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ESTIMATION OF THE EFFICIENCY OF PARTIAL CASE OF HEAT AND MASS TRANSFER PROCESSES BETWEEN HEAT PUMPS AND MOVING SUBSTANCE, PART 3

A theoretical model for evaluating the efficiency of the partial case of heat and mass transfer processes between a moving substance and thermoelectric heat pumps with their heat exchange parts, in which the moving substance (or at least part of this moving substance) is brought into thermal contact with heat absorbing and heat dissipating heat exchangers , which can operate in modes that may differ from the mode of maximum energy efficiency, in particular, taking into account the amounts of materials required for the manufacture of these thermoelectric heat pumps. The results of the corresponding theoretical estimation are given. Bibl. 9, Tabl. 1, Fig. 2.
Key words: heat pump, moving substance, heat and mass transfer, efficiency, energy efficiency, thermoelectric heat pump, thermoelements, thermoelectric material.

Introduction

This paper (part 3) is a continuation of the previous works [1, 2] (part 1 and part 2). In this part 3 we will use abbreviations that were introduced in [1, 2], in the same sense as in [1, 2]. We will also use the word combination the investigated process in order to indicate the process which corresponds to the investigated method of heat and mass transfer [1 - 5]. In [1, 2], mathematical expressions were obtained for estimating the efficiency of the investigated method of heat and mass transfer and examples of appropriate calculations were given, in particular, for the case of using thermoelectric heat pumps (THPs) that operate in the modes if maximum energy efficiency. According to the data of these calculations, there exists a possibility to increase the energy efficiency of the investigated method of heat and mass transfer owing to increased number of heat pumps (HPs) that are used in this case. At the same time, in [1, 2], the details of how the amount of materials from which the HPs are made changes with a change in the number of HPs.

The purpose of this work is to create theoretical prerequisites for an approximate quantitative estimation of the efficiency (primarily energy efficiency) of the investigated method of heat and mass transfer using THPs that can operate in the modes that may differ from the maximum energy efficiency, in particular, with regard to the amount of materials of which these THPs are made. To achieve this purpose, the objectives of this work are to create an appropriate estimation model, obtain mathematical expressions for estimation calculations and obtain examples of relevant calculations.

Description of estimation model

Consider the following example of *the investigated processes*. Consider the processes involving moving substance (MS) and at least one THP (all HPs used in these processes are THPs based on thermocouples), in which according to Fig. 2 [1] MS in its input flow is cooled by all individual THPs.

Let the useful effect of these processes be to maintain the temperature difference of the MS in its input flow between the positions 1.0 and 1.n according to Fig. 2 [1] (for some MS the inlet temperature in position 1.0). This useful effect is carried out due to the total power consumption of all THPs W^{TTH} (and more directly due to the total cooling capacity of all THPs Q_{cool}^{TTH}). In this work (part 3) we will not take into account the energy consumption to create a MS flow [6].

Consider an individual *i*-th THP.

We use a well-known ratio to determine the coefficient of performance of the *i*-th THP ε_i^{TE} [7, 8]:

$$\varepsilon_i^{TE} = \frac{Q_{cool,i}^{TE}}{W_i^{TE}},$$ (3.1)

where

$$Q_{cool,i}^{TE} = \alpha_i I_i T_{cool,i}^{TE} - \frac{1}{2} I_i^2 r_i - k_i \left(T_{hot,i}^{TE} - T_{cool,i}^{TE} \right);$$ (3.2)

$$W_i^{TE} = I_i^2 r_i + \alpha_i \left(T_{hot,i}^{TE} - T_{cool,i}^{TE} \right) I_i;$$ (3.3)

$Q_{cool,i}^{TE}$ – is total cooling capacity of thermoelements of the *i*-th THP; W_i^{TE} is total electrical power consumed by thermoelements of the *i*-th THP; α_i is total differential Seebeck coefficient of material of thermoelements of the *i*-th THP; I_i is strength of current flowing through thermoelements of the *i*-th THP; r_i is total electrical resistance of thermoelements of the *i*-th THP; k_i is total thermal conductivity of thermoelements of the *i*-th THP; $T_{hot,i}^{TE}$ is temperature of heat-releasing junctions of thermoelements of the *i*-th THP; $T_{cool,i}^{TE}$ is temperature of heat-absorbing junctions of the *i*-th THP.

