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ON ONE IMPLEMENTATION OF A NATURAL APPROACH TO SOLVING THE TRAVELING SALESMAN PROBLEM

The work constructs a new computational scheme for solving the traveling salesman problem as a transport-type problem. The technology used is based on a modified distribution method. The concept of a three-element replacement operation is introduced, which is used to formulate the necessary condition for the optimality of a solution. The search for the optimal solution begins with a random cyclic route. The solution is then improved by applying a three-dimensional replacement operation. There is a transition from the current cyclic solution to a new cyclic solution with a better value of its cost. The search for an optimal solution ends when the three-dimensional replacement operation fails after a given number of iterations. Computational experiments have shown the possibility of obtaining an acceptable solution for problems of a sufficiently large size. This work represents a further development of the natural approach to solving the traveling salesman problem.

Keywords: *modified distribution method, three-element replacement operation, natural approach, traveling salesman problem.*

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Introduction. The traveling salesman problem has been the object of numerous studies, which resulted in works concerning the development of heuristic methods, the branch and bound method, the genetic method, the ant colony method, the use of neural networks, etc. [1–4]. The versatility of approaches to solving the traveling salesman problem is relevant for creating a complete theory of it.

The paper presents one of the implementations of the idea of using a modified distribution method (potentials method) to solve the traveling salesman problem. This approach is natural, since the traveling salesman problem is a transport-type problem.

Direct application of the modified distribution method to solve the traveling salesman problem is impossible, since the corresponding transport problem is degenerate and, in addition, due to the requirement that the solution be cyclic. Eliminating degeneracy and taking into account the cyclicity of an admissible solution is a prerequisite for the development of algorithms that use computer technology for solving the traveling salesman problem using the specified method.

1. Modification of the potentials method in solving the traveling salesman problem

The use of the potentials method as a type of method of sequential improvement of the plan in relation to the transport problem is possible by modifying the corresponding stages of solving the transport problem, including the generation of the initial feasible solution, calculation of the evaluation matrix, assessment of the optimality of the solution, and transition to a new feasible solution.

The necessary changes, in view of the requirement of cyclicity and non-degeneracy of the solution to the traveling salesman problem as a transport problem, are set out further in Algorithms 1–3 (sometimes using MATLAB language tools [5]), are as follows.

At the initial stage, a cyclic solution (route) is generated. The degeneracy of the solution is eliminated by a special method of introducing *n*–1 fictitious zeros ε (ε is a fairly small value of the transported goods).

The potentials are calculated simultaneously with the introduction of fictitious zeros. The introduction of a fictitious zero in position (*i*, *j*) is associated with the simultaneous calculation of the potential v_j for column j and the potential u_i for row *i*, which ensures the non-degeneracy of the initial cyclic solution. Then the evaluation matrix, $m_{ij} = u_i + v_j - c_{ij}$, is calculated.

The use of the matrix *m* to assess the optimality of the current solution, in the case of solving the traveling salesman problem, is carried out not according to the criterion of non-positive elements of m, but in a different way due to the possibility of breaking the duality [3].

At each iteration of the algorithm, a transition occurs from a cyclic solution (route) (*xv*, *yv*), using the replacement of its three constituent elements (points), to a new cyclic solution (*xv*2, *yv*2).

Definition. The operation of transition from one cyclic solution, with the replacement of three elements in it, to another cyclic solution, will be called a three-element replacement operation.

The use of a three-element replacement operation is intended to move from one cyclic solution to another cyclic solution, as in the fbsol function:

[*fopt*, *xv*2, *yv*2, *cucr*]=***fbsol*** (*n*, *c*, *x*, *xv*, *yv*, *muv*, *Inf*).

With the completion of Algorithm 1, the best cyclic solution is determined, found in a certain number of iterations with different initial cyclic solutions.

2. Description of algorithms and functions

The description of the main algorithm 1 includes a function for generating the initial route and the initial cyclic solution, a function for calculating the evaluation matrix, and a function for transitioning from the initial cyclic solution to a new cyclic solution using a three-element replacement operation.

