables having wide variation intervals. It is quite difficult to trace the relation between all affecting factors and generalize this data in a space of natural physical variables. Problem dimensionality can be reduced considerably and brought to a generalized form through use of similarity theory methods [12].

In [7] it was shown that the problem of thermoelectric energy conversion can be reduced to a small-size system of generalized variables having clear physical meaning and normalized variation intervals:

$$N = f(I_o, \Theta, J, Ki, Bi). \tag{1}$$

Here, $N = Nh/\lambda T_p$ is dimensionless power of thermoelement; $I_o = zT_p$ is the Ioffe criterion characterizing thermoelectric material properties. Dimensionless temperature of thermoelements $\Theta = T/T_p$ is a definable parameter used to calculate the energy characteristics of TEC. Dimensionless current $J = jeh/\lambda$, the Kirpichev criterion $Ki = qh/\lambda T_p$ and the Biot criterion $Bi = \alpha h/\lambda$ characterize duty parameters defining the state of the system.

Special attention should be given to a choice of problem defining temperature T_p which assigns the temperature scale. As a defining temperature for thermoelectric generator (TEG) use is made of maximum acceptable device operating temperature $T_p = T_{\max}$; as a defining temperature for thermoelectric heat pump (THP) the heat sink temperature $T_p = t_o$ is assumed. Such choice of scale restricts the range of possible temperatures to the values of $\Theta \leq 1$.

The examined criteria have a clear physical meaning. The Ioffe criterion characterizes the limiting opportunities of TEC. It can be shown that in TEG mode full dimensionless power of thermoelement in short circuit mode $N_o = I_o\Delta\Theta^2$. That is, the Ioffe criterion characterizes the power of thermoelement of single dimensions with a unit step temperature (at $\Delta\Theta = 1$ $I_o = N_o$). The following relations hold for an idealized schematic of TE heat pump: $J_{opt} = I_o$ and $Ki_{\max} = 0.5I_o$, i.e. the Ioffe criterion characterizes extreme modes of THP (J_{opt} is dimensionless current density assuring the mode of maximum cooling capacity of THP). Taking into account that $qh/\lambda = \Delta T$, the Kirpichev criterion $Ki = \Delta T/T_p$ can be treated as the Carnot efficiency in the temperature range considered. Dimensionless current density is the ratio between maximum possible values of the Peltier heat ($Q_p = jeT_p$) and the flux of net thermal conductivity ($Q_\lambda = \lambda/hT_p$), i.e. $J = Q_p/Q_\lambda$. The Biot criterion is known to represent the ratio between thermal resistances of thermal conductivity $R_\lambda = h/\lambda$ and heat transfer $R_\alpha = 1/\alpha$. In their physical meaning criteria *Ki, J* and *Bi* represent a generalized description of the main system units, namely heat source, thermoelectric converter and heat sink. Having optimal solutions in generalized variables, one can always transfer their results to a concrete project, having received a description of generalized criteria as a function of primary data for each of the system units.

Idealized model of TEC

The relation between TEC characteristics and duty parameters is traced most obviously in the idealized mathematical model of thermoelement which comes down to one-dimensional problem of thermal conductivity under the first-kind boundary conditions [7]:

$$\frac{\partial^2\Theta}{\partial Y^2} + \frac{J^2}{I_o} = 0, \tag{2}$$

$$\begin{cases} \Theta(0) = \Theta_o; \\ \Theta(1) = \Theta_h; \end{cases} \tag{3}$$

Solution of equation (2) is of the form:

$$\Theta(Y) = C_1 + C_2 Y - \frac{J^2}{2I_o} Y^2, \tag{4}$$

where $C_1 = \Theta_o$; $C_2 = \Theta_h - \Theta_o + \frac{J^2}{2I_o}$; $Y = y/h$ is dimensionless coordinate.

In such formulation it is supposed that on the heat-releasing thermoelement junction there is an ideal heat exchange ($Bi_h \to \infty$), and heat flux on the heat-absorbing junction Ki corresponding to the assigned conditions can be found from the heat balance:

$$Ki + Q_\lambda + Q_p = 0. \tag{5}$$

Here, $Q_\lambda = -\Theta'(0)$ is thermal conductivity flux and $Q_p = J\Theta(0)$ is the Peltier heat. Hence, with regard to (4), we obtain the main relation for an ideal schematic of TEC:

$$Ki = J\Theta_o - \frac{J^2}{2I_o} - \Delta\Theta. \tag{6}$$

This expression includes two duty parameters Ki and J. In heat pump mode, parameter J is controlling, and Ki – controlled. In energy generator mode, on the contrary, device characteristics are defined by the density of heat flux Ki which is responsible for dimensionless current J.

Heat pump mode

From (6) it follows that dependence of heat flux on supply current for THP mode has the form of a square parabola, i.e. the range of problem definition is restricted by the range of currents at the ends of which cooling capacity goes to zero. The boundary conditions for J are determined as the roots of equation (6):

$$J_{1,2} = I_o\Theta_o \pm \sqrt{(I_o\Theta_o)^2 - 2I_o\Delta\Theta}; \tag{7}$$

From (7) it follows that the problem has physical meaning at $(\Theta_o^2 - 2I_o\Delta\Theta) \geqq 0$.

Maximum cooling capacity of thermoelement is achieved at current density

$$J_{opt} = I_o\Theta_o. \tag{8}$$

Thus,

$$Ki_{\max} = \frac{I_o\Theta_o^2}{2} + \Theta_o - \Theta_h. \tag{9}$$

Maximum difference between thermoelement junctions in cooling mode is achieved on condition of $Ki_{\max} = 0$. In so doing,

$$\Theta_{\min} = \frac{\sqrt{1 + 2I_o\Theta_h} - 1}{I_o}. \tag{10}$$

Accordingly, maximum possible temperature difference on thermoelement is equal to:

$$\Delta\Theta_{\max} = 1 - \Theta_{o\min}. \quad (11)$$

Temperature distributions in thermoelement for different supply currents from the permissible range (7) are represented in Fig. 1 *a*.