The coefficient of performance of the *i*-th THP that works in the investigated process according to Fig. 2 [1] and with regard to assumption 6 [1] ($d = const$) ε_i^{TTH}:

$$\varepsilon_i^{TTH} = \frac{Q_{cool,i}^{TTH}}{W_i^{TTH}} = \frac{T_{cool,(i-1)}^{PP} - T_{cool,i}^{PP}}{\left(T_{hot,(i-1)}^{PP} - T_{hot,i}^{PP} \right) - \left(T_{cool,(i-1)}^{PP} - T_{cool,i}^{PP} \right)},$$ (3.4)

where $Q_{cool,i}^{TTH}$ is cooling capacity of the *i*-th THP; W_i^{TTH} is power consumed by the *i*-th THP; $T_{hot,i}^{MS}$ is temperature of MS immediately before its thermal contact with heat-releasing heat-exchange part (HE) of the *i*-th THP; $T_{hot,(i-1)}^{MS}$ is temperature of MS immediately after its thermal contact with heat-releasing HE of

the *i*-th THP; $T_{cool,i}^{MS}$ is temperature of MS immediately after its thermal contact with heat-absorbing HE of the *i*-th THP; $T_{cool,(i-1)}^{MS}$ is temperature of MS immediately before its thermal contact with heat-absorbing HE of the *i*-th THP.

Let $Q_{cool,i}^{TTH} = Q_{cool,i}^{TE}$ and $W_i^{TTH} = W_i^{TE}$. Then, on the basis of expressions (1) and (4) one can write:

$$\varepsilon_i^{TE} = \varepsilon_i^{TTH}, \tag{3.5}$$

$$\frac{T_{cool,(i-1)}^{MS} - T_{cool,i}^{MS}}{\left(T_{hot,(i-1)}^{MS} - T_{hot,i}^{MS}\right) - \left(T_{cool,(i-1)}^{MS} - T_{cool,i}^{MS}\right)} = \frac{\alpha_i I_i T_{cool,i}^{TE} - \frac{1}{2}I_i^2 r_i - k_i\left(T_{hot,i}^{TE} - T_{cool,i}^{TE}\right)}{I_i^2 r_i + \alpha_i\left(T_{hot,i}^{TE} - T_{cool,i}^{TE}\right)I_i}. \tag{3.6}$$

We will assume that heat transfer from MS to heat-absorbing junctions of thermoelements of the *i*-th THP is carried out through the medium characterized by the corresponding resistance of heat transfer (thermal resistance) $R_{cool,i}$, and heat transfer from heat-releasing junctions of thermoelements of the *i*-th THP to MS is carried out through the medium heat transfer resistance (thermal resistance). We will also assume that there are no other additional factors that could affect the heat transfer between the MS and the *i*-th THP. Then we can write the following equations, which, in particular, reflect the relationship between the junction temperatures of thermoelements of the *i*-th THP and MS (in the corresponding positions of its motion):

$$T_{cool,i}^{MS} - T_{cool,i}^{TE} = Q_{cool,i}^{TE} R_{cool,i}, \tag{3.7}$$

$$T_{cool,i}^{MS} - T_{cool,i}^{TE} = \left(\alpha_i I_i T_{cool,i}^{TE} - \frac{1}{2}I_i^2 r_i - k_i\left(T_{hot,i}^{TE} - T_{cool,i}^{TE}\right)\right) R_{cool,i} \tag{3.8}$$

(expression (3.8) was obtained with the use of expression (2));

$$T_{hot,i}^{TE} - T_{hot,(i-1)}^{MS} = Q_{hot,i}^{TE} R_{hot,i}, \tag{3.9}$$

$$T_{hot,i}^{TE} - T_{hot,(i-1)}^{MS} = \left(\alpha_i I_i T_{hot,i}^{TE} + \frac{1}{2}I_i^2 r_i - k_i\left(T_{hot,i}^{TE} - T_{cool,i}^{TE}\right)\right) R_{hot,i}, \tag{3.10}$$

where $Q_{hot,i}^{TE}$ is total calorific power of thermoelements of the *i*-th THP

$Q_{hot,i}^{TE} = \alpha_i I_i T_{hot,i}^{TE} + \frac{1}{2}I_i^2 r_i - k_i\left(T_{hot,i}^{TE} - T_{cool,i}^{TE}\right)$ [7, 8]).

As a characteristic of MS flow we will use heat capacity losses of MS (the rate of losing MS, if the amount of MS is expressed in the units of its heat capacity) V_C^{MS}, (J/K)/s or W/K:

$$V_C^{MS} = \frac{C^{MS}}{\Delta\tau}, \tag{3.11}$$

where C^{MS} is heat capacity of MS that takes part in the corresponding process during time interval $\Delta\tau$ (in some position of MS motion, for instance, according to Fig. 2 [1] – in position 1.1; in the context of this work we will consider that V_C^{MS} is the same in all positions of MS motion).

