CYBERNETICS and COMPUTER TECHNOLOGIES

УДК 519.1
DOI:10.34229/2707-451X.23.4.6

D. TERZI

A NATURAL APPROACH TO SOLVING THE TRAVELING SALESMAN PROBLEM

The research concerns the solution of the traveling salesman problem as a transport-type problem. The technology of the potentials method is used for the first time. Such a natural approach, due to the cyclicity and degeneracy of the solution to the traveling salesman problem, requires significant modification of the corresponding stages of the transport problem. An efficient algorithm for generating an initial cyclic solution has been developed; a criterion for the optimality of a solution is formulated; an algorithm for transition from the initial cyclic solution to another cyclic solution was constructed. To conduct mass computational experiments, an algorithm for constructing traveling salesman problems with a previously known optimal solution was used.

Keywords: *travelling salesman problem, method of potentials, optimality criterion, cyclic substitution, route, algorithm.*

Introduction. One of the tasks of researching transport-type operations is the traveling salesman problem [1–3]. It is natural to use the method of potentials to solve it. Consider the traveling salesman problem with cost matrix C of order n in the following interpretation. There are n points of departure for cargo G_i, $i = 1,...,n$ and shipment volumes for each point of consumption a_i, $i = 1,...,n$. Known need for cargo b_j, $j = 1,...,n$ for each of n destinations H_j, $j = 1,...,n$. The shipping cost for each option is c_{ij}, $i = 1,...,n$, $j = 1,...,n$.

It is necessary to calculate a transportation plan with minimal transportation costs under the condition that all a_i and b_j are equal to one unit cargo, and the set of pairs of (i, j) of points of departure i and destinations j of cargo constitute a cyclic permutation. One of the methods for solving the transport problem is the method of potentials [4–6], which includes the following steps: drawing up an initial feasible solution; calculation of potentials; checking the plan for optimality; search for the maximum value of the evaluation matrix; drawing up a cycle of reallocation of resources; determination of the minimum element in the redistribution contour and redistribution of resources along the contour; obtaining a new feasible solution. These stages are modified taking into account the restrictions of cyclicity and degeneracy of an admissible solution to the traveling salesman problem. The algorithm is carried out in several iterations until the optimal solution is found.

1. Algorithmization of the solution of the traveling salesman problem using the technology of the method of potentials. When solving the traveling salesman problem by the method of potentials, it is necessary to take into account the differences between the traveling salesman problem and the usual transport problem at all stages of the algorithm. At the first stage of drawing up the initial plan, it is necessary to take into account that all volumes of sending $a_i = 1$, $i = 1,...,n$ and cargo needs at destinations $b_j = 1$, $j = 1,...,n$. All deliveries for each option, i.e. part of the x_{ij} variables, $i=1,...,n$ and $j=1,...,n$, that make up the feasible basic solution x of the traveling salesman problem, as in the transport problem, must be positive values.

Next we will consider basic solutions consisting of 2*n*–1 variables.

One part of the decision variables *x* is equal to one, we will say that this is a set of ones. These variables make up the traveling salesman route. The set of ones is given by the cyclic substitution

$$(x,y)=\begin{pmatrix} x_1x_2...x_n \\ y_1y_2...y_n \end{pmatrix}. \tag{1}$$

If $r=x_i$, $s=y_j$, , then $x_{rs}=1.$ The number of such variables, based on the specifics of the problem, is *n*.

The other part of the variables of the feasible solution *x*, in the amount of *n*–1, are equal to zero, and these zeros are not ordinary, but fictitious, in order to prevent the degeneracy of the solution. Let's call this set the set of fictitious zeros and denote it by $(x\varepsilon, y\varepsilon)=\begin{pmatrix} x\varepsilon_1 x\varepsilon_2...x\varepsilon_{n-1} \\ y\varepsilon_1 y\varepsilon_2...y\varepsilon_{n-1} \end{pmatrix}$.

If $r=xe_i$, $s=ye_j$, then $x_{rs}=\varepsilon,$ where ε is a sufficiently small positive number.

