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## EFFICIENCY IMPROVEMENT OF SECTIONAL THERMOELECTRIC HEAT RECUPERATORS

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*The paper deals with a physical model of a thermoelectric sectional heat recuperator. Its mathematical description is given and a computer model is developed. The simulation of the recuperator is done for the hot gas temperature range from 150 to 600°C. Dependences of optimal temperatures of recuperator sections on the inlet gas temperature are established. The number of thermoelectric converters in each section for optimal temperature distribution in the sections is determined. The specific cost of each section of thermoelectric heat recuperator in the above temperature range is calculated.*

**Key words:** heat recuperator, thermoelectric generator, computer simulation.

### Introduction

*General characterization of the problem.* Annual energy consumption in the world is on the order of 13 TW. According to predictions, by the end of the century these figures will be tripled by population growth and industrial development [1-3]. At the same time, the majority of industrial process equipment, thermal machines (turbines, internal combustion engines, etc.), while in operation, dissipate huge amount of thermal waste [4]. This heat is not used at all, and, moreover, leads to adverse implications for the environment, namely its thermal pollution. This situation is due to the fact that thermal waste temperatures are in the range of 50-700 °C, and the use of heat engines for this temperature range, especially below 400 °C, in most cases is unreasonable. Such thermal waste recovery is a relevant task of thermoelectricity.

*Analysis of the literature.* Analysis shows [5] that 90% of waste thermal energy is released by industrial facilities at surface temperatures up to 300 °C (Fig. 1). Exactly thermoelectric method of

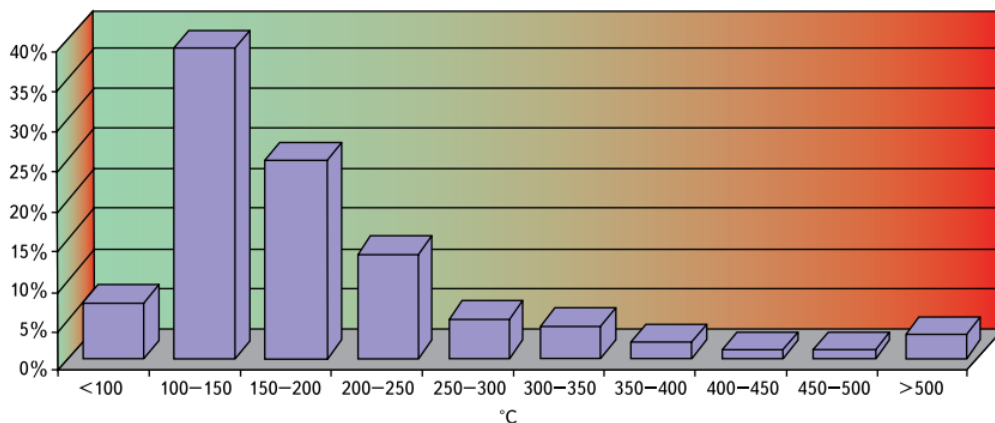


Fig. 1. Temperature distribution on the surfaces of commercial units [4].

direct thermal into electric energy conversion is beneficial for heat recovery at such temperatures [6- 9].

All kinds of practical applications of thermoelectric recuperators of waste thermal energy from internal combustion engines, gas turbines and various industrial furnaces are described in [10-17]. These works are concerned with one-section recuperators, which is not always efficient. The results of optimization of a thermoelectric sectional recuperator using exhaust heat from automobile engine are presented in [19-20]. Calculations have established possible efficiency improvement of thermal energy recovery up to 40% with the use of optimal number of generator sections. However, calculations in these works were performed for the temperature range of 400-800 °C, which, as was shown earlier, does not correspond to the most common temperatures of heat recovery.

The purpose of this work is efficiency increase of thermoelectric heat recuperator via optimization of the number of sections in the temperature range from 150 to 600 °C.

### Physical model of a thermoelectric sectional heat recuperator

A physical model of a thermoelectric sectional heat recuperator is represented in Fig. 2.