For instance, if we know the specific mass heat capacity of MS c_m^{MS} and mass losses of MS M^{MS}, then the heat capacity losses of MS can be determined by the formula:

$$V_C^{MS} = c_m^{MS} M^{MS}. \tag{3.12}$$

With regard to assumption 6 [1] and the information given above, we write the equation, which, in particular, reflects the relationship between the total cooling capacity of thermocouples of the *i*-th THP, the change in MS temperature as a result of its thermal contact with heat-absorbing HE of the *i*-th THP and the heat capacity losses of MS:

$$V_C^{MS}\left(T_{cool,(i-1)}^{MS} - T_{cool,i}^{MS}\right) = \alpha_i I_i T_{cool,i}^{TE} - \frac{1}{2}I_i^2 r_i - k_i\left(T_{hot,i}^{TE} - T_{cool,i}^{TE}\right). \tag{3.13}$$

With regard to assumption 6 [1] and the information given above, we write the equation which, in particular, reflects the relationship between total calorific power of thermoelements of the *i*-th THP, change in MS temperature as a result of its thermal contact with heat-releasing HE of the *i*-th THP and heat capacity losses of MS:

$$V_C^{MS}\left(T_{hot,(i-1)}^{MS} - T_{hot,i}^{MS}\right) = \alpha_i I_i T_{hot,i}^{TE} + \frac{1}{2}I_i^2 r_i - k_i\left(T_{hot,i}^{TE} - T_{cool,i}^{TE}\right). \tag{3.14}$$

With regard to assumption 6 [1] and the information given above, we write the equation which, in particular, reflects the relationship between the total power consumption of thermoelements of the *i*-th THP, total change in MS temperature as a result of its thermal contact with heat-absorbing and heat-releasing HEs of the *i*-th THP and heat capacity losses of MS:

$$V_C^{MS}\left(\left(T_{hot,(i-1)}^{MS} - T_{hot,i}^{MS}\right) - \left(T_{cool,(i-1)}^{MS} - T_{cool,i}^{MS}\right)\right) = I_i^2 r_i + \alpha_i\left(T_{hot,i}^{TE} - T_{cool,i}^{TE}\right)I_i. \tag{3.15}$$

In this work (part 3) we will assume that the values $R_{hot,i}$, $R_{cool,i}$, α_i, r_i, k_i are constant (their possible temperature and other dependences will be disregarded) for an individual case.

Equations (3.6), (3.8), (3.10), (3.13), (3.14), (3.15) can be used for estimation calculations of the operating modes of individual THPs and the investigated processes in general.

For the above described example we will use an indicator of the energy efficiency of the investigated process ω_{cool}:

$$\omega_{cool} = \frac{Q_{cool}^{\mathrm{TTH}}}{W^{TTH}} = \frac{\Delta T_{cool}^{PP}}{\Delta T_{hot}^{PP} - \Delta T_{cool}^{PP}} \qquad (3.16)$$

(the right-hand side of this expression was obtained with the use of expressions (1), (23) and (24) [1]), where ΔT_{cool}^{MS} in conformity with the diagram in Fig. 2 [1] is the temperature difference of MS which is formed as a result of cooling MS in its input flow by all individual THPs; ΔT_{hot}^{MS} in conformity with the diagram in Fig. 2 [1] is the temperature difference of MS which is formed as a result of heating MS in its output flow by all individual THPs.

Note that when in the investigated process only one THP is used, then ω_{cool} is equal to the coefficient of performance of this single THP ε_1^{TTH}.

On the change in the amount of materials

The technical implementation of the investigated method of heat and mass transfer can be carried out using a suitable device. The cost, weight, size and other characteristics of such a device may depend on the quantities (e.g. masses) of the materials of which THPs are made, for example, in particular, on the amount of thermoelectric material of which the thermocouple legs are made (e.g. bismuth telluride material) and on the amount of material of which HEs are made (for example, in the first place, the aluminum-based material of which the heat exchangers are made). With a change in the number of THPs n, which are used to implement the investigated method of heat and mass transfer, the above amounts of this or other material can remain unchanged, decrease or increase. As characteristics of this, you can use the coefficient of change in the amount of thermoelectric material (with a corresponding change in the implementation of the investigated method of heat and mass transfer) and the coefficient of change in the amount of material of which the heat exchangers $\varphi_{l_0}^{l_x}$ are made (with a corresponding change in the implementation of the investigated method of heat and mass transfer):

$$\gamma_{l_0}^{l_x} = \frac{m_{l_x}^{TM}}{m_{l_0}^{TM}}; \qquad (3.17)$$

$$\varphi_{l_0}^{l_x} = \frac{m_{l_x}^{MHE}}{m_{l_0}^{MHE}}, \qquad (3.18)$$

where $m_{l_0}^{TM}$ is the amount of thermoelectric material, expressed in appropriate units (for example, mass, expressed in kilograms) in the case (variant) of the implementation of the investigated method of heat and mass transfer, with which the comparison is carried out (in the initial case); l_0 is designation of this initial case (as a variant – its number in Table 3.1); $m_{l_x}^{TM}$ is the corresponding amount of thermoelectric material (expressed in the same units) in the case (variant) of implementation of the investigated method of heat and mass transfer which is considered (compared); l_x is designation of this compared case (as a variant – its number in Table 1); $m_{l_0}^{MHE}$ is the amount of material of which the heat exchangers are made, expressed in