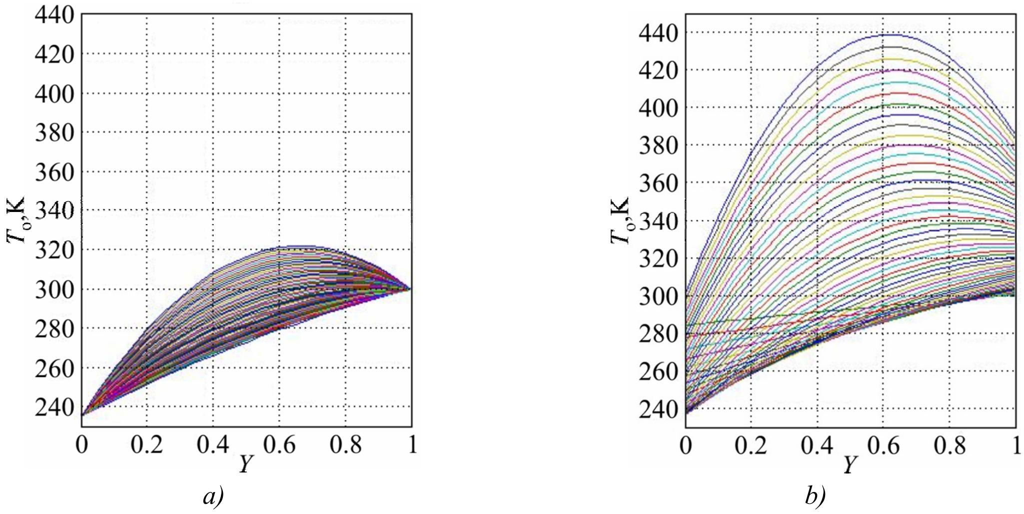


Fig. 1. Temperature distributions in thermoelement, THP mode, 0 ≦ J ≦ $J_{max.}$
(a – idealized model; b – boundary conditions of II/III kind).

THP efficiency *E* is determined as the ratio between net cooling capacity *Ki* and power *N* consumed by energy source. Consumed power can be determined as

$$N = \frac{J^2}{I_o} + J\Delta\Theta. \quad (12)$$

With regard to (6) we will get:

$$E = \frac{J\Theta_o - \dfrac{J^2}{2I_o} - \Delta\Theta}{\dfrac{J^2}{I_o} + J\Delta\Theta}. \quad (13)$$

The ratio between *E* and *Ki* in the acceptable range of currents at fixed junction temperatures is illustrated in Fig. 2.

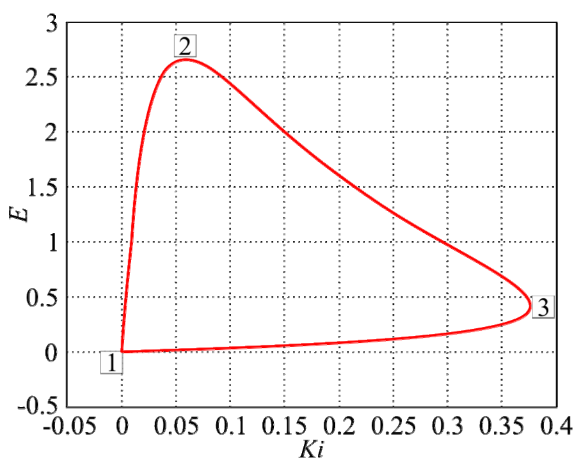


Fig. 2. The ratio between cooling capacity and efficiency of THP in the assigned temperature range.

This set of solutions restricts the range of definition of thermoelectric cooling problem under assumed restrictions. As is seen from the represented data, on the set under consideration one can separate two typical ranges. In the areas 1 – 2 and 1 – 3 one can always find a change in governing parameters resulting in a simultaneous improvement of criteria *E* and *Ki*. This range is called the range of efficient solutions. The area 2 – 3 is related to the range of weakly efficient solutions, since any improvement of one criterion results in the degradation of another. Nevertheless, it is obvious that optimal solutions of the problem should be sought exactly in the range of weakly efficient solutions, since it covers a set of solutions with maximum values of THP efficiency criteria. Having constructed for the range of temperatures ($\Theta_{\text{omin}} - \Theta_h$) the sets of solutions similar to that represented in Fig. 1, we will obtain a vivid presentation of the range of definition of the problem under consideration (Fig. 3 *a*).