A cyclic permutation of $j_1 j_2 j_3 ... j_k j_{k+1} ... j_n$ corresponds to a traveling salesman route, i.e. a sequence of detour points, which corresponds to a type substitution

$$\begin{pmatrix} j_1 j_2 j_3 ... j_k j_{k+1} ... j_n \\ j_2 j_3 ... j_k j_{k+1} ... j_n j_1 \end{pmatrix}. \tag{2}$$

The substitution structure (2) shows one of the ways to generate *n* units of the initial cyclic feasible solution to the traveling salesman problem: generating a random permutation $j_1 j_2 j_3 ... j_k j_{k+1} ... j_n$, i.e. row numbers *x*, and the formation of a permutation of $j_2 j_3 ... j_k j_{k+1} ... j_n j_1$, representing the numbers of the *y* columns, as in (1). So, we determine the location of the units (*x*, *y*). The following algorithm 1 is used to generate a set of fictitious zeros (*xe*, *ye*) that provide the determination of all 2*n* potentials.

1.1. Algorithm 1

1. We determine the matrix *x*, in which the numbers *M* are located diagonally – sufficiently large positive numbers, and in the positions determined by the set (*x*, *y*), there are units; further, we supplement the matrix *x* to the matrix of an admissible solution;

2. Determine the first potential $u_1=0$ and the second potential $v_j=c_{1j}$, where *j* is the number of the column in which there is one in the first row of the matrix *x*, $x_{1j}=1$;

3. Further in the cycle for *k*, *k* = 1,2,…,*n*–1, each time, we determine two potentials u_{i_k} and v_{j_k}, we also enter one fictitious zero in the row with a random number i_k from the row numbers that do not yet contain a fictitious zero, and in the column j_{k-1}:

$$\begin{gathered} u_{i_k}=c_{i_k j_{k-1}}-v_{j_{k-1}}, \\ x_{i_k j_{k-1}}=\varepsilon, \\ v_{j_k}=c_{i_k j_k}-u_{i_k}. \end{gathered} \tag{3}$$

Remark 1. The indices of the last and first potentials determine the position of the additional *n*-th fictitious zero, which can be used as previously defined fictitious zeros.

Remark 2. Algorithm 1, as a rule, generates a cyclic substitution from positions of *n* fictitious zeros. If the first attempt does not result in a cyclic substitution, you can make the next attempt until one is obtained. Another variant of Algorithm 1 for obtaining a cyclic substitution is to enumerate possible options for calculating some 2–3 last potentials.

The generation of fictitious zeros essentially determines the quality of the initial solution, i.e. remoteness of the initial solution from the optimal one.

Theorem 1. Let (*xn*, *yn*) be any initial cyclic route of the traveling salesman, and (*xo*, *yo*) be the optimal cyclic route. There is a set $(x\varepsilon, y\varepsilon)$ of *n*–1 fictitious zeros such that, given an admissible solution *x*, consisting of *n* elements (*xn*, *yn*) and *n*–1 fictitious zeros $(x\varepsilon, y\varepsilon)$, it is possible to construct a transition cycle from the initial (*xn*, *yn*) to the optimal route (*xo*, *yo*).

Proof. Suppose we have generated an initial cyclic route

$$(xn, yn) = \begin{pmatrix} xn_1 xn_2 ... xn_n \\ yn_1 yn_2 ... yn_n \end{pmatrix}, \tag{4}$$

and the optimal route has the form

$$(xo, yo) = \begin{pmatrix} xo_1 xo_2 ... xo_n \\ yo_1 yo_2 ... yo_n \end{pmatrix}, \tag{5}$$

in which possibly *s* positions of units (4) coincide with *s* positions of units of the optimal route (5).

Since the sequence of writing pairs $\begin{pmatrix} xn_i \\ yn_i \end{pmatrix}$ and $\begin{pmatrix} xo_j \\ yo_j \end{pmatrix}$ in (4) and (5), respectively, is not significant, we can assume, for s not equal to zero, that the sets $\begin{pmatrix} xn_1 xn_2 ... xn_s \\ yn_1 yn_2 ... yn_s \end{pmatrix}$ and $\begin{pmatrix} xo_1 xo_2 ... xo_s \\ yo_1 yo_2 ... yo_s \end{pmatrix}$ match. For the time being, we exclude these *s* units from consideration, and for the remaining

$$\begin{pmatrix} xn_{s+1} xn_{s+2} ... xn_n \\ yn_{s+1} yn_{s+2} ... yn_n \end{pmatrix} \text{ and } \begin{pmatrix} xo_{s+1} xo_{s+2} ... xo_n \\ yo_{s+1} yo_{s+2} ... yo_n \end{pmatrix}.$$

Let's make a chain from the first unit of the initial route to the unit along the line of the optimal route, and then along the column from the unit of the optimal route to the unit of the initial route.