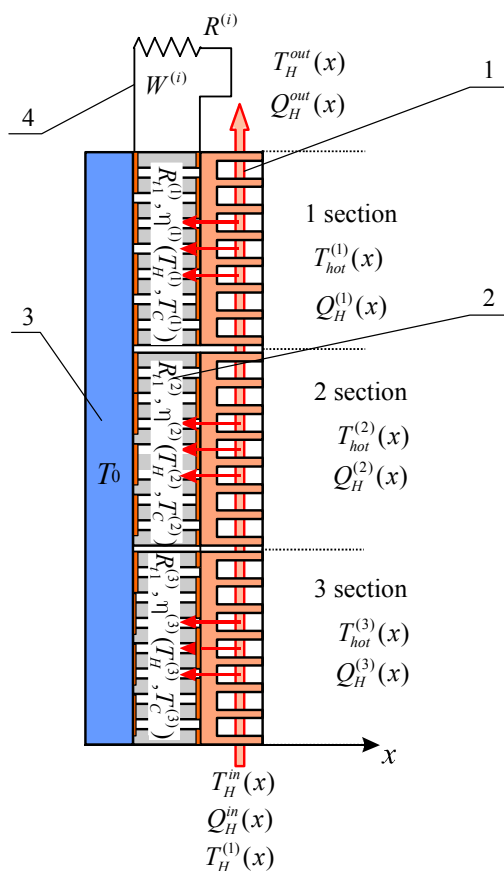


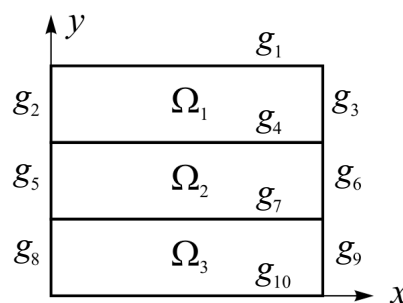
Fig. 2. Physical model of a thermoelectric sectional heat recuperator:  
 1 – hot heat exchanger; 2 – thermopiles; 3 – cold heat exchanger;  
 4 – matched electric load of section.

Each recuperator section consists of hot heat exchanger (1), thermopile (2) with thermal resistance  $R_{t2}^{(i)}$  and efficiency  $\eta(T_H, T_0)$ ; cold heat exchanger (3) with temperature  $T_0$ . Matched electric load  $R^{(i)}$  is connected to thermopiles of each recuperator section (4). The inlet hot gas flow has temperature  $T_H^{in}$  and thermal power  $Q_H^{in}$ . The hot gas gives up part of heat  $Q_H^{(i)}(x)$  at temperature  $T_{hot}^{(i)}(x)$  to the hot heat exchanger. Gas flow at recuperator inlet has temperature  $T_H^{out}$  and thermal power  $Q_H^{out}$ . Heat from the hot heat exchanger is passed to thermopile, heating its hot side to temperature  $T_H^{(i)}(x)$ . Two variants of heat exchange with the cold side of thermopiles are considered. In one case, the cold side temperature  $T_0$  is constant, and in the other heat exchange of cold heat sink 3 with the ambient is taken into account. To calculate maximum possible recuperator power, heat losses will be ignored.

### **Mathematical and computer description of the model**

To optimize the thermoelectric sectional heat recuperator, one should find the distribution of temperatures and heat flows in the thermopiles of each section. Such calculations for the represented model are possible only with the use of computer simulation.

Let us consider one of generator sections and conventionally divide it into three areas  $\Omega_1 - \Omega_3$  with the limits  $g_1 - g_{10}$  (Fig. 3). Area  $\Omega_1$  is the hot heat exchanger with heat carrier, area  $\Omega_2$  is the thermopile, and area  $\Omega_3$  is the hot and cold heat exchangers.



*Fig. 3. Computer representation of the areas and boundaries of recuperator sections.*

In area  $\Omega_1$  there is mass transfer of the hot heat carrier. The thermal conductivity equation for this area is of the form:

$$-\nabla(\kappa_H(T)\nabla T) = -\rho_H(T)C_H(T)\bar{v}\nabla T, \quad (1)$$

where  $\rho_H$  is density,  $C_H$  is heat capacity,  $\kappa_H$  is gas thermal conductivity,  $v_H$  is gas motion velocity. The boundary conditions for area  $\Omega_1$  take into account the continuity of gas flow with and between the sections, heat flow through heat exchangers.