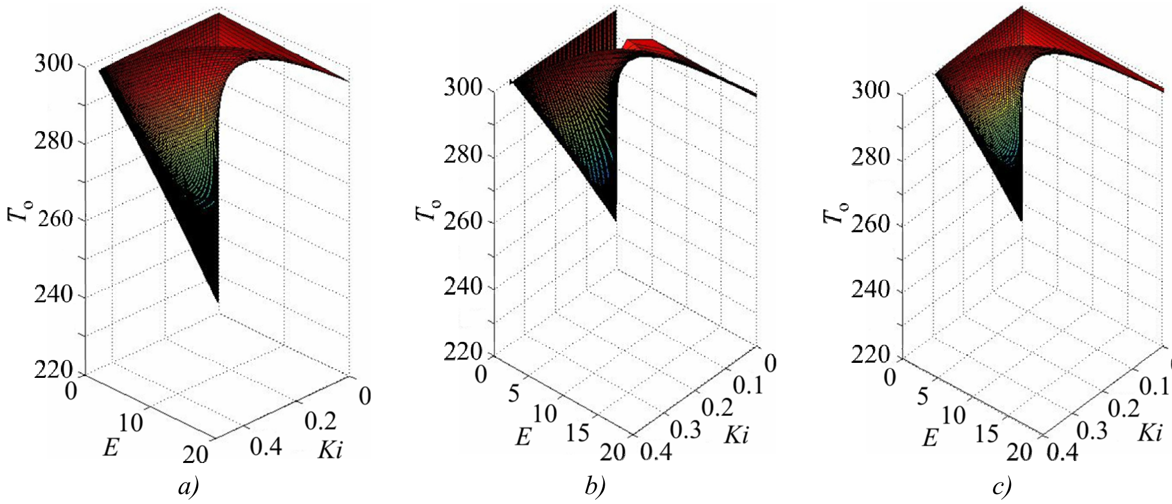


Fig. 3. The range of definition of thermoelectric cooling problem in the space of criteria T_o, Ki, E. (a – boundary conditions of I/I kind; b – of II/III kind; c – of III/III kind).

Electric energy generator mode

In electric energy generator mode the definable parameter is dimensionless current density which in the problem formulation considered (with the assigned material properties and known temperature conditions) depends only on load factor $m = R_n/R$:

$$J = \frac{I_o \Delta\Theta}{2(m+1)}. \tag{14}$$

Here, R_n is load resistance; $R = 2(\rho h/s)$ thermocouple resistance.

Thermocouple power is equal to

$$N = I_o \Delta\Theta^2 \frac{m}{2(m+1)^2} \tag{15}$$

From the last expression it follows that maximum TEG power is achieved at $m_{opt} = 1$:

$$N_{\max} = \frac{I_o \Delta\Theta^2}{8}. \tag{16}$$

The thermoelement efficiency is equal to the ratio between net power and heat flux on the heat-absorbing junction

$$n = \frac{N}{Ki}. \tag{17}$$

The range of change in the net power is determined by expression (16). The corresponding range of change in dimensionless heat fluxes $Ki_{\min} \div Ki_{max}$ meeting the problem conditions can be determined by substituting (14) into (6) at $m = 0$ (short circuit current, $J = J_{\max}$) and at $m = \infty$ (idle current, $J = 0$):

$$Ki_{\min} = \Delta\Theta(\frac{I_o\Theta_o}{2} - \frac{I_o\Delta\Theta}{8} - 1); \quad (18)$$

$$Ki_{\min} = -\Delta\Theta.$$

The behaviour of permissible range of dimensionless heat flux density as a function of temperature of the heat-absorbing junction is illustrated in Fig. 4.

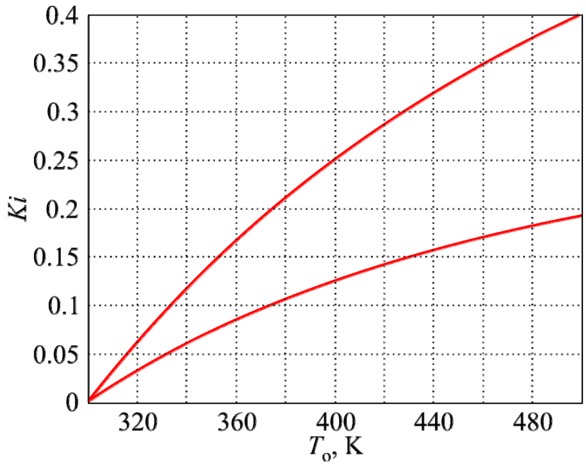


Fig. 4. Acceptable values of Ki in the range of temperatures $T_p \geqq T_o \geqq t_o$.

With regard to (6) from (17) we obtain

$$n = \frac{I_o\Delta\Theta^2 \dfrac{m}{2(m+1)^2}}{J\Theta_o - \dfrac{J^2}{2I_o} - \Delta\Theta}. \quad (19)$$

The ratio between the specific power and thermoelement efficiency with a change in load factor within $0 \leqq m \leqq \infty$ is represented in Fig. 5.

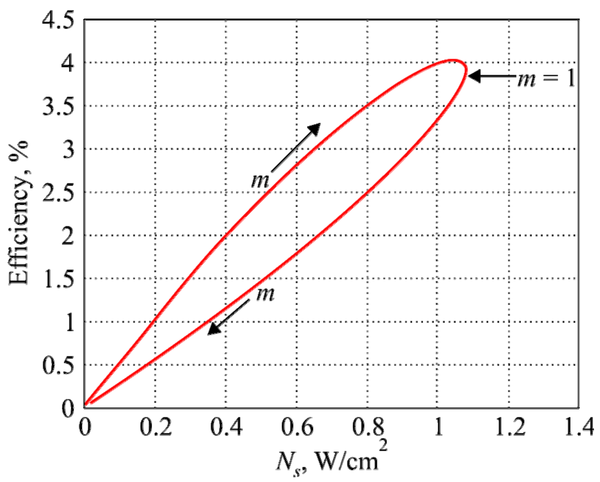


Fig. 5. The ratio between power and efficiency of TEG with a change in load factor within $0 \leqq m \leqq \infty$ ($z = 0.003$; $dT = 100$ K, $h = 1$ mm).

As it follows from the data given in the figure, the area corresponding to the region of $0 \leqq m \leqq 1$ is a set of efficient solutions, as long as with increase in *m,* both efficiency criteria of the problem are increased. The region $1 \leqq m \leqq \infty$ can be characterized as a set of inefficient solutions, since with increase in *m* both criteria are degraded.

Having constructed similar sets for the entire range of operating temperatures, we get a set of possible solutions of the problem under consideration in the space of basic technical and economic criteria (Fig. 6).