Having made such a transition *n*–*s* times from the unit of the initial route to the unit of the optimal route, we close the loop. Again, referring to the arbitrariness of the sequence of writing the positions of units in (4) and (5), we can write the resulting cycle as follows:

$$\begin{pmatrix} xn_{s+1} \\ yn_{s+1} \end{pmatrix} \begin{pmatrix} xo_{s+1} \\ yo_{s+1} \end{pmatrix} \begin{pmatrix} xn_{s+2} \\ yn_{s+2} \end{pmatrix} \begin{pmatrix} xo_{s+2} \\ yo_{s+2} \end{pmatrix} ... \begin{pmatrix} xn_n \\ yn_n \end{pmatrix} \begin{pmatrix} xo_n \\ yo_n \end{pmatrix}. \tag{6}$$

Now we will single out one of the even positions, for example, $\begin{pmatrix} xo_j \\ yo_j \end{pmatrix}$, as a free variable, and in the remaining even positions (they are in the amount of *n*–*s*–1) from (6) we will put fictitious zeros in the matrix *x*. At the end, we introduce *s* more fictitious zeros, arbitrarily chosen, but complementing all even positions (6) to a cyclic set $(x\varepsilon, y\varepsilon)$. At the end, we introduce *s* fictitious zeros, arbitrarily chosen, but complementing all even positions (6) to the cyclic set $(x\varepsilon, y\varepsilon)$.

The admissible solution of the transport problem constructed in this way, starting from the free variable $\begin{pmatrix} xo_j \\ yo_j \end{pmatrix}$ and going through the cycle (6), transforms the initial route into the optimal route, consisting of *s* units in coinciding positions and *n*–*s* units in even positions (6).

Let us demonstrate Theorem 1 by an example. Assume that the initial route, i.e. set of units (sequence of bypass points) $(xn, yn) = \begin{pmatrix} 83675124 \\ 36751248 \end{pmatrix}$ and the second optimal route $(xo, yo) = \begin{pmatrix} 63785124 \\ 37851246 \end{pmatrix}$. The common positions of the routes are (1,2), (2,4) and (5,1), s = 3.

For the remaining positions of units in both routes, we build a cycle. Let's start, for example, from position (3,6) and further along the row, and then along the column, etc., we get a cycle

(3,6), (3,7), (6,7), (6,3), (8,3), (8,5), (7,5), (7,8), (4,8), (4,6).

Let us replace the transition (3,6) with (3,7), (6,7) with (6,3), (8,3) with (8,5), (7,5) with (7,8) and (4,8) to (4,6). As a result of such replacements, taking into account the coinciding s positions, we get the transition from the cyclic route (xn, yn) to the optimal cyclic route (xo, yo).

Theorem 1 shows the possibility of solving the traveling salesman problem, as a transport type problem, method of potentials. To further explain the corresponding Algorithm 2, we will continue to follow the steps of solving the usual transportation problem in relation to solving the traveling salesman problem.

At the second stage, according to the admissible solution x of the traveling salesman problem as a transport problem, i.e. for x, compiled by Algorithm 1 from a cyclic route (xn, yn) and a set of fictitious zeros $(x\varepsilon, y\varepsilon)$, allowing you to calculate all $2n$ potentials u_i and v_j, the matrix of estimates is determined

$$m = \{m_{ij}\}, \tag{7}$$

where $m_{ij} = u_i + v_j - c_{ij}$, $i, j = \overline{1,n}$.

The so-defined evaluation matrix m, we will say that corresponds to the route (xn, yn) and set of fictitious zeros $(x\varepsilon, y\varepsilon)$.