The Joule heat is released in area  $\Omega_2$  (thermopile). For area  $\Omega_2$  the thermal conductivity equation is given below:

$$-\nabla(\kappa_{TE}(T)\nabla T) = Q_J, \quad (2)$$

where  $\kappa_{TE}$  is effective thermal conductivity of thermopile,  $Q_J$  is specific power of the Joule heat released in the thermopile.

The boundary conditions for area  $\Omega_2$  take into account the interaction between thermopile and heat exchangers.

Area  $\Omega_3$  in this model of TEG is considered to be thermostated with temperature  $T_0$ .

A combination of the boundary conditions is mathematically expressed as follows:

$$g_1: \quad q_1^{(i)}(x) = 0, \quad (3)$$

$$g_2, g_3: \quad Q_H^{in(1)} = Q_H^{in}, \quad Q_H^{in(i+1)} = Q_H^{out(i)}, \quad Q_H^{out(N)} = Q_H^{out}, \quad (4)$$

$$g_4: \quad Q_H^{(i)}(x) = (T_H^{(i)}(x) - T^{(i)}(x)) / R_{t1}^{(i)} \quad (5)$$

$$g_5, g_6: \quad q_4^{(i)}(y) = 0, \quad (6)$$

$$g_7: \quad Q_C^{(i)}(x) = (T_C^{(i)}(x) - T^{(i)}(x)) / R_{t2}, \quad (7)$$

$$T(x) = T_0, \quad (7)$$

$$\Omega_3, g_8, g_9, g_{10}: \quad T(x, y) = T_0. \quad (8)$$

A combination of equations (1)-(2) with the boundary conditions (3-8) permits to find temperature field  $T(x, y)$  in TEG and to determine the distribution of temperatures  $T_H^{(i)}(x)$  along the hot sides of section thermopiles.

Then the power of each section can be found from the following expression:

$$W^{(i)} = \int Q_H^{(i)}(x) \eta(T_H^{(i)}(x), T_C^{(i)}(x)) dx. \quad (9)$$

Total generator power

$$W_{TEG} = W^{(1)} + W^{(2)} + W^{(3)}. \quad (10)$$

Thermoelectric generator efficiency

$$\eta_{TEG} = \frac{W_{TEG}}{Q_H^{in}}. \quad (11)$$

To calculate the recuperator electric power with regard to provision of heat removal system, it is necessary to know the efficiency of air-liquid heat exchanger

$$Q_{cool} = f(W_{cool}, T_L, T_A), \quad (12)$$

where  $Q_{cool}$  is thermal power of heat removal system,  $W_{cool}$  is electric supply power of heat removal system,  $T_L$  is liquid temperature,  $T_A$  is air temperature. Such dependence was obtained

from the experimental studies of heat exchanger [18].

The effective efficiency of the recuperator is introduced by the expression:

$$\eta_{ef} = (W_{TEG} - W_{cool}) / Q_{in}. \quad (13)$$

A system of equations (1)-(2) with the boundary conditions (3)-(8) was solved by finite element method [21] on a two-dimensional mesh.