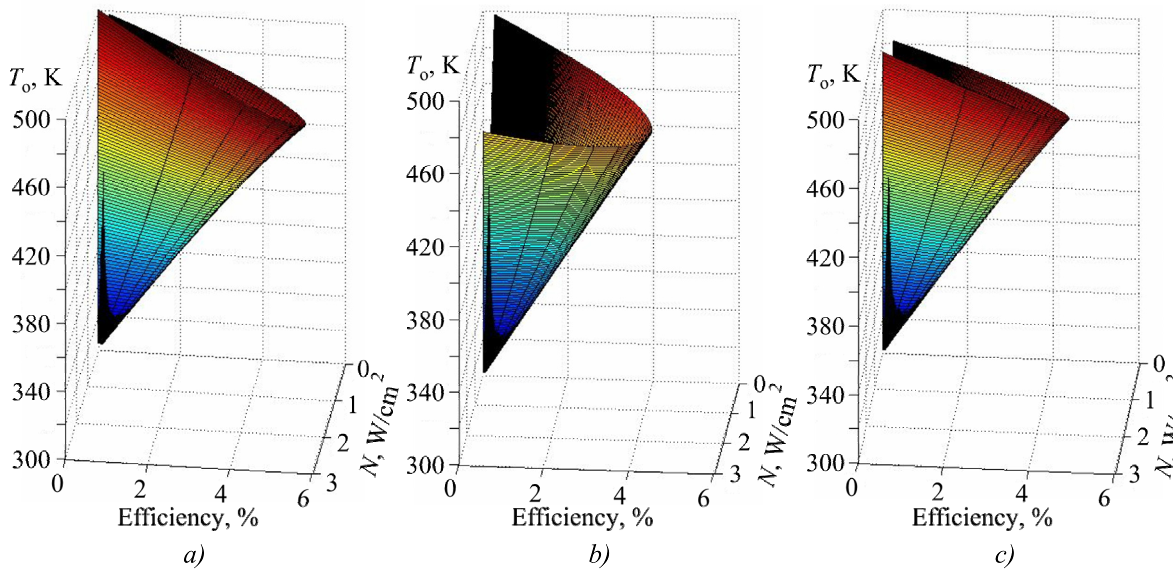


Fig. 6. The range of definition of thermoelectric energy conversion problem.
(a – boundary conditions of I/I kind; b – of II/III kind; c – of III/III kind).

Mathematical model of TEC under the boundary conditions of II/III kind

The above ideal model of TEC under the boundary conditions of I kind gives a vivid presentation of the basic regularities and ratios between criteria in THP and TEG modes, but it is unsuitable for the calculation of real devices, as long as similar abstraction cannot be implemented in practice. In fact, the temperature of TEC junctions is a function of state and is determined by conditions of heat transfer in a system. A more general and real problem formulation is assignment of the boundary conditions of II kind on the heat-absorbing thermoelement junctions (the specific density of heat flux is known) and the boundary conditions of III kind (convective heat exchange conditions) on the heat-releasing thermoelement junctions. In this case the boundary conditions of problem (3) are of the form:

$$\Theta'(0) - J\Theta(0) + Ki(0) = 0, \tag{20}$$

$$Bi[\Theta(1) - \vartheta] + \Theta'(1) - J\Theta(1) = 0.$$

Substituting solution (4) into (20), we get the following system of equations for the determination of integration constants C_1, C_2:

$$C_1 J - C_2 = Ki; \tag{21}$$

$$C_1(J - Bi) + C_2(J - Bi - 1) = \frac{J^2}{2I_o}(J - Bi) - Bi\vartheta - \frac{J^2}{I_o}.$$

Unlike idealized problem formulation, in the case under consideration the temperature mode is completely determined by conditions of heat exchange between thermoelement junctions.

Heat pump mode

Typical temperature distributions along thermoelement height on condition of $\Theta_o \leq 1$ are represented in Fig. 1 *b*. As it appears from the above data, the effect of heat exchange conditions is manifested in the reduction of cooling depth as compared to idealized schematic of THP.

It follows from (4) that $C_1 = \Theta_o$. Taking this into account, from (21) we obtain the following expression for finding acceptable values of current density in THP mode:

$$aJ^3 + bJ^2 + cJ + d = 0, \tag{22}$$

where

$$a = -\frac{1}{2I_o};$$

$$b = \Theta_o + \frac{Bi_h}{2I_o} + \frac{1}{I_o};$$

$$c = -\Theta_o Bi_h - Ki;$$

$$d = Ki\left(Bi_h + 1\right) - Bi_h(\Theta_o - \vartheta_h).$$

At $\Theta_o = 1$ the roots of this equation restrict the range of current densities wherein the condition $\Theta_o \leq 1$ is met. The problem has a physical meaning in the case when the discriminant of equation (22) $D \geqq 0$. At $D = 0$ the equation has only one real root – this case is limiting for the assumed restrictions. Thus, permissible combinations of problem criteria $\{I_o, Ki, Bi_h, \vartheta_h\}$ restricting the area of possible solutions can be determined from the condition:

$$D = -4b^3 d + b^2 c^2 - 4ac^3 + 18abcd - 27a^2 d^2 = 0. \tag{23}$$

The values of acceptable ranges of *J* are determined by the ratio between criteria *Ki* and Bi_h. To each value of Bi_h corresponds an acceptable range of values $0 \leqq Ki \leqq Ki_{max}$, wherein the problem has a physical meaning, i.e. wherein the condition $\Theta_o \leq 1$ can be met. Dependence of maximum possible dimensionless cooling capacity Ki_{max} on the intensity of heat exchange Bi_h is given in Fig. 7 *a*. At Bi_h = const by means of (23) one can determine the range of possible values of $J = f(Ki)$, (Fig. 7 *b*). From the data given in the figure it follows that the range of efficient problem solutions is matched by the values of current in the range of $0 \leqq J \leqq 1/2\,J_{max}$. Maximum cooling depth $\Theta_o = \Theta_{min}$ can be achieved under conditions of $Bi_h \to \infty$, $Ki = 0$.