At the stage of estimating the optimality of the solution to the traveling salesman problem, the following theorem is used.

Theorem 2. Let (xn, yn) be a cyclic route and $(x\varepsilon, y\varepsilon)$ be the set of fictitious zeros used to calculate the evaluation matrix m. In order for (xn, yn) to be an optimal solution to the traveling salesman problem, it is necessary that the sum S of estimates on this route be nonpositive, i.e.

$$S = \sum_{i=1}^{n} m_{xn_i yn_i} \le 0 . \tag{8}$$

Proof. Let us assume that (xn, yn) is the optimal solution and at the same time $S > 0$.

Let us calculate the difference between the value of the objective function on the route (xn, yn) and the sum of the estimates on this route,

$$\sum_{i=1}^{n} c_{xn_i yn_i} - S = \sum_{i=1}^{n} u_{xn_i} + \sum_{i=1}^{n} v_{yn_i} - \sum_{i=1}^{n} m_{xn_i yn_i} =$$

$$= \sum_{i=1}^{n} u_{xn_i} + \sum_{i=1}^{n} v_{yn_i} - (\sum_{i=1}^{n} u_{xn_i} + \sum_{i=1}^{n} v_{yn_i} - \sum_{i=1}^{n} c_{xn_i yn_i}) = \sum_{i=1}^{n} c_{xn_i yn_i} .$$

We have obtained a contradiction, which means that the sum of estimates S must not be positive.

Theorem 3. Condition (8) is sufficient for the optimality of (xn, yn) provided that $(x\varepsilon, y\varepsilon)$ is any set of fictitious zeros.

Proof. According to the admissible solution *x*, formed from two sets (*xn*, *yn*) and $(x\varepsilon, y\varepsilon)$, for the corresponding transport problem, we calculate the matrix of estimates *m*. Assume that the optimal route is not (*xn*, *yn*), but some other route (xd, yd) with $S \le 0$ with the same set of fictitious zeros $(x\varepsilon, y\varepsilon)$.

Then the following equalities hold

$$\sum_{i=1}^{n} c_{xn_i yn_i} - S = \sum_{i=1}^{n} u_{xn_i} + \sum_{i=1}^{n} v_{yn_i} - \left(\sum_{i=1}^{n} u_{xd_i} + \sum_{i=1}^{n} v_{yd_i} - \sum_{i=1}^{n} c_{xd_i yd_i} \right) = \sum_{i=1}^{n} c_{xd_i yd_i} .$$

Hence from the optimality of (xd, yd) and from the fact that $S \le 0,$ follows the optimality of the route (*xn*, *yn*).

Thus, the stage of searching for the maximum value of the assessment matrix when solving the transport problem is replaced by searching for the maximum value of the sum of the assessment matrix on a given cyclic route (*xn*, *yn*) when solving the traveling salesman problem.

Other stages of solving the transport problem by the method of potentials, namely, drawing up a cycle of resource redistribution, determining the minimum element in the redistribution contour and redistributing resources along the contour, obtaining a new feasible solution, are replaced, when solving the traveling salesman problem, with the route search stage (*xn*, *yn*) with the maximum value of the sum of estimates on it. If, as a result of the analysis of the estimates of the matrix m, there is an (xd, yd) route with a total estimate $S > 0$, then such a route is better than the initial one.

As for solving any transport type problem, it is necessary to use some way to generate an initial solution. Considering the specifics of the traveling salesman problem, the following efficient algorithm 2 is proposed.

1.2. Algorithm 2

1. Ordering the matrix with in ascending order. We get data set *D* in the form

$$c_{i_r i_s}, i_r, i_s,\ r \ne s,\ r, s = \overline{1, n} .$$

2. Construction of a cyclic permutation of $(i_1, i_2, ..., i_n)$ for each element of the $c_{i_r i_s}$ from *D*.

The first transition (i_1, i_2) is determined from the equality $c_{i_1 i_2} = c_{i_r i_s}$ whence it follows $i_1 = i_r$ and $i_2 = i_s$.

3. If the first *t*–1 pairs of $(i_1, i_2), ..., (i_{t-1}, i_t)$ are defined, then the next pair of (i_t, i_{t+1}) is determined from the condition: the $c_{i_t i_{t+1}}$ element is the first element (in the original ordered set *D*) for which the i_{t+1} is not equal to any of the $i_1, i_2, ..., i_t$ numbers.