Algorithm 1 and its constituent functions

a) An initial random cyclic route is formed (*xv*, *yv*),

xv = ***randperm***(*n*); *yv* = [*xv*(2:*n*), *xv*(1)],

auxiliary matrix *X*,

X(1:*n*,1:*n*) = 0; *for i* = 1:*n* *X*(*xv*(*i*), *yv*(*i*)) = 1; *X*(*i*,*i*) = *Inf*;

and solution

xvop = [*xv*]; *yvop* = [*yv*] *with fop* = *f*,

where $f = \sum_{i=1}^{n} c(xv(i), yv(i))$;

b) Use the ***fmdr*** function,

[*u*, *v*, *x*, *j*1, *puv*] = ***fmdr*** (*n*, *X*, *c*),

with parameters *n*, *X* and *c* for simultaneous calculation of an admissible solution *x* and the corresponding potentials u_i and v_j, *i* = 1,2,..., *n*, *j* = 1,2,..., *n*. The fmdr function ensures that the solution *x* is ondegenerate and that all 2*n* potentials can be determined. We calculate the matrix of estimates *muv* and *tm* – the number of positive elements *tm* in it:

[*muv*, *tm*]=***MatrPot*** (*n*, *x*, *c*, *u*, *v*).

If the evaluation matrix does not contain positive elements, *tm* = 0, then solution *x* is optimal. Such a case may occur, for example, when the optimal solution to the assignment problem with matrix *c* is a cyclic solution;

c) Otherwise, which happens more often when duality breaks, when *tm* > 0, we continue searching for the best solution *x* using the function ***fbsol***,

$$[fopt, xv2, yv2, cucr] = \boldsymbol{fbsol}\,(n, c, x, xv, yv, muv, Inf);$$

The output parameters mean the best cyclic solution ($xv2$, $yv2$) obtained at the current iteration, with the value of the objective function fopt after cucr improvements of the cyclic solution.

If $fopt < fop$, then we form the best solution for all iterations

$$xvop = [xv2];\ yvop = [yv2]$$

with $fop = fopt$;

d) If $cucr = 0$, calculations end, otherwise start the next iteration, in which as the initial solution we take the best solution found

$$xv = [xvop],\ yv = [yvop]$$

and go to step b).

Let us describe the content of the functions used.

Algorithm 2 for constructing a non-degenerate cyclic solution

The formation of the initial cyclic solution x is carried out by the function fmdr,

$$[u, v, x] = fmdr(n, X, c).$$

a) At the first iteration, having formed a random cyclic substitution (xv,yv) and its matrix representation X, we determine the first zero potential $u(1)$ and the k-th potential $v(k) = c(1,k)$, where $X(1, k) = 1$;

b) At the next iterations, in one of the rows with a random number r, $r = 2,...,n$, for which the potential $u(r)$ has not yet been determined, we introduce one fictitious zero in column k and row r, $X(r,k) = \varepsilon$, and calculate the potential $u(r) = c(r,k) – v(k)$. We find in row r the number s of the element $X(r,s)$ equal to one, and set $v(s) = c(r,s) – u(r)$;

c) Taking as k the number of the last column s with the already determined potential $v(s)$, we go to step b).

Algorithm 3. Transition from a cyclic solution (*xv*, *yv*) to a new cyclic solution

Let the admissible cyclic solution x and the evaluation matrix muv be known.

a) For each zero element $x(i, j)$, which corresponds to a positive value of the evaluation matrix muv, we find in row i and column jc a matrix element x equal to one, $x\ (i, jc) = 1$. Also, using column j we find the row ic with the identity element, $x(ic, j) = 1$.We put $a1 = i$, $b1 = jc$, $a2 = ic$, $b2 = j$;

b) In the loop, for each $i1 = 1 : n$, with $x(xv(i1), yv(i1)) = 1$ and $(xv(i1), yv(i1))$ not matching $(a1,b1)$ and $(a2,b2)$ determine

$$a3 = xv(i1),\ b3 = yv(i1);$$

$$ci(1) = a1;\ cj(1) = b1;\ ci(4) = a2;\ cj(4) = b1;\ ci(7) = a3;\ cj(7) = b1;$$

$$ci(2) = a2;\ cj(2) = b2;\ ci(5) = a3;\ cj(5) = b2;\ ci(8) = a1;\ cj(8) = b2;$$

$$ci(3) = a3;\ cj(3) = b3;\ ci(6) = a1;\ cj(6) = b3;\ ci(9) = a2;\ cj(9) = b3;$$

$$sc0 = c(a1,b1) + c(a2,b2) + c(a3,b3);$$

$$sc1 = c(a2,b1) + c(a3,b2) + c(a1,b3);$$

$$sc2 = c(a3,b1) + c(a1,b2) + c(a2,b3);$$

$$[msc,isc]=min([sc1\ sc2]);$$

c) If $sc0 > msc$ and $msc < Inf$, we determine the solution ($xv1$,$yv1$),

$$xv1 = [xv];\ yv1 = [yv],$$

and then we adjust it depending on at what isc the minimum value is reached; when isc=1