appropriate units (for instance, mass, expressed in kilograms) in the case (variant) of implementation of the дinvestigated method of heat and mass transfer, with which the comparison is carried out (in the initial case); l_0 is designation of this initial case (as a variant – its number in Table); $m_{n_{ch}}^{MHE}$ is the corresponding amount of material of which the heat exchangers are made (expressed in the same units) in the case (variant) of implementation of the investigated method of heat and mass transfer, which is considered (compared); l_x is the designation of this compared case (as a variant – its number in Table).

Let us make the following assumptions.

10. Suppose that with a change in the implementation of the investigated method of heat and mass transfer, which may be accompanied by a change in the number of THPs, the following relations are fulfilled, which are related to the amount of thermoelectric material (provided that all thermoelements of the i-th THP are electrically connected in series, and in terms of heat flows - in parallel):

$$\alpha_{i;x} = \frac{n_0}{n_x}\gamma_{l_0}^{l_x}\alpha_{i;0}; \tag{3.19}$$

$$r_{i;x} = \frac{n_0}{n_x}\gamma_{l_0}^{l_x}r_{i;0}; \tag{3.20}$$

$$k_{i;x} = \frac{n_0}{n_x}\gamma_{l_0}^{l_x}k_{i;0}, \tag{3.21}$$

where n_0 is the number of THPs in the case (variant) of implementation of the investigated method of heat and mass exchange, with which the comparison is carried out (in the initial case); n_x is the number of THPs in the case (variant) of implementation of the investigated method of heat and mass transfer which is considered (compared); $\alpha_{i;x}$ is total differential Seebeck coefficient of the material (legs) of thermoelements of the i-th THP in the case (variant) of implementation of the investigated method of heat and mass transfer, which is considered (compared); $\alpha_{i;0}$ is total differential Seebeck coefficient of the material (legs) of thermoelements of the i-th THP in the case (variant) of implementation of the investigated method of heat and mass exchange, with which a comparison is carried out (in the initial case); $r_{i;x}$ is total electrical resistance of thermoelements of the i-th THP in the case (variant) of implementation of the investigated method of heat and mass transfer, which is considered (compared); $r_{i;0}$ is total electrical resistance of thermoelements of the i-th THP in the case (variant) of implementation of the investigated method of heat and mass transfer, with which a comparison is carried out (in the initial case); $k_{i;x}$ is thermal conductivity of thermoelements of the i-th THP in the case (variant) of implementation of the investigated method of heat and mass transfer, which is considered (compared); $k_{i;0}$ is thermal conductivity of thermoelements of the i-th THP in the case (variant) of implementation of the investigated

method of heat and mass transfer, with which a comparison is carried out (in the initial case).

11. Suppose that with a change in the implementation of the investigated method of heat and mass transfer the following relations are fulfilled, which are related to the amount of material of which the heat exchangers are made (provided that all thermoelements of the *i*-th THP are connected in parallel in terms of heat flows):

$$R_{hot,i;l_x} = \frac{n_x}{n_0 \varphi_{l_0}^{l_x}} R_{hot,i;l_0} \; ; \tag{3.22}$$

$$R_{cool,i;l_x} = \frac{n_x}{n_0 \varphi_{l_0}^{l_x}} R_{cool,i;l_0} \; , \tag{3.23}$$

where $R_{hot,i;l_x}$ is total resistance of heat transfer from heat-releasing thermoelement junctions of the *i*-th THP to MS in the case (variant) of implementation of the investigated method of heat and mass exchange which is considered (compared); $R_{hot,i;l_0}$ is total resistance of heat transfer from heat-releasing thermoelement junctions of the *i*-th THP to MS in the case (variant) of implementation of the investigated method of heat and mass exchange, with which a comparison is carried out (in the initial case); $R_{cool,i;l_x}$ is total resistance of heat transfer from MS to heat-absorbing thermoelement junctions of the *i*-th THP in the case (variant) of implementation of the investigated method of heat and mass exchange, which is considered (compared); $R_{cool,i;l_0}$ is total resistance of heat transfer from MS to heat-absorbing thermoelement junctions of the *i*-th THP in the case (variant) of implementation of the investigated method of heat and mass exchange, with which a comparison is carried out (in the initial case).

Hereinafter, in this work (part 3) we will use assumptions 10 and 11.