4. From all *n*(*n*–1) cyclic permutations constructed in steps 2 and 3, a permutation with a minimum value of the objective function is selected, which is taken as the initial solution.

Based on the foregoing, Algorithm 3 is constructed to find a solution to the traveling salesman problem.

1.3. Algorithm 3

1. Input of initial data: cost matrices *c* and its dimension *n*.

2. Generation of the initial route in the form of a cyclic substitution (*xn*, *yn*), for example, using algorithm 2. Simultaneously, we form a matrix *x* of size *n* with elements equal to zero, except for $x(xn_i, yn_i) = 1$, $i = \overline{1, n}$.

3. We determine *f*, the cost of the route, i.e. the value of the objective function on the substitution (xn, yn), $f = \sum_{i=1}^{n} c_{xn_i yn_i}$ and remember the values of *f* and (xn, yn), $fop=f$, $xop=xn$, $yop=yn$.

4. We perform a predetermined number of iterations of finding a better route than (*xop*. *yop*).

4.1. Given (*xn*, *yn*) and *c*, we generate a set of fictitious zeros $(x\varepsilon, y\varepsilon)$ so that by the matrix *x*, complemented by the values $x(x\varepsilon_i, y\varepsilon_i) = \varepsilon, i = \overline{1,n}$ it was possible to calculate all 2*n* potentials $u_i, v_j, i, j = \overline{1,n}$, ε is a fictitious zero, a sufficiently small positive number.

4.2. For the values of *n*, *c*, *u*, *v* now determined, we calculate the matrix of estimates

$$m = \{m_{ij}\},\ i, j = \overline{1,n},$$

$$m_{ij} = \begin{cases} u_i + v_j - c_{ij},\ i \neq j \\ -M,\ i = j \end{cases},$$

where *M* is a sufficiently large positive number.

In the evaluation matrix *m*, all $m(xn_i, yn_i)$, $i = \overline{1,n}$ and $m(x\varepsilon_j, y\varepsilon_j)$, $j = \overline{1, n-1}$ elements are equal to zero.

The remaining elements are positive or negative numbers. If there are no positive elements, then the route (*xn*, *yn*) is optimal, and the calculations end. In the presence of both positive and negative elements, as in solving the transport problem, a search is made for the best route.

4.3. Search for a route with a positive sum of ratings.

4.3.1. After sorting all the values of the assessment matrix *m*, except for the diagonal ones, in descending order, we get the following data:

$$y_k,\ iy_k,\ jy_k,\ k=1, 2, \ldots, n(n-1), \tag{9}$$

where $y_k = m(iy(k), jy(k))$. Let's put k=0.

4.3.2. For the *k*+1 iteration, a cyclic substitution

$$(xn, yn) = \begin{pmatrix} i_1 i_2 \ldots i_{s-1} i_s \ldots i_n \\ i_2 i_3 \ldots i_s i_{s+1} \ldots i_1 \end{pmatrix} \tag{10}$$

is constructed, where the first (i_1, i_2) pair coincides with the (iy_r, jy_r) pair from (9).

Let's assume that the first *s*–1 pairs $(i_1, i_2), (i_2, i_3), \ldots, (i_{s-1}, i_s)$ in (10) are defined. Next couple (i_s, i_{s+1}), at *www*, coincides with the pair (iy_r, jy_r) from (9) under the conditions that

a) iy_r does not match any of $i_1, i_2, \ldots, i_{s-1}$;

b) jy_r does not match any of $i_1, i_2, \ldots, i_s$.

For $s = n$ pair (i_s, i_{s+1}) coincides with pair (i_n, i_1) with i_n and i_1 already defined.

4.3.3. For the constructed route (*xn*, *yn*), the score *S* is calculated as the sum of the elements of the evaluation matrix *m*,

$$S = \sum_{i=1}^{n} m(xn_i, yn_i).$$

If $S > 0$, then go to item 3, otherwise go to item 4.3.2.

5. If at step 4.3.2 it was not possible to find the best route, with an estimate of *S* > 0, then go to step 2, to perform the next iteration.