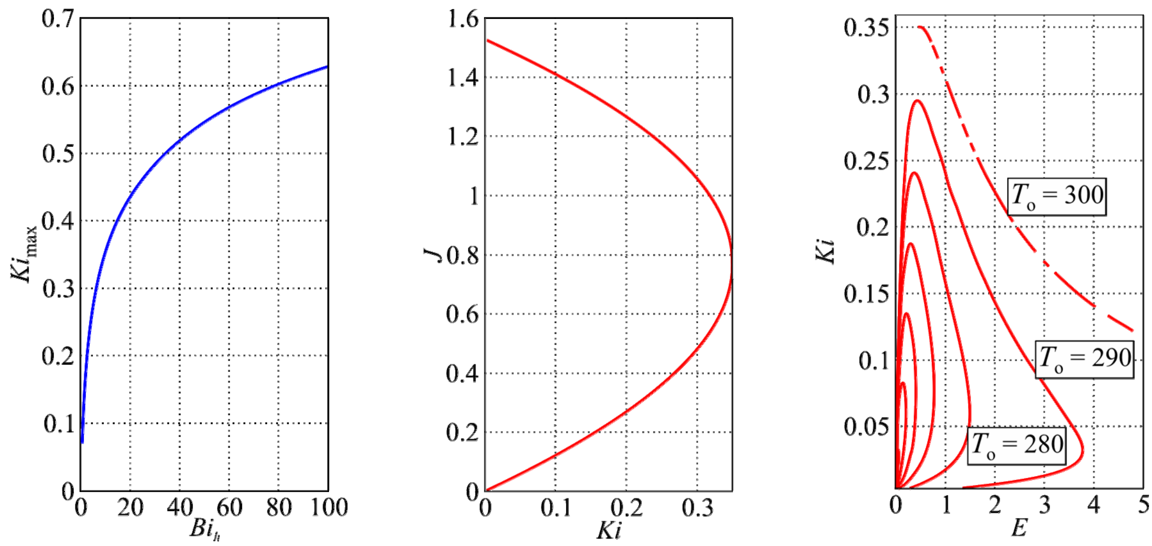
The efficiency of THP is determined as

$$E = \frac{Ki}{\dfrac{J^2}{I_o} + J\Delta\Theta} \tag{24}$$

and depends on cooling depth Θ_o, cooling capacity *Ki* and current density *J*.

The ratio between *E* and *Ki* for conditions under consideration is given in Fig. 7 *c*.

Having constructed an array of solutions similar to that represented in Fig. 7 *c*, for the acceptable ranges of changes in the temperature of the heat-absorbing junction we will obtain a plurality of limiting opportunities of thermoelectric cooling problem in the space of criteria *Ki, E* and Θ_o (Fig. 3 *c*).



a) b) c)

Fig. 7. a) Dependence of maximum possible cooling capacity Ki_{max} on Bi_h criterion.
b) The range of acceptable values of current density J as a function of heat flux Ki at Bi_h = 10.
c) The ratio between efficiency E and cooling capacity Ki.

Electric energy generator mode

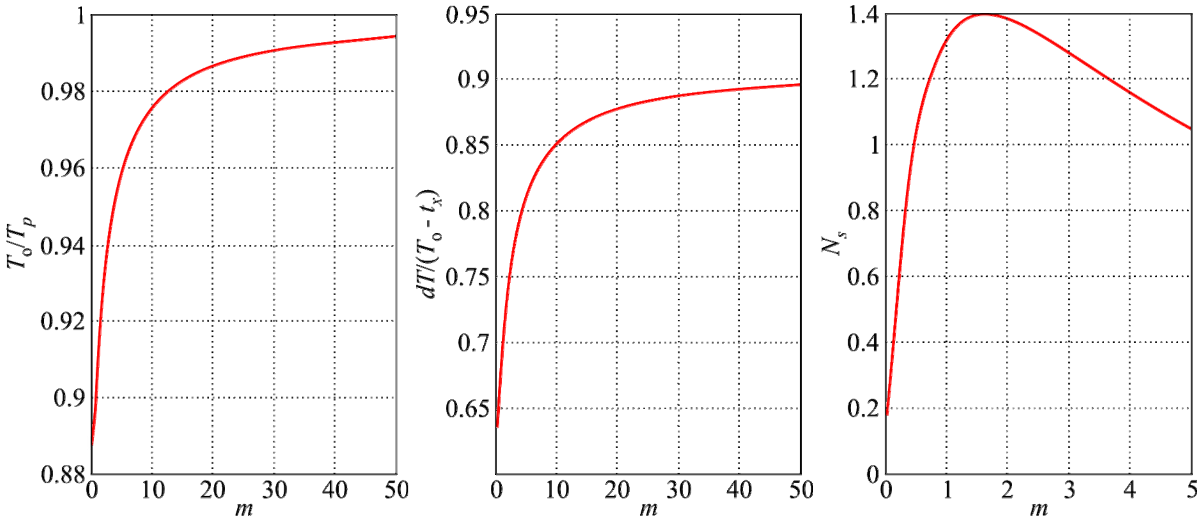
For electric energy generator mode the temperature restriction of the heat-absorbing junction is active, i.e. for the achievement of maximum efficiency the condition $\Theta_o = 1$ should be met. With regard to the fact that maximum temperature of the heat-absorbing junction is achieved in idle mode (current equal to zero), assuming in (22, 23) $J = 0$, one can obtain the expression restricting acceptable ratios of *Ki* and Bi_h for the problem formulation considered:

$$Ki_{\max} = \frac{Bi_h(1-\vartheta_h)}{Bi_h+1}, \tag{25}$$

from the last expression it follows that in the limiting case ($Bi_h \to \infty$) the dimensionless density of heat flux is restricted by the value