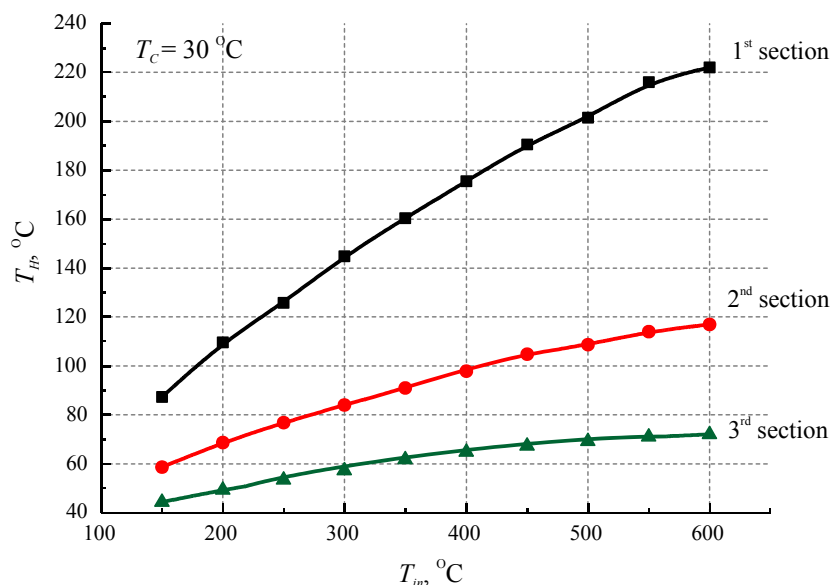
Further optimization consists in a search for optimal hot temperatures of sections through variation of thermal resistances of sections in order to achieve maximum integral efficiency of thermoelectric recuperator.

### Computer simulation results

For the calculation of the efficiency and power there were selected *Bi-Te* based thermoelectric materials whose figures of merit are among the best in the temperature range under consideration [21].

Computer simulation of thermoelectric sectional recuperator of thermal energy yielded the following results.

During the first step, optimization of the hot temperatures of recuperator sections was performed. The presence of such optimum is due to the impact of two competing factors. The reduction of section thermal resistance leads to increase in thermal flow through thermoelectric converter, hence to increase in the section electric power. On the other hand, this leads to thermopile hot temperature decrease, and, accordingly, its efficiency decrease. Fig. 4 presents the results of optimization of the hot temperature of sections versus the inlet gas temperature. Fig. 5 shows a relative number of one-type thermoelectric modules in a section for the achievement of optimal temperature distribution.



*Fig. 4. Optimal hot temperature of sections versus the inlet gas temperature.*

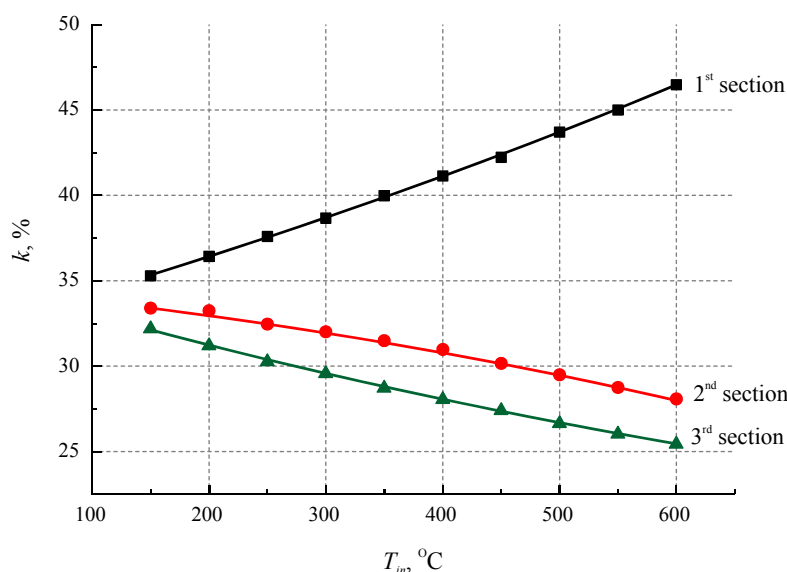


Fig. 5. Relative number of thermoelectric modules in a section for achievement of optimal temperature distribution.

The next simulation step was to determine the efficiency of thermoelectric modules (Fig. 6) and heat recuperator as a whole (Fig. 7) versus the inlet gas temperatures.

Fig. 6 shows the efficiency of thermoelectric modules from each recuperator section versus the inlet gas temperatures.