6. Upon completion of the specified number of iterations, at the output we get the route with the minimum cost, obtained as a result of selection from the routes generated by the algorithm.

At the stages of constructing the initial cyclic solution and moving from the initial to the new cyclic solution, the following algorithm is applied, along with Algorithm 3, in relation to the original matrix *c* and the evaluation matrix *m*.

Algorithm 4. Let a cyclic permutation $I=(i_1,i_2,i_3,\dots,i_n)$ be given. Let's form pairs

$$(i_1,i_2),(i_2,i_3),(i_3,i_4),(i_4,i_5),\dots,(i_{n-1},i_n),(i_n,i_{n+1}),\quad i_{n+1}=i_1, \tag{11}$$

which make up the route, and take from them any three pairs

$$(i_{k_1},i_{s_1}),(i_{k_2},i_{s_2}),(i_{k_3},i_{s_3}). \tag{12}$$

Let's replace pairs (12) with pairs

$$(i_{k_1},i_{s_2}),(i_{k_2},i_{s_3}),(i_{k_3},i_{s_1}) \tag{13}$$

substitute them into (11) and get a new route

$$(j_1,j_2),(j_2,j_3),(j_3,j_4),(j_4,j_5),\dots,(j_{n-1},j_n),(j_n,j_{n+1}),\quad j_{n+1}=j_1, \tag{14}$$

that corresponds to a cyclic permutation corresponds to a cyclic permutation $J=(j_1,j_2,j_3,\dots,j_n)$.

Comparing values $f_1=\sum_{k=1}^{n}c(i_k,i_{k+1})$ and $f_2=\sum_{k=1}^{n}c(j_k,j_{k+1})$ and choose the best of solutions *I* and *J*, minimum for matrix *c*. In the case of applying algorithm to matrix *m*, we select the maximum value.

Calculations according to Algorithm 3 were performed with traveling salesman problems, which were generated with a pre-selected optimal solution. The following question was considered. Assume that for the traveling salesman problem A with cost matrix *c*, the optimal solution is (*xa*, *ya*). What needs to be changed in matrix *c* to get the traveling salesman problem *B* with cost matrix *d* and optimal solution (*xb*, *yb*)? We formulate the answer in the form of Theorem 4 and give its constructive proof.

Theorem 4. Let the *c* matrix of dimension *n* consist of non-negative integers. On the basis of matrix *c*, one can construct a traveling salesman problem with matrix *d* with a pre-selected optimal solution (*xb*, *yb*).

Proof. Given the matrix *c* and the solution (*xb*, *yb*), we determine (for example, by algorithm 1) a set of *n*–1 fictitious zeros. Let us compose a matrix *x*, in which some of the elements are equal to one, $x(xb,yb)=1$, $i=\overline{1,n}$, another part of the elements from fictitious zeros, another part of the elements from fictitious zeros, $x(x\varepsilon_j,y\varepsilon_j)=\varepsilon$, $j=\overline{1,n-1}$, ε – a small enough positive number. We set the diagonal elements of the matrix *x* to be equal to the *M* – a large enough positive number $x(i,i)=M, i=\overline{1,n}$. The remaining elements of the matrix *x* are equal to zero. From the matrix *c* and *x*, we calculate the potentials $u_i, i=\overline{1,n}$, and $v_j, j=\overline{1,n}$, and the corresponding evaluation matrix *m*,

$$m=m(i,j)=\begin{cases}u_i+v_j-c_{ij}, i\neq j,\\ -M,\ \ i=j\end{cases},\ i,j=\overline{1,n}.$$

The matrix *d* of the traveling salesman problem with the optimal solution to (xb,yb) is determined by the formula

$$d=d(i,j)=\begin{cases}u_i+v_j+\alpha_{ij},\ at\ i\neq j, x(i,j)=0, m(i,j)>0\\ c_{ij},\ \text{otherwise}\end{cases},$$

where α_{ij} are non-negative integer random numbers.

The optimality of the (xb, yb) solution follows from the fact that, according to the optimality criterion of Theorem 2, it is impossible to obtain a positive total estimate for any cyclic solution.

Let us give one example and the results of some experiments with Algorithm 3.