Results of estimation calculations and their peculiarities

The initial data and the results of the corresponding calculations related to example under study are presented (in abbreviated form) in Table. Column headings of Table contain, sequentially, from top to bottom, a textual description of the corresponding quantities, their symbolic designation (if any) and dimension (if any), which are separated by dotted lines. In Table 1, the initial data and calculated results are designated by different colours (the initial data – in this colour, and calculated results – in this, different colour). Case numbers of example under study for which the value V_C^{MS} is identical are designated by the same colours.

For all the cases of example under study the total temperature difference of MS in its input flow according to expression (23) [1] and diagram in Fig. 2 [1] is identical and equal to 5 K :

$$\Delta T_1^{MS} = 5 \; K \; . \tag{3.24}$$

Initial data and some results of corresponding estimation calculations of the efficiency of the investigated method of heat and mass exchange with the use of THP for the case of cooling MS in its input flow by all individual THPs (according to Fig. 2 [1]; according to assumptions 1, 2, 4-8 [1]; $T_{1.n}^{PP} = T_{2.n}^{PP} = 298.15\,K$)

Case № of example under study	Total number of THPs	Coefficient of change in the amount of thermoelectric material	Coefficient of change in the amount of material of heat echnagers	Total differential Seebeck coefficient of material (legs) of thermoelements of each individual i-th THP	Total electrical resistance of thermoelements of each individual i-th THP	Strength of current flowing through each thermoelement of each individual i-th THP	Total thermal conductivity of thermoelement legs of each individual i-th THP	Total resistance of heat transfer from heat-releasing thermoelement junctions of each individual i-th THP to MS	Total resistance of heat transfer from MS to heat-absorbing thermoelement junctions of each individual i-th THP	Heat capacity losses of MS (heat capacity rate of \ms)	Total temperature difference of MS in its input flow	Temperature difference at $R_{cool,n}$, K	Temperature difference on thermoelements of n-th THP	Coefficient of performance of thermoelemnts of n-th THP	Indicator of energy efficiency of the investigated process
l	n	$\gamma_{l_0}^{l_x}$	$\varphi_{l_0}^{l_x}$	α_i	r_i	I_i	k_i	$R_{hot,i}$	$R_{cool,i}$	V_C^{MS}	ΔT_1^{MS}	$\Delta T_{cool,n}^{R}$	ΔT_n^{TE}	ε_n^{TE}	ω_{cool}
				V	Ohm	A	W/K	K/W	K/W	W/K	K	K	K		
$1^{(1)}$	1	1	1	0.048	2.6	1.617	0.34	0.1	0.1	3.152	5	1.5762	11.36	2.051	2.051
2	1	2	1	0.096	5.2	0.863	0.68	0.1	0.1	3.152	5	1.5762	10.12	3.343	3.343
3	1	16	1	0.768	41.6	0.364	5.44	0.1	0.1	3.152	5	1.5762	11.829	1.789	1.789
4	2	1	1	0.024	1.3	1.49	0.17	0.2	0.2	3.152	5	1.5835	7.306	2.517	2.492
5	2	2	1	0.048	2.6	0.768	0.34	0.2	0.2	3.152	5	1.5844	6.601	4.458	4.411
6	2	2	2	0.048	2.6	0.713	0.34	0.1	0.1	3.152	5	0.791	4.712	5.337	5.288
7	2	0.87	0.87	0.021	1.128	1.765	0.148	0.23	0.23	3.152	5	1.825	8.255	2.074	2.051
8	2	0.9	0.9	0.022	1.17	1.689	0.153	0.222	0.222	3.152	5	1.760	7.989	2.183	2.160
9	8	1	1	0.006	0.325	1.403	0.042	0.8	0.8	3.152	5	1.586	4.560	2.923	2.878
10	16	1	1	0.003	0.162	1.3895	0.021	1.6	1.6	3.152	5	1.565	5.677	2.997	2.948
11	16	4	4	0.012	0.65	0.313	0.085	0.4	0.4	3.152	5	0.396	1.155	14.51	14.32
12	16	16	16	0.048	2.6	0.082	0.34	0.1	0.1	3.152	5	0.0991	0.521	50.44	49.71
13	16	32	16	0.096	5.2	0.047	0.68	0.1	0.1	3.152	5	0.0994	0.520	71.02	69.72
14	16	32	32	0.096	5.2	0.045	0.68	0.05	0.05	3.152	5	0.050	0.419	81.05	79.69
15	16	0.78	0.78	0.002	0.127	1.882	0.017	2.042	2.042	3.152	5	2.028	5.492	2.089	2.051
16	16	0.9	0.9	0.003	0.146	1.579	0.019	1.778	1.778	3.152	5	1.764	4.648	2.582	2.538
17	48	48	48	0.048	2.6	0.0272	0.34	0.1	0.1	3.152	5	0.033	0.172	153.5	151.2

