

Modeling of Pt-catalyst surface influence on characteristics of oxygen and carbon monoxide molecules

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A simple mathematical model of Pt-catalyst surface influence on both carbon monoxide and oxygen molecules behavior is considered. It is shown that an electric field, which is located in a near-surface layer of the Pt-catalyst, leads to a significant change of both the dipole moment and the equilibrium interatomic distance.

Keywords: *diatomic molecule in electric field, catalyst phenomenon, semi-bounded metal*

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1. Introduction

A significant number of theoretical and experimental researches are devoted to investigation of carbon monoxide (CO) oxidation processes kinetics on a metal catalysts surface (based on platinum, palladium, ruthenium) [1]. Nevertheless, the research of these processes remains urgent both from the viewpoint of experimental investigations at nanocatalytic level and theoretical viewpoint because a consequent microscopic theory of reactive diffusion processes has not been investigated enough yet. Without understanding catalytic CO oxidation mechanisms on a microscopic scale, the development of a mathematical model of a quantitative description of catalytic CO oxidation process is impossible.

The catalytic CO oxidation mechanisms (Langmuir-Hinshelwood (LH) [2], Eley-Rideal (ER) [3]) were investigated in [4]. According to the LH mechanism both reactants (CO and O_2) are adsorbed on a metal catalyst surface in the beginning and then between adsorbed CO_{ad} and O_{ad} under thermodynamically advantageous conditions a fusion reaction with a quick desorption process of the resultant product (CO_2) from the catalyst surface occur. The CO oxidation process through the ER mechanism occurs when oxygen atoms are adsorbed on the catalyst surface and a CO molecule is located in a subsurface gas zone. The energy barrier of the reaction according to such a mechanism is 0.72 eV and according to LH mechanism it is larger (0.8 – 1.05 eV). However, experimental investigations show that for CO oxidation processes on a platinum catalyst, LH mechanism is more acceptable. Although, the minimum barrier energy of a reaction for ER mechanism is less than for LH mechanism, such factors as the CO molecule structure, its orientation near a catalyst surface begin to play a determinant role. Detailed microscopic analysis of CO oxidation mechanisms on a platinum catalyst surface was conducted in [5–7] and an impurity effect on a catalytic CO oxidation was considered in [8].

ZGB model [9] and its generalization [10] a basis for theoretical investigation of catalytic CO oxidation kinetics being considered. This model assumes that a CO oxidation reaction occurs only on a catalyst surface, moreover, an adsorption of oxygen atoms occurs in consequence of a O_2 molecule decomposition near the catalyst surface. The CO molecule is adsorbed by the surface and lies on it without decomposing into atoms. However, as shown in [11], near a metal catalysts surface there exists a strong electrostatic field (with the intensity 40 V/nm), which localization is the same as the atomic

number of the metal catalyst. The influence of this field on a diatomic molecule (CO or O_2) can lead to its decomposition into atoms that henceforth can be adsorbed by the surface of metal catalysts.

In this work, a simple mathematical model of a diatomic molecule near a metal catalyst surface is developed and investigated. It is shown that existing near-surface electric field affects an equilibrium distance R between molecule atoms (leads to its increase), that implies a possibility of molecule decomposition into atoms with a consequent adsorption of O_2 molecule by the surface.

2. Problem formulation

Let us consider a motion of a diatomic molecule (CO or O_2) near a metal catalyst surface. The diatomic molecule contains atoms with atomic numbers Q_1 and Q_2 respectively, and set of electrons which general number is $Q_1 + Q_2$.

Whereas the metal catalyst is macroscopically large comparing to a size of the molecule, further let us assume that it is located in the lower half-space $Z < 0$ ($OXYZ$ is the Cartesian coordinate system connected to the catalyst) with a flat surface $Z = 0$ (Fig. 1).

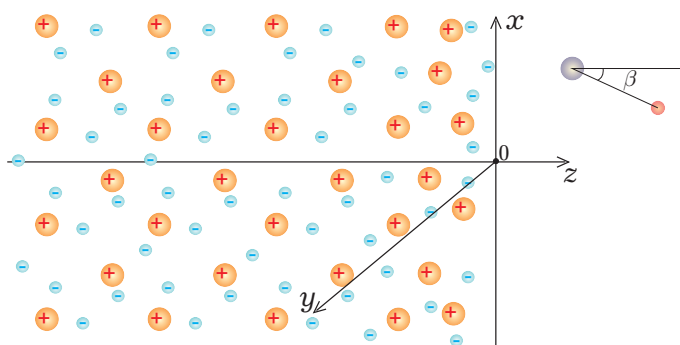


Fig. 1. Diatomic molecule near metal catalyst surface.