$$xv1(j1) = ci(i1+3),\ yv1(j1) = cj(i1+3),$$

at $isc = 2$

$$xv1(j1) = ci(i1+6),\ yv1(j1) = cj(i1+6).$$

From all new solutions ($xv1$, $yv1$), the solution with the minimum cost is selected, which is considered the result of Algorithm 3,

$$fopt = f; \; xv2 = [xv1]; \; yv2 = [yv1].$$

3. Rationale for algorithms

Calculations according to Algorithm 1 are made with matrix X, which represents an admissible non-degenerate solution to the traveling salesman problem as a transport-type problem with unit volumes of consumption of a homogeneous product for n consumers and unit volumes of product production at n production points.

The matrix X is formed from n elements and n–1 dummy zeros, with one 1 and one dummy zero in each column and each row, except for one column and one row, which contain only one 1.

The cyclic route is interpreted as a cyclic substitution ($x1$, $y1$) or as a matrix X containing identity elements at positions ($x1$, $y1$) and dummy elements at positions ($x\varepsilon$, $y\varepsilon$)

$$X(x1(i), y1(i)) = 1, \; i = 1, 2, \ldots, n, \; X(x\varepsilon(j), y\varepsilon(j)) = \varepsilon, \; j = 1, 2, \ldots, n-1.$$

Three-element replacement operation means transforming one cyclic route to another cyclic route as follows. Let the zero element $X(i0, j0)$, $i0 \neq j0$, of the matrix X correspond to a positive value of the evaluation matrix muv. Let's take three unit elements of matrix X, for example from rows $i1$, $i2$ and $i3$ and columns $j1$, $j2$ and $j3$

$$X(i1, j1) = X(i2, j2) = X(i3, j3) = 1.$$

The three-element replacement operation consists of zeroing the values of X in ($i1$, $j1$), ($i2$, $j2$) and ($i3$, $j3$),

$$X(i1, j1) = X(i2, j2) = X(i3, j3) = 0$$

and entering three elements in new positions:

$$X(i2, j1) = X(i3, j2) = X(i1, j3) = 1 \text{ when } s0 < s1 \leq s2 \quad (1)$$

or

$$X(i3, j1) = X(i1, j2) = X(i2, j3) = 1 \text{ when } s0 < s2 < s1, \quad (2)$$

where

$$s0 = c(i1, j1) + c(i2, j2) + c(i3, j3),$$
$$s1 = c(i2, j1) + c(i3, j2) + c(i1, j3), \quad (3)$$
$$s2 = c(i3, j1) + c(i1, j2) + c(i1, j3).$$

Let

$$(i1, j1), (i2, j2), (i3, j3), \ldots, (in-1, jn-1), (in, jn) \quad (4)$$

optimal route for the traveling salesman problem.

Theorem. For any three pairs

$$(ip, jq), (ir, js), (iu, jv) \quad (5)$$

from the optimal route (4), inequality

$$c(ip, jq) + c(ir, js) + c(iu, jv) \leq min\, (c(ir, jq) + c(iu, js) + c(ip, jv), \quad (6)$$
$$c(iu, jq) + c(ip, js) + c(ir, jv))$$

is true.

Proof. The three-element replacement operation converts the current cyclic route into a new cyclic route with a lower cost. If route (4) is optimal, then inequality (6) must be true. Otherwise, if

$$c(ip, jq) + c(ir, js) + c(iu, jv) > c(ir, jq) + c(iu, js) + c(ip, jv),$$

then in route (4) all three elements of matrix X in positions (5) can be replaced by elements X in positions (ir, jq), (iu, js), (ip, jv) and obtain a new cyclic optimal a route different from the original one and with a lower cost. Similarly, in the second possible case, when

$$c(ip, jq) + c(ir, js) + c(iu, jv) > c(iu, jq) + c(ip, js) + c(ir, jv),$$

in (4) we replace elements (5) with (iu, jq), (ip, js) and (ir, jv). In both cases we get a contradiction that the original plan (4) is not optimal.

Remark 1. Theorem 1 can be used as a criterion for stopping the execution of Algorithm 1.

Remark 2. At iterations of Algorithm 1, it is possible to simultaneously carry out two or more three-element replacement operations.