$$Ki_{\max} \le 1-\vartheta_h \tag{26}$$

Under real conditions the restriction (26) is also active, i.e. to assure maximum efficiency, the condition $Ki = 1 - \vartheta_h$ should be met. The overrunning of this limit of heat flux is impermissible, since at nonstandard switching off the load there is considerable (by 15...20 %) temperature rise of the heat-absorbing layers of TEG (Fig. 8 *a*) that can result in construction damage. This condition restricts markedly the operating temperature difference which in the range of optimal loads is reduced by 20...30 % as compared to the available one (Fig.8 *b*). Since load mode has a pronounced effect on temperature distribution in thermoelements, and the problem becomes highly nonlinear, the mode of maximum TEG power is achieved at $\Delta\Theta < \Delta\Theta_{max}$ (Fig. 9). In so doing, the modes of maximum power and maximum efficiency coincide, since at given heat flux density the maximum efficiency is achieved at $N = N_{max}$.



a) b) c)

Fig. 8. Effect of load factor on TEG characteristics: a) –dependence of relative temperature of the heat-absorbing junction on m; b) – dependence of relative operating temperature difference on m; c) – dependence of specific power N_s, W/cm^2, on m.

The foregoing implies that in the general case the defining criterion which restricts technical and economic characteristics of TEG at given properties of thermoelectric material is the Biot criterion which characterizes heat removal conditions, namely it limits both the losses in temperature head on cooling of TEG and the range of acceptable densities of heat flux on the side of heat source.

The range of definition of the problem of thermoelectric power conversion in formulation considered is given in Fig. 3 *b*.

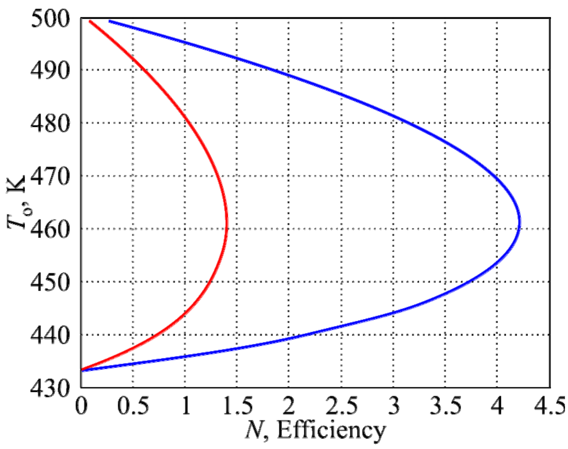


Fig. 9. The ratio between temperature T_o, power and efficiency of TEG at $0 \leqq m \leqq \infty$.
(– efficiency, %; – specific power N, W/cm^2).

The case of using constant-power heat sources (Ki = const), such as radioisotope heat sources, has unique features. As a rule, such sources assure relatively low heat flux densities, but have no restrictions on temperature conditions (within reasonable limits) [7, 8]. At first sight, for such TEG it is optimal to provide for a mode with maximum permissible temperature of heat-absorbing junction ($\Theta_o = 1$). However, in fact, in this case also the decisive influence on the characteristics of TEG is produced by cooling conditions (Bi_h criterion). Figs. 10 *a, b* show the dependences of junction temperatures and temperature differences on Bi_h criterion for the case $Ki < Ki_{max}$ = const. Significant is the fact that TEG power tends to maximum on achieving minimum possible under conditions concerned temperature value of the heat-absorbing junction T_o (Fig. 10 *c*). That is, increase in temperature difference (and power) is possible only due to reduction of losses from heat exchange irreversibility. The limiting power of TEG can be achieved only under conditions of $Ki = Ki_{max}$, $Bi_h \to \infty$.

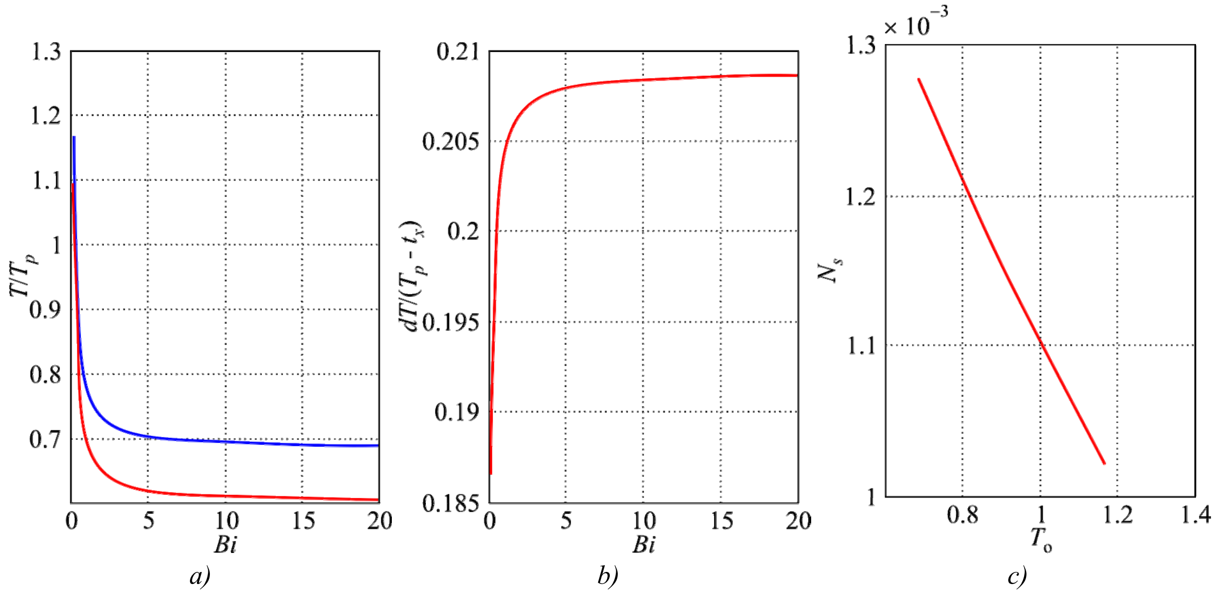


Fig. 10. Characteristics of TEG at $Ki < Ki_{max}$: a) – dependence of junction temperatures on Bi_h (– T_o/T_p ; – T_h/T_p); b) – dependence of relative operating temperature difference $dT/(T_p - t_x)$ on Bi_h; c) – the ratio between the specific power of TEG N_s and the temperature of the heat-absorbing junction T_o/T_p.