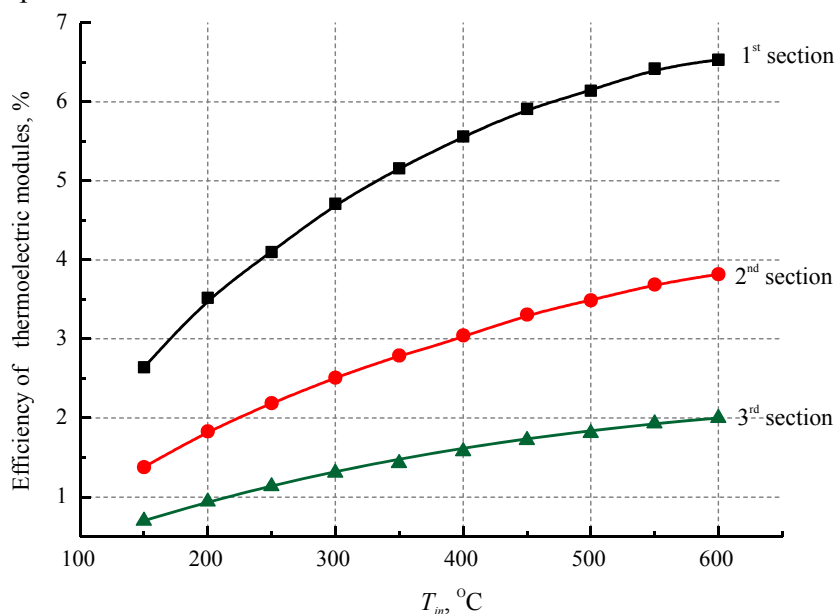
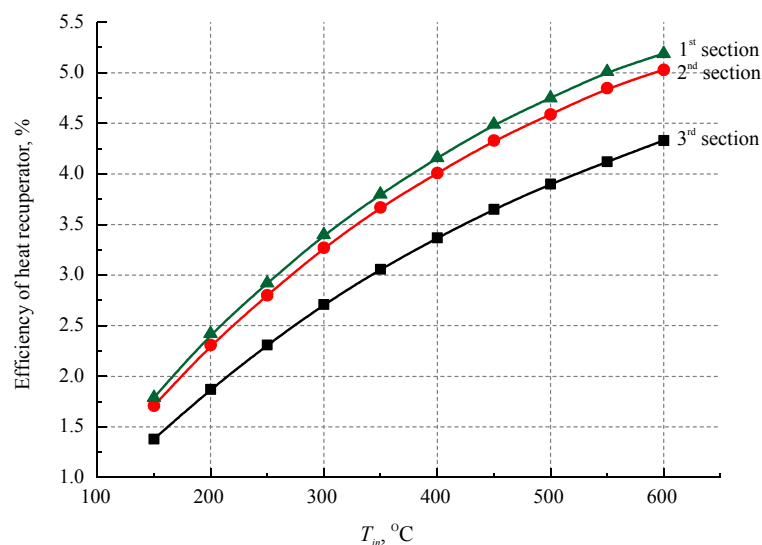


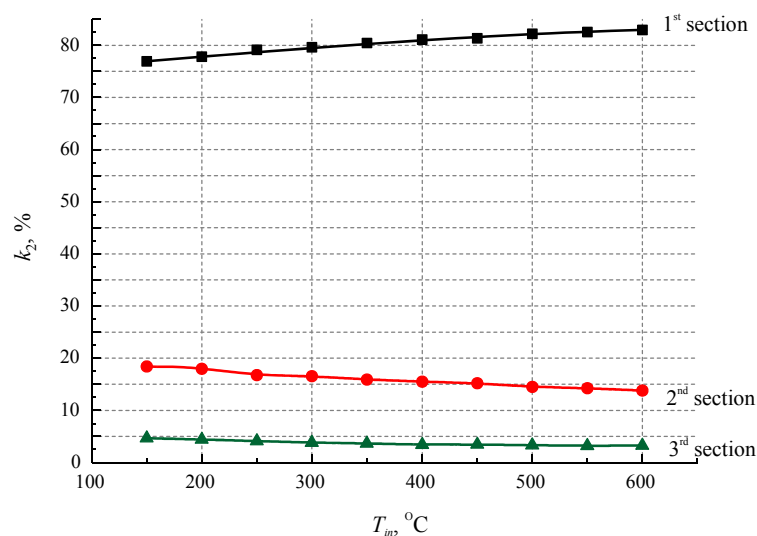
Fig. 6. The efficiency of thermoelectric modules of sections versus the inlet gas temperatures.