Table (continued)

l	n	$\gamma_{l_0}^{l_x}$	$\varphi_{l_0}^{l_x}$	α_i	r_i	I_i	k_i	$R_{hot,i}$	$R_{cool,i}$	V_C^{MS}	ΔT_1^{MS}	$\Delta T_{cool,n}^R$	ΔT_n^{TE}	ε_n^{TE}	ω_{cool}
				V	Ohm	A	W/K	K/W	K/W	W/K	K	K	K		
18	1	1	1	0.048	2.6	0.314 (2)	0.34	0.1	0.1	0.445	5	0.223	6.267	6.360	6.360
19	1	1	1	0.048	2.6	0.350 (3)	0.34	0.1	0.1	0.536	5	0.268	6.374	6.293	6.293
20	2	1	1	0.024	1.3	0.274	0.17	0.2	0.2	0.536	5	0.270	3.300	11.31	11.18
21	16	1	1	0.003	0.1625	0.214	0.02125	1.6	1.6	0.536	5	0.269	0.881	21.12	20.839
22	16	0.314	0.314	0.00094	0.051	0.699	0.0067	5.098	5.098	0.536	5	0.859	2.216	6.383	6.293
23	64	1	1	0.00075	0.041	0.207	0.0053	6.4	6.4	0.536	5	0.269	0.632	22.816	22.516
24	64	8	8	0.006	0.325	0.027	0.0425	0.8	0.8	0.536	5	0.034	0.147	160.4	158.1
25	1	1	1	0.048	2.6	4.502 (4)	0.34	0.1	0.1	5.669	5	2.834	26.841	0.484	0.484
26	1	1	1	0.048	2.6	5.455 (5)	0.34	0.1	0.1	5.343	5	2.671	35.21	0.309	0.309
27	2	1	1	0.024	1.3	3.086	0.17	0.2	0.2	5.343	5	2.694	13.08	1.009	0.994
28	16	1	1	0.003	0.1625	2.846	0.02125	1.6	1.6	5.343	5	2.704	8.201	1.219	1.191
29	16	0.828	0.828	0.002483	0.1345	4.838	0.01759	1.93	1.93	5.343	5	3.327	14.008	0.519	0.495
30	16	0.801	1	0.002404	0.1302	4.928	0.01703	1.6	1.6	5.343	5	2.751	11.724	0.521	0.497

[1] coefficients of change in the amount of materials for all other cases were calculated with respect to the amount of materials in this case (the amount of materials in this case is initial); [2] in the mode of maximum energy efficiency of THP working in the investigated process; [3] corresponds to the mode of maximum energy efficiency of THP thermoelements working at the temperature values of their heat-absorbing and heat-releasing junctions that correspond to this case [4] in the mode of maximum cooling capacity of THP working in the investigated process; [5] corresponds to the mode of maximum cooling capacity of THP thermoelements working at the temperature values of their heat-absorbing and heat-releasing junctions that correspond to this case.

In the 1st case of example under study only one THP is used, the thermoelements of which work in the intermediate mode between the modes of maximum energy efficiency and the mode of maximum cooling capacity. The parameters of this process were calculated with the use of a system of 4 equations (3.8), (3.10), (3.13), (3.14) (it is also possible to use a system of 4 equations (3.8), (3.10), (3.13), (3.15), or a system of 4 equations (3.8), (3.10), (3.14), (3.15) or a system of 4 equations (3.6), (3.8), (3.10), (3.13), or a system of 4 equations (3.6), (3.8), (3.10), (3.14), or a system of 4 equations (3.6), (3.8), (3.10), (3.15)). In so doing, the values of $T_{hot,(i-1)}^{PP}$, $T_{cool,i}^{TE}$, $T_{hot,i}^{TE}$, V_C^{PP} are unknown, and other values are known. The initial data and the results of calculations for this case are given in the row of table 3.1, which corresponds to this case.

In the following cases of the considered example in which several THPs are used, these several THPs are identical (the values of α_i, r_i, k_i, $R_{hot,i}$ and a $R_{cool,i}$ for all these THPs are identical) and also for all these THPs the strength of current flowing through them, I_i is the same.

Fig. 3.1 shows some peculiarities of cases 18 and 19 of example under study. Plot *a* in Fig. 3.1 was built on the basis of a function that reflects a dependence of the coefficient of performance of respective *i*-th THP on the strength of current flowing through thermoelements of this *i*-th THP $\varepsilon_i^{TTH}(I_i)$, which, in turn, was obtained on the basis of equations, such as (3.6), (3.8), (3.10), (3.13) and expression (3.4), provided that $\Delta T_1^{MS} = const = 5\,K$. Coordinates of point **18** in Fig. 3.1 (which corresponds to case 18 of example under study) were obtained, in particular, with the use of condition $\left(\varepsilon_i^{TTH}(I_i)\right)' = 0$. Plots *b* and *c* in Fig. 3.1 were built on the basis of expression (3.1) (with the use of expressions (2) and (3)).