Mathematical model of TEC under boundary conditions of III/III kind

Quite common is a schematic whereby heat transfer on the heat-absorbing junction of TEC is due to convective heat exchange. In this case the junction temperature is limited by heat carrier temperature. For the analysis of similar schematics it is necessary to use a mathematical model of thermoelement under the boundary conditions of III kind on the heat-absorbing and heat-releasing junctions:

$$Bi_o[\vartheta_o - \Theta(0)] + \Theta'(0) - J\Theta(0) = 0; \qquad (27)$$

$$Bi_h[\Theta(1) - \vartheta_h] + \Theta'(1) - J\Theta(1) = 0.$$

Substituting (4) into (27), we get the following system of equations for the determination of integration constants C_1, C_2:

$$C_1(J + Bi_o) - C_2 = Bi_o\vartheta_o; \qquad (28)$$

$$C_1(Bi_h - J) + C_2(Bi_h - J + 1) = Bi_h\vartheta_h + \frac{J^2}{I_o}(1 + \frac{Bi_h - J}{2}).$$

Heat pump mode

For the schematic under consideration the heat flux on the heat-absorbing junction is equal to:

$$q = \alpha_o(t_o - T_o), \qquad (29)$$

or in the dimensionless form:

$$Ki = Bi_o(\vartheta_o - T_o). \qquad (30)$$

From the last relation it follows that heat pump mode is implemented on condition of

$$T_o \leq \vartheta_o. \tag{31}$$

For arbitrary combinations of problem parameters the range of permissible currents $J_{min} \leq J \leq J_{\max}$ that satisfy condition (31) will be found as the roots of equation

$$\vartheta_o - \Theta_o(J, I_o, Bi_o, Bi_h, \vartheta_o, \vartheta_h) = 0. \tag{32}$$

At the ends of this current interval the temperature of the heat-absorbing junction is equal to the temperature of liquid being cooled, and cooling capacity goes to zero (Fig. 11 *a, b*). It is noteworthy that the intensity of heat exchange on the heat-absorbing junction of Bi_o does not affect the values of acceptable supply currents and their range depends only on the heat carrier temperatures ϑ_o, ϑ_h and heat exchange conditions on the heat-releasing junction of Bi_h. It is due to the fact that changes in Bi_o are compensated by respective changes in T_o according to (30).

Similar to the schematic considered above, the cooling capacity of THP is limited by cooling depth and heat exchange conditions on the heat-absorbing junction of Bi_h. The specific feature of schematic under study is that cooling depth is assigned by the temperature of cooled heat carrier ϑ_o, and cooling capacity, in conformity with (30), is a function of temperature difference between the heat carrier and the heat-absorbing junction. It restricts maximum cooling depth of THP and places the following limitation on the acceptable temperature ratio between the cooled heat carrier and the heat-absorbing junction:

$$\vartheta_o - \Theta_o \geqq \frac{Ki}{Bi_o}. \tag{33}$$

From the last equation it follows that heat flux on the heat-absorbing junction (net cooling capacity) is varied in conformity with a change in junction temperature (Fig. 11 *a, b*). The ratio between cooling capacity and efficiency is given in Fig. 11 *c*, from which it follows that optimal modes of THP correspond to the region of weakly efficient solutions with the values of supply current $J \leq J_{opt.}$

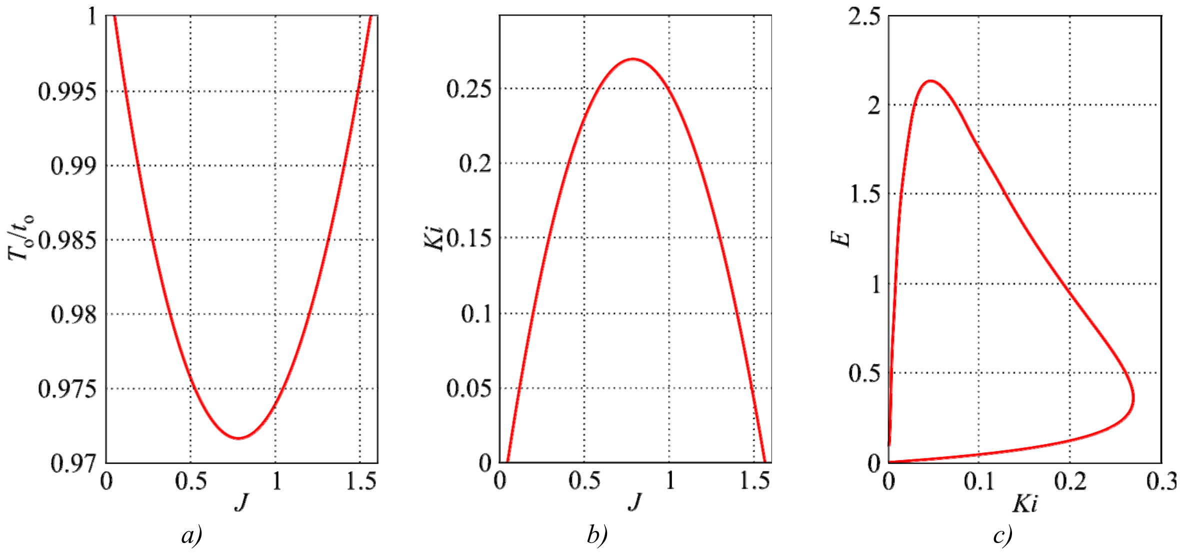


Fig. 11. Characteristics of THP of heat-exchange type: a) – dependence of relative temperature of the heat-absorbing junction (T_o/t_o) on J; b) – dependence of THP heat capacity on J; c) – the ratio between cooling capacity Ki and THP efficiency E.