As is seen from Fig. 7, using the second section in thermoelectric heat recuperator results in efficiency increase by ~ 16%, and the third section – by as low as 4%.

Contribution by percentage of each section of thermoelectric heat recuperator to its total power is given in Fig. 8. As it follows from the figure, contribution by percentage of the first recuperator section to total power is 75-85%, the second section – 15-20%, the third – about 5%.



*Fig. 7. The efficiency of thermoelectric sectional recuperator versus the inlet gas temperatures.*

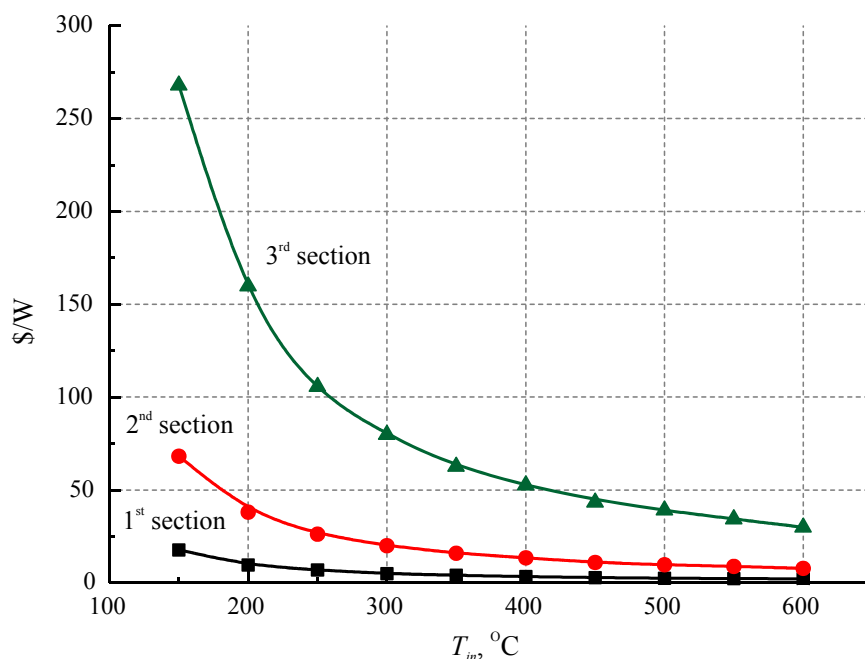


*Fig. 8. Contribution by percentage of each recuperator section to its total power.*

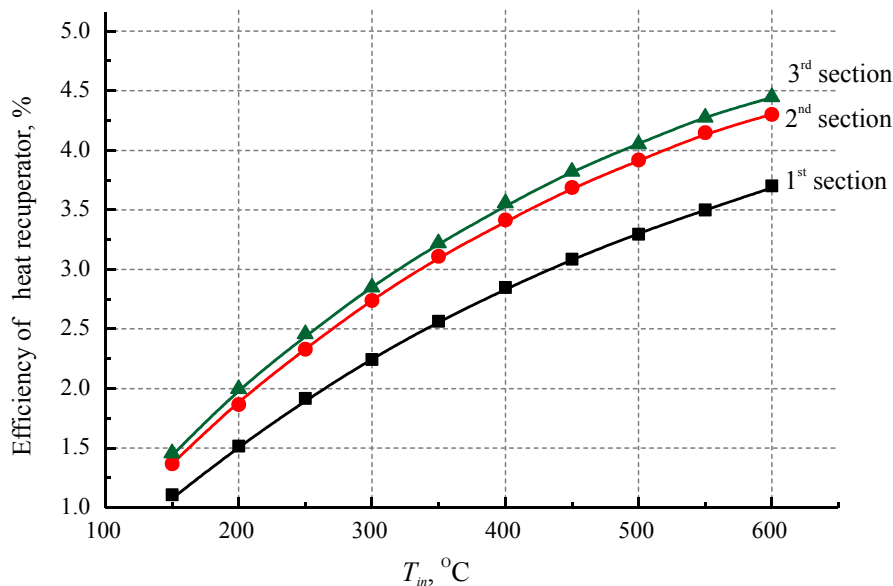
To estimate the economic viability of using sections in thermoelectric heat recuperator, the specific cost of the sections was calculated (Fig. 9) based on the results obtained in [22]. As is seen from the figure, using the third section in the temperature range under consideration is economically unviable. Using the second section is worthwhile at the hot gas temperatures from 400 °C.