The results of calculations for case 19 were obtained using a system of 5 equations, for instance, (6), (8), (10), (13) and a commonly known expression to determine the strength of current flowing through thermoelements that work in the mode of maximum energy efficiency $I_i^{\varepsilon_{i,max}^{TE}}$ [9]:

$$I_i^{\varepsilon_{i,max}^{TE}} = \frac{\alpha_i(T_{hot,i}^{TE} - T_{cool,i}^{TE})}{r_i\left(\sqrt{1 + 0.5\dfrac{\alpha_i^2}{k_i r_i}(T_{hot,i}^{TE} + T_{cool,i}^{TE})} - 1\right)} \,. \tag{3.25}$$

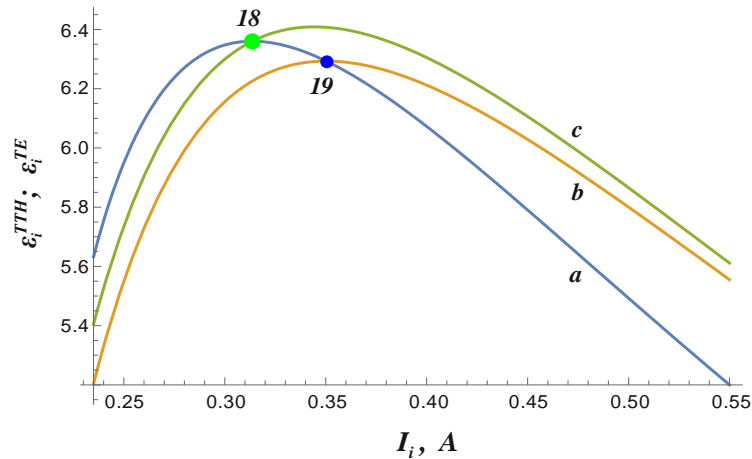In this case 19 $I_i = I_i^{\varepsilon_{i,max}^{TE}}$.

Fig. 3.1. Figure showing some peculiarities of cases 18 and 19 of example under study:
$a - plot\ \varepsilon_i^{TTH}(I_i)$ ($\Delta T_1^{MS} = const = 5\,K$, $V_C^{MS} \neq const$); $b - plot\ of\ \varepsilon_i^{TE}$ versus I_i for fixed
temperature values of heat-absorbing and heat-releasing thermoelement junctions of the i-th THP
that correspond to case 19 of example under study; $c - plot\ of\ \varepsilon_i^{TE}$ versus I_i for fixed temperature values
of heat-absorbing and heat-releasing thermoelement junctions of the i-th THP that correspond to case
18 of example under study; point **18** corresponds to case 18 of example under study;
point **19** corresponds to case 19 of example under study.

Fig. 3.2 shows some peculiarities of cases 25 and 26 of example under study. Plot *a* in Fig. 3.2 was built on the basis of a function showing a dependence of cooling capacity of corresponding *i*-th THP on the strength of current flowing through thermoelements of this *i*-th THP $Q_{cool,i}^{TTH}(I_i)$, which, in turn, was obtained on the basis of equations, such as (3.6), (3.8), (3.10), (3.13) and the expression to determine cooling capacity of the *i*-th THP by its effect on MS:

$$Q_{cool,i}^{TTH} = V_C^{PP}\left(T_{cool,(i-1)}^{PP} - T_{cool,i}^{PP}\right)$$ (3.26)

(with regard to the respective assumptions) provided that $\Delta T_1^{MS} = const = 5\,K$.

Coordinates of point **25** in Fig. 3.2 (which corresponds to case 25 of example under study) were obtained, in particular, with the use of condition $\left(Q_{cool,i}^{TTH}(I_i)\right)' = 0$. Plots *b* and *c* in Fig. 3.2 were built on the basis of Eq.(2).
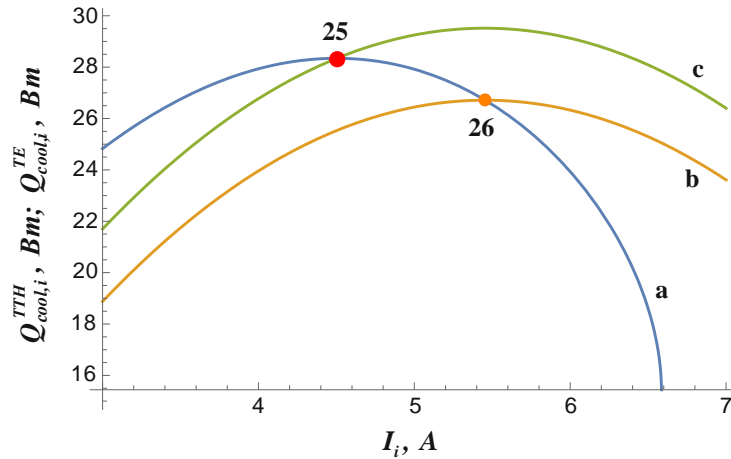