Electric energy generator mode

In TEG mode, the main limiting factor is the temperature of heating heat carrier ϑ_o. Its maximum acceptable value with regard to (33, 26) is determined as:

$$\vartheta_{o\max} \leqq 1 + \frac{1-\vartheta_h}{Bi_o}. \tag{34}$$

At $\vartheta_o = \vartheta_{o\max}$ the temperature of the heat-absorbing junction in idle mode is equal to T_p, and TEG characteristics achieve maximum possible values. At $\vartheta_o \leqq \vartheta_{o\max}$ the situation is similar to the case with constant-power heat source, namely the governing influence on TEG characteristics is produced by heat exchange conditions on the heat-releasing junctions of thermoelements. However, by virtue of the fact that heat flux and temperature of the heat-absorbing thermoelement junctions depend on load conditions, the acceptable region of schematic under consideration has a greater similarity to characteristics of an idealized TEG (Fig. 12). A set of limiting opportunities of heat-exchange-type TEG is represented in Fig. 6 *c*.

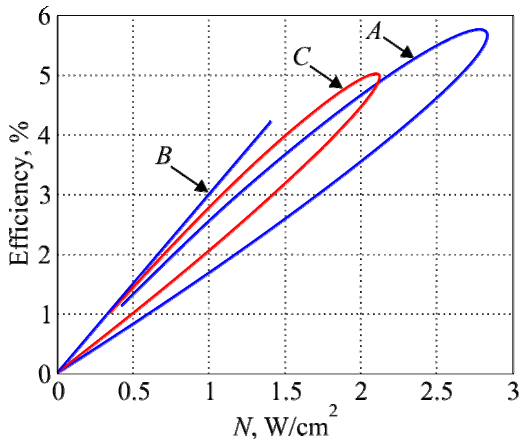


Fig. 12. The ratio between power and efficiency of TEG for different schematics (T_p = 500 K, $0 \le m \le \infty$) A – boundary conditions of I/I kind; B – of II/III kind; C – of III/III kind).

Conclusions

The above results demonstrate the advisability of using system approach for the analysis of TEC characteristics. Solution of thermoelectric energy conversion problem in the generalized form allows clear determination of the effect of the basic system units on the characteristics of thermoelectric devices. Comparative analysis of the sets of limiting opportunities of various TEC schematics allows revealing their peculiarities and obtaining quantitative estimates of each. The obviousness of comparison methods under study is well illustrated in Fig. 12 and Fig. 13 showing characteristics of considered TEC schematics in the space of basic technical and economic criteria (power – efficiency for TEG and cooling capacity – efficiency for THP). Analysis of such characteristics allows objective estimation of actual device losses as compared to an ideal schematic and the specific features of each schematic. The optimal solutions obtained in generalized variables are universal, since they can be transferred to any combination of primary initial data and give unambiguous information on possible characteristics of real TEC with known restrictions on governing parameters (thermoelectric material properties, heat exchange conditions, temperature conditions and device geometry restrictions).

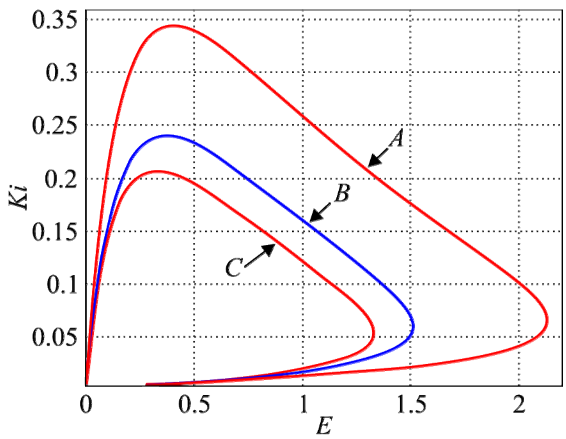


Fig. 13. Comparison of various THP schematics with identical cooling depth (T_o = 280 K). A – boundary conditions of I/I kind; B – of II/III kind; C – of III/III kind.

Symbols

e is the Seebeck coefficient, V/K; σ is electric conductivity, $(\Omega\cdot cm)^{-1}$; λ is thermal conductivity, W/cmK; *z* is thermoelectric figure of merit, K^{-1}; *h* is thermoelement height, cm; *q* is specific density of heat flux, W/cm^2; α is heat transfer coefficient, W/cm^2K; *j* is current density, A/cm^2; t_h is heat sink temperature, K; t_o is heat source temperature, K; $\Theta = T/T_p$ is dimensionless temperature; T_p is governing temperature, K (for TEG – maximum acceptable temperature $T_p = T_{max}$; for THP – heat sink temperature $T_p = t_h$); $\vartheta_h = t_h/T_p$; $\vartheta_o = t_o/T_p$; ΔT, $\Delta\Theta$ is temperature difference; *N* is specific power, W/cm^2. In the calculations the following set of initial data was used for illustrations: $z = 3.1\cdot10^{-3}$ K^{-1}; $h = 0.1$ cm; $Bi_o = Bi_h = 10$; $t_h = 300$ K; for TEG mode $T_p = 500$ K; for THP mode $T_p = 300$ K.

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