4. Computational experiments

The computational experiments carried out were carried out to determine the applicability of the new approach to solving the traveling salesman problem using the modified distribution method technology. The software developments are presented in the Appendix. The experiments were carried out based on the hypothesis that if the algorithm gives consistently good results for problems of size n up to the order of 20–25, then, spending time proportional to n, we will obtain an acceptable solution for large n. As a result of experiments, it turned out that this approach is realistic for obtaining an acceptable solution in a small number of iterations, no more than n. Problems with a known optimal solution and problems generated using the algorithm from [5] were solved, when a solution is specified and a problem with this optimal solution is constructed. Algorithm 1 was used. Starting with a random route, three-element replacement operations were performed repeatedly to generate new cyclic solutions. If the new solution is better than the current best solution, it becomes the new best solution. The algorithm's calculations end when further improvements using three-element replacement operations are not possible.

Conclusion

Many areas of the economy are developing along the path of greater automation. Algorithms with good performance are important. More advanced algorithms are necessary and relevant in cases where it is necessary to carry out analysis with large amounts of information in a short execution time. The proposed implementation of a new approach to solving the traveling salesman problem, based on an adaptation of the modified distribution method, has a number of advantages over well-known algorithms: is a more natural approach in terms of expanding the possibilities for solving the traveling salesman problem as a transport-type problem; Due to the sequential application of three-element replacement operations or the simultaneous execution of several three-element replacement operations, there is a rapid approach to an acceptable solution without large requirements for RAM.

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On One Implementation of a Natural Approach to Solving the Traveling Salesman Problem

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Introduction. The relevance of the traveling salesman problem is associated with the need to develop computational schemes for use in situations that require the analysis of information of a sufficiently large volume. The versatility of research on the traveling salesman problem makes it possible to reveal many properties and identify the necessary conditions for the optimality of the solution. One of the approaches to theoretical and practical research is based on computer technology of the modified distribution method

Purpose. The goal of the work is to develop conditions for the optimality of the solution and the concept of three-element replacement operations to construct the corresponding algorithm and obtain an acceptable solution for a small number of operations used.

Results. The concept of a three-element replacement operation is introduced. A necessary condition for the optimality of this solution and a special method for constructing an admissible cyclic solution to the transport problem corresponding to the traveling salesman problem are given. The search for an optimal solution can be represented as a transition from one cyclic solution to another cyclic solution using three-element replacement operations. Such replacement operations are performed for zero elements of an admissible solution with a positive value of the evaluation matrix.

Conclusions. Due to the emergence of discrete optimization problems with the need to analyze a sufficiently large amount of information, the development of new computational schemes that give consistently good results is required. Research in this direction is carried out on the example of the traveling salesman problem, for which, with the use of three-element replacement operations, it is possible to obtain an acceptable solution to the problems.

Keywords: modified distribution method, three-element replacement operation, natural approach, traveling salesman problem.

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Про одну реалізацію природного підходу до вирішення задачі комівояжера

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Вступ. Актуальність задачі комівояжера пов'язана з необхідністю розробки обчислювальних схем для використання в ситуаціях, що потребують аналізу інформації досить великого обсягу. Багатогранність досліджень щодо задачі комівояжера дозволяє розкривати багато властивостей і виявляти необхідні умови оптимальності рішення. Один із підходів теоретичних та практичних досліджень ґрунтується на комп'ютерній технології модифікованого розподільчого методу.

Мета роботи. Розробка умови оптимальності рішення та поняття триелементних операцій заміни для побудови відповідного алгоритму та отримання прийнятного рішення за невелику кількість застосовуваних операцій.

Результати. Введено поняття триелементна операція заміни. Наводиться необхідна умова оптимальності даного рішення та спеціальний спосіб конструювання допустимого циклічного рішення транспортної задачі, що відповідає задачі комівояжера. Пошук оптимального рішення здійснюється як перехід від одного циклічного рішення до іншого циклічного рішення триелементними операціями

заміни. Такі операції заміни здійснюються для нульових елементів допустимого рішення з позитивним значенням матриці оцінок.

Висновки. У зв'язку з виникненням задач дискретної оптимізації з необхідністю аналізу досить великого обсягу інформації, потрібна розробка нових обчислювальних схем, що дають стабільно хороші результати. Дослідження в цьому напрямку проводяться на прикладі задачі комівояжера, для якої із застосуванням триелементних операцій заміни, можливе отримання прийнятного розв'язання задач.

Ключові слова: модифікований метод розподілу, триелементна операція заміни, природний підхід, задача комівояжера.