Fig. 3.2. Figure showing some peculiarities of cases 25 and 26 of example under study:
*a –plot $Q_{cool,i}^{TTH}(I_i)$ ($\Delta T_1^{MS} = const = 5\,K$, $V_C^{MS} \neq const$); b – plot of $Q_{cool,i}^{TE}$ versus I_i for fixed temperature values of heat-absorbing and heat-releasing thermoelement junctions of the i-th THP that correspond to case 26 of example under study; c – plot of $Q_{cool,i}^{TE}$ versus I_i for fixed temperature values of heat-absorbing and heat-releasing thermoelement junctions of the i-th THP that correspond to case 25 of example under study; point **25** corresponds to case 25 of example under study; point **26** corresponds to case 26 of example under study.*

The results of calculations for case 26 of example under study were obtained using a system of 5 equations, such as (3.6), (3.8), (3.10), (3.13) and a commonly known expression to determine the strength of current flowing through thermoelements that work in the mode of maximum cooling capacity $I_i^{Q_{cool,i,max}^{TE}}$ [8]:

$$I_i^{Q_{cool,i,max}^{TE}} = \frac{\alpha_i T_{cool,i}^{TE}}{r_i}.$$ (3.27)

In this case 26 $I_i = I_i^{Q_{cool,i,max}^{TE}}$.

Conclusions

From the calculations described in this paper it can be concluded that increase in the quantity of THPs used in the above described processes, in particular, can:

1) create the possibility of reducing the amount of thermoelectric material and/or the amount of material of heat exchangers of which all THPs are made, with constant energy efficiency of the respective process;

2) create the possibility of increasing the energy efficiency of the respective process with constant amount of thermoelectric material and/or material of heat exchangers of which all THPs are made;

3) create the possibility of increasing the energy efficiency of the respective process with a change (in particular, increase) in the amount of thermoelectric material and/or material of heat exchangers of which all THPs are made.

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Submitted 28.08.2019

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ОЦІНКА ЕФЕКТИВНОСТІ ЧАСТИННОГО ВИПАДКУ ПРОЦЕСІВ ТЕПЛОМАСООБМІНУ МІЖ ТЕПЛОВИМИ НАСОСАМИ І РУХОМОЮ РЕЧОВИНОЮ
Частина 3

Представлена теоретична модель для оцінки ефективності роботи частинного випадку процесів тепломасообміну між рухомою речовиною і термоелектричними тепловими насосами з їх теплообмінними частинами, при якому рухому речовину (або принаймні частину цієї рухомої речовини) приводять у тепловий контакт з теплопоглинальною і тепловиділяючою теплообмінними частинами принаймні двох термоелектричних теплових насосів, які можуть працювати в режимах, які можуть відрізнятися від режиму максимальної енергоефективності, зокрема, з врахуванням кількостей матеріалів, що необхідні для виготовлення цих термоелектричних теплових насосів. Наведені результати відповідної теоретичної оцінки. . Бібл. 9, табл. 1, рис. 2.

Ключові слова: тепловий насос, рухома речовина, тепломасообмін, ефективність, енергоефективність, термоелектричний тепловий насос, термоелементи, термоелектричний матеріал.

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ОЦЕНКА ЭФФЕКТИВНОСТИ ЧАСТНОМ СЛУЧАЕ ПРОЦЕССОВ ТЕПЛОМАССООБМЕНУ МЕЖДУ ТЕПЛОВЫХ НАСОСОВ И ДВИЖЕНИЕМ ВЕЩЕСТВОМ
Часть 3

Представлена теоретическая модель для оценки эффективности работы в частном случае процессов тепломассообмена между движущимся веществом и термоэлектрическими тепловыми насосами с их теплообменными частями, при котором движущееся вещество (или хотя бы часть этого движущегося вещества) приводят в тепловой контакт с теплопоглощающей и тепловыделяющей теплообменными частями по крайней мере двух термоэлектрических тепловых насосов, могущих работать в режимах, отличающихся от режима максимальной энергоэффективности, в частности, с учетом количеств материалов, необходимых для изготовления этих термоэлектрических тепловых насосов. Приведены

результаты соответствующей теоретической оценки.Библ. 9, табл. 1, рис. 2.

Ключевые слова: тепловый насос, подвижне вещество, тепломассообмен, КПД, энергоэффективность, термоэлектрический тепловой насос, термоэлементы, термоэлектрический материал.

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