2. Computational experiments

Example. Let the cost matrix, dimension n=8, for the traveling salesman problem be

c = [*M* 76 43 38 51 42 19 80; 42 *M* 49 26 78 52 39 87; 48 28 *M* 36 53 44 68 61; 72 31 29 *M* 42 49 50 38; 30 52 38 47 *M* 64 75 82; 66 51 83 51 22 *M* 37 71; 77 62 93 54 69 38 *M* 26; 42 58 66 76 41 52 83 *M*],

Starting route $(xn, yn) = \begin{pmatrix} 65174832 \\ 51748326 \end{pmatrix}$ has a value of $f = \sum_{i=1}^{n} c(xn_i, yn_i) = 309.$ We generate *n*–1 fictitious zeros, for example, by algorithm 1. $(x\varepsilon, y\varepsilon) = \begin{pmatrix} 2345678 \\ 3158467 \end{pmatrix}$, then we calculate the potentials and the evaluation matrix *m*,

m = [-*M* -160 -41 -17 -59 -37 0 -92; -59 -*M* 0 42 -39 0 27 -52; 0 0 -*M* 97 51 73 63 39; -86 -65 23 -*M* 0 6 19 0; 0 -42 58 68 -*M* 35 38 0; -100 -105 -51 0 0 -*M* 12 -53; -108 -113 -58 0 -44 0 -*M* -5; -42 -78 0 9 15 17 0 -*M*];

At the first iteration, from the analysis of the matrix *m*, we obtain a positive sum of estimates $S = \sum_{i=1}^{n} m(xn_i, yn_i) = 2$ on the $(xo, yo) = \begin{pmatrix} 17653248 \\ 76532481 \end{pmatrix}$ route. In this case, the new value of the objective function is equal to the difference between the previous value of *f* and the sum of the estimates *S*, $f - S = 307.$ A new iteration is carried out with the best solution. If *S* <= 0, then we start a new iteration with the previous solution. So, after the 13th iteration, we arrive at the optimal solution of the traveling salesman problem (xo, yo) with *aaa* value *f*=251. This means that by calculating the evaluation matrix *m* for the optimal solution (xo, yo) and any set of fictitious zeros , we will not get any solution of the traveling salesman problem (xn, yn) with a positive total estimate $S = \sum_{i=1}^{n} m(xn_i, yn_i),$ despite the presence of positive elements in the assessment matrix *m*, as in solving the transport problem.

In the experiments, the numbers of α_{ij} were taken from the range from zero to 50. For the optimal solutions obtained for problems with $n \le 16$ in no more than 100 iterations, the $\Delta f = \left(f - f_{opt} \right) / f_{opt}$ score did not exceed fifty percent. With an increase in the number of iterations, the number of solved problems for which $\Delta f = 0$ increases.

Conclusions.

The results of computational experiments show that the use of potential method technology to solve the traveling salesman problem, as a special transport problem, is a new promising direction for searching for a high-quality solution. Since the traveling salesman problem belongs to the class of *NP* difficult-to-solve problems and it is not yet known whether the equality *P*=*NP* holds, then all algorithms for solving it, in general, should be considered approximate, heuristic.

References

1. Grötschel M. The Travelling Salesman Problem and its Applications. *Combinatorial Optimization at Work*. Block Course at TU Berlin, October 4 – 15, 2005. https://co-at-work.zib.de/berlin/download/CD/Talks/02M1-TSP.pdf
2. Taillard E.D. Design of Heuristic Algorithms for Hard Optimization, With Python Codes for the Travelling Salesman Problem. Springer, 2023. https://doi.org/10.1007/978-3-031-13714-3
3. Roberti R., Toth P. Models and algorithms for the Asymmetric Traveling Salesman Problem: An experimental comparison. *EURO Journal on Transportation and Logistics*. 2012. **1** (1–2). P. 113–133. https://doi.org/10.1007/s13676-012-0010-0
4. Hasan-Shawopn M.F., Raval N. Operations research: Traveling Salesman Problem. Technical Report. Wernigerode, Germany, 2023. 57 p. https://www.researchgate.net/publication/369973661
5. Golstein E.G., Yudin D.B. Problems of linear programming of transport type. M.: Nauka, 1969. 382 p. (in Russian)
6. Sharma J.K. Operations Research: Theory and Applications. Macmillan Publishers India Limited, 2009. 976 p. https://books.google.com.ua/books?id=1EZxJHO32swC