Appendix

The program for solving the traveling salesman problem using modified distribution method technology

The main part of the MATLAB code for Algorithm 1 connects the functions *randperm*() – generation of a random permutation of order *n*, *CelFunc*() – calculation of the route cost, *fmdr*() – determination of the potentials *u* and *v*, as well as representation of an admissible (cyclic) solution in the form of a matrix *X*, *MatrPot*() – calculation of the evaluation matrix, *fbsol*() – solution search function.

```
function TSP     % Main part - algorithm 1
  xv=randperm(n); yv=[xv(2:n) xv(1)]; M=Inf;  % Initial solution
  f=CelFunc(n,c,xv,yv);
  fop=f; xvop=[xv]; yvop=[yv]; iop=0;
  for it=1:It
       X(1:n,1:n)=0;
       for i=1:n X(xv(i),yv(i))=1; X(i,i)=M; end
       for np=1:Np
           [u,v,x]=fmdr(n,X,c);
           if puv==2*n
              for i=1:n x(i,i)=0; end
              break;
           end
       end  % np
        [muv,t,tm]=MatrPot(n,x,c,u,v);  % t-max muv in position (r,k)
        if t<=0  break; end
        for i=1:n muv(i,i)=-Inf; end
        [fopt,xv2,yv2,cucr]=fbsol(n,c,x,xv,yv,muv,Inf);
        if fopt<fop
             fop=fopt; xvop=[xv2]; yvop=[yv2]; iop=it;
        end
        if cucr==0 break; end
         xv=[xvop]; yv=[yvop];
   end  % it
   fprintf('\n\n The best decision fop=%d за iop=%d \n (xvop,yvop):', fop,iop);
  for ic=1:n fprintf(' (%d,%d)', xvop(ic),yvop(ic)); end
```

The ***fbsol*** function that searches for a solution.

```
function [fo pt,xv2,yv2,cucr]=fbsol(n,c,x,xv,yv,muv,Inf)
    %The function corresponds to algorithm 3
    fopt=Inf; xv2(1:n)=0; yv2(1:n)=0;
    ctc=0; ccr=0; cucr=0;  % Computational process parameters
```

```
for i=1:n
   for j=1:n
      if i~=j && x(i,j)==0    % && muv(i,j)>0
        for jc=1:n
          if x(i,jc)==1 a1=i; b1=jc; break; end
        end
        for ic=1:n
          if x(ic,j)==1 a2=ic; b2=j; break; end
        end

       for ic=1:n
          if xv(ic)~=a1 && yv(ic)~=b1 && xv(ic)~=a2 && yv(ic)~=b2
          ctc=ctc+1;
          a3=xv(ic); b3=yv(ic);
        ci(1)=a1; cj(1)=b1; ci(4)=a2; cj(4)=b1; ci(7)=a3; cj(7)=b1;
        ci(2)=a2; cj(2)=b2; ci(5)=a3; cj(5)=b2; ci(8)=a1; cj(8)=b2;
        ci(3)=a3; cj(3)=b3; ci(6)=a1; cj(6)=b3; ci(9)=a2; cj(9)=b3;
        sc0=c(a1,b1)+c(a2,b2)+c(a3,b3);
        sc1=c(a2,b1)+c(a3,b2)+c(a1,b3);
        sc2=c(a3,b1)+c(a1,b2)+c(a2,b3);
        [msc,isc]=min([sc1 sc2]);
        if sc0>msc && msc<Inf
          X=[x]; xv1=[xv]; yv1=[yv];
          for i1=1:3
             ac=ci(i1); bc=cj(i1);
             for j1=1:n
                if ac==xv1(j1) && bc==yv1(j1)
                  if isc==1 xv1(j1)=ci(i1+3); yv1(j1)=cj(i1+3); end
                  if isc==2 xv1(j1)=ci(i1+6); yv1(j1)=cj(i1+6); end
                  break;
                end
              end
          end
          for ic=1:n X(xv(ic),yv(ic))=0; X(xv1(ic),yv1(ic))=1; end
          % X after improvement
          ccr=ccr+1; fsc=CelFunc(n,c,xv1,yv1);
          if fsc<=fopt    % fsc - Best solution per iteration
            fopt=fsc; xv2=[xv1]; yv2=[yv1]; cucr=cucr+1;
          end
        end %sc0>msc
      end
    end % if i~=j ... muv(i,j)>0
  end  % j
end  %  i
if fopt<M
  fprintf('\n fopt=%d ctc=%d ccr=%d cucr=%d', fopt,ctc,ccr,cucr);
  for ig=1:n fprintf(' (%d,%d)',xv2(ig),yv2(ig)); end
end
if fopt>=Inf
  f=CelFunc(n,c,xv,yv); fprintf('\n f=%d ',f);
  for ig=1:n fprintf(' (%d,%d)',xv(ig),yv(ig)); end
end
```