To estimate the economic viability of using sections in thermoelectric heat recuperator, the specific cost of the sections was calculated (Fig.9) based on the results obtained in [22]. As is seen from the figure, using the third section in the temperature range under consideration is economically unviable. Using the second section is worthwhile at the hot gas temperatures from 400 °C.

Moreover, estimation was made of the energy and economic features of thermoelectric heat recuperator with regard to energy expenditures on heat removal. Fig. 10 shows the efficiency of thermoelectric sectional recuperator versus the inlet gas temperature with regard to expenditures on heat removal.



*Fig. 9. Specific cost of recuperator sections.*



*Fig. 10. The efficiency of a thermoelectric sectional recuperator versus the inlet temperature with regard to heat removal expenditures.*



Fig. 11 shows the specific cost of recuperator sections with regard to heat removal expenditures. As is seen from the figures, the efficiency of heat recuperation in this case is reduced by ~ 15-20%, and the specific cost is accordingly increased by the same value.

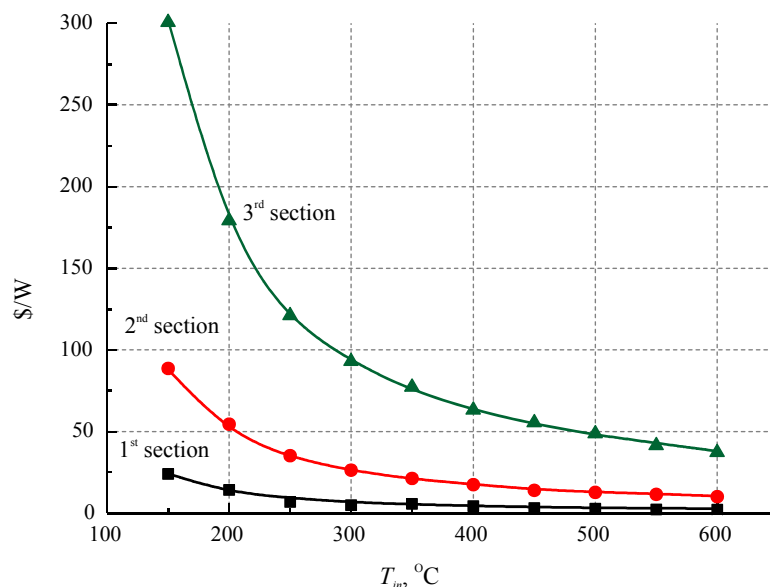


Fig. 11. Specific cost of recuperator sections with regard to heat removal expenditures.

## Conclusions

1. Dependences of the optimal temperatures of recuperator sections on the inlet gas temperature in the range from 150 to 600 °C are established. For the first section they make from 90 to 220 °C, for the second – from 60 to 120 °C, for the third– from 40 to 70 °C.
2. The number of thermoelectric converters for each section necessary to achieve optimal temperature distribution in the sections is determined. For low inlet gas temperatures ( $\square$  150 °C) the number of thermal converters in the sections is about the same. With a rise in temperatures, the share of thermal converters in the first section is increased.
3. Using the second section of thermoelectric heat recuperator in the range of inlet gas temperatures from 150 to 600 °C leads to its efficiency increase by ~ 16%, and the third – by as low as 4%.
4. The specific cost of each section of thermoelectric heat recuperator in the above temperature range is calculated. It is established that the specific cost of the third section is an order higher than the cost of the first section, which makes its application inadvisable. Using the second section is worthwhile at the hot gas temperatures from 400 °C
5. The efficiency of heat recuperation with regard to energy expenditures on heat removal is reduced by ~ 15-20%, and the specific cost is accordingly increased by the same value.

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