Received 21.11.2023

Dmitri Terzi,
doctor of science, senior researcher, associated professor of
Moldova State University, Chisinau, Moldova.
https://orcid.org/0000-0002-1518-4012
terzidg@gmail.com

MSC 90C10, 90C27, 90C90

Dmitri Terzi

A Natural Approach to Solving the Traveling Salesman Problem

Moldova State University, Chisinau, Moldova
Correspondence: terzidg@gmail.com

Introduction. The traveling salesman problem is a transport-type problem. It is natural to use a method based on the technology for solving transport problems to solve it. The cyclicity and degeneracy of the solution to the traveling salesman problem requires significant modification of the corresponding stages of solving the transport problem (drawing up an initial feasible solution; checking the plan for optimality; obtaining a new feasible solution).

Purpose. Development of a natural approach to solving the traveling salesman problem. Description of the structure of a set of traveling salesman problems that have a predetermined optimal solution. Algorithmic formation of such problems for the purpose of conducting mass computing experiments.

Results. The paper presents new results and computational experiments with a developed natural algorithm for solving the traveling salesman problem, based on the technology for solving transport problems, including a new effective method for generating an initial cyclic solution, an algorithm for transitioning from the initial cyclic to another, also cyclic, solution. An algorithm has been developed for constructing the traveling salesman problem with an optimal solution given in advance, which allows for a better understanding of the structure of traveling salesman problems.

Conclusions. The results of computational experiments show that the use of potentials method technology for solving the traveling salesman problem, as a special transport problem, is a promising direction for searching for a high-quality solution. The developed algorithms and programs expand the possibilities of solving the traveling salesman problem. The time it takes to solve a problem depends significantly on the size of the problem. In this regard, it is essential to automatically generate the traveling salesman problem with a given optimal solution, which allows you to conduct mass experiments and draw conclusions.

Keywords: travelling salesman problem, method of potentials, optimality criterion, cyclic substitution, route, algorithm.

УДК 519.1

Д.Г. Терзі

Природний підхід до розв'язання задачі комівояжера

Державний університет Молдови, Кишинів
Листування: terzidg@gmail.com

Вступ. Задача комівояжера відноситься до задач транспортного типу. Природно до її розв'язання застосувати метод, заснований на технології розв'язування транспортних задач. Циклічність і виродженість розв'язку задачі комівояжера вимагає суттєвої модифікації відповідних етапів розв'язування транспортної задачі (складання початкового допустимого розв'язку; перевірка плану на оптимальність; отримання нового допустимого розв'язку).

Мета роботи. Розробка природного підходу до розв'язання задачі комівояжера. Опис структури множини задач комівояжера, що мають заданий наперед оптимальний розв'язок. Алгоритмічне формування таких задач із проведенням масових обчислювальних експериментів.

Результати. У роботі представлені нові результати та обчислювальні експерименти з розробленим природним алгоритмом розв'язання задачі комівояжера, заснованим на технології розв'язування транспортних задач, що включає новий ефективний спосіб генерації початкового циклічного розв'язку, алгоритм переходу від початкового циклічного до іншого. Розроблено алгоритм побудови задачі комівояжера із заданим наперед оптимальним розв'язком, що сприяє кращому розумінню структури задач комівояжера.

Висновки. Результати обчислювальних експериментів показують, що використання технології методу потенціалів для розв'язання задачі комівояжера, як спеціальної транспортної задачі, є перспективним напрямом пошуку якісного розв'язку. Розроблені алгоритми та програми розширюють можливості розв'язання задачі комівояжера. Час розв'язання задачі суттєво залежить від розмірності задачі. У цьому плані істотним є автоматичне формування задачі комівояжера із заданим оптимальним розв'язком, що дозволяє проводити масові експерименти та зробити висновки.

Ключові слова: задача комівояжера, метод потенціалів, критерій оптимальності, циклічна підстановка, маршрут, алгоритм.