As shown in [13], near the surface of metal there exists a non-uniform electric field with the intensity $E(\mathbf{r})$.

In dipole approximation [14], the energy W of diatomic molecule in an internal electric field of the intensity $E(\mathbf{r})$ can be expressed as follows:

$$W = W_0 + \frac{1}{2} \int_V \varphi(\mathbf{r}) \rho(\mathbf{r}) d\mathbf{r} - \int_V (\mathbf{d}(\mathbf{r}), \mathbf{E}(\mathbf{r})) d\mathbf{r}. \quad (1)$$

Here W_0 is a molecule energy at the moment when electric field is absent, $\rho(\mathbf{r})$ is the electric charge density, $\varphi(\mathbf{r})$ and $\mathbf{E}(\mathbf{r})$ are the potential and the intensity of the electric field respectively, $\mathbf{d}(\mathbf{r})$ is the dipole moment of the molecule, $(,)$ is a scalar product.

The diatomic molecule density of the electric field is

$$\rho(\mathbf{r}) = \rho^e(\mathbf{r}) + Q_1 e \delta(\mathbf{r} - \mathbf{R}_1) + Q_2 e \delta(\mathbf{r} - \mathbf{R}_2). \quad (2)$$

Here \mathbf{R}_1 and \mathbf{R}_2 are coordinates of the nucleus atoms of the molecule in Cartesian coordinate system tied to the catalyst, e is electron charge, $\delta()$ is Dirac delta function [14].

Let us assume that the dipole moment of the molecule is a constant vector. It allows us to write the expression for the dipole moment in the presence of the electric field in the following form

$$\mathbf{d}(\mathbf{r}) = (d_0 + \varepsilon_0 \alpha \mathbf{E}) \delta(\mathbf{r} - \mathbf{R}^d) \mathbf{n}, \quad (3)$$

where d_0 is an absolute value of the dipole moment, ε_0 is the vacuum permittivity, α is the molecule polarization, \mathbf{n} is a unit direction vector (in our case under influence of a field, created by the surface, the direction is perpendicular to the surface). Let us consider that dipole moment is concentrated in the molecule center of the charge \mathbf{R}^d , which we denote by

$$\mathbf{R}^d = \frac{Q_1 \mathbf{R}_1 + Q_2 \mathbf{R}_2}{Q_1 + Q_2}.$$

For the chosen model of catalyst in case of a plane surface, the electrostatic potential is $\varphi(\mathbf{r}) \equiv \varphi(z)$, and $E(\mathbf{r}) \equiv E(z)$ [15], where z is a coordinate of the Oz axis that is perpendicular to the surface of catalyst.

Considering this, let us rewrite the expression (3) as follows:

$$W - W_0 = \frac{1}{2} \int_V \varphi(z) \rho^e(\mathbf{r}) d\mathbf{r} - \frac{1}{2} e (Q_1 \varphi(z_1) + Q_2 \varphi(z_2)) - d_{0z} E(z_d) - \varepsilon_0 \alpha E^2(z_d), \quad (4)$$

where d_0 is a dipole moment projection of the molecule onto Oz axis.

3. Calculation of the electronic density, the electrostatic potential and intensity

The electronic density of molecules has been calculated using a software package Firefly (old name is PC GAMESS) [18]. In Fig. 2, it is shown scalar surfaces of the electronic density of CO molecule and in Fig. 3 of O_2 . Note that the scalar surfaces of the electronic density for CO coincide with results of analogical calculations given in [16].

The electrostatic potential $\varphi(z)$ in our model (Fig. 1) is a solution of Poisson's equation [15]:

$$\frac{d^2 \varphi(z)}{dz^2} = -4\pi en (F_1(z) - F_{ion}(z)) \quad (5)$$

or

$$\varphi(z) = \varphi(-\infty) + 4\pi en \int_{-\infty}^z dz' (z' - z) (F_1(z') - F_{ion}(z')). \quad (6)$$

Here $F_1(z)$ is a unary distribution function of electrons, $nF_{ion}(z) \equiv \frac{2N}{5L} e\theta(-z - m_d)$ is a unary distribution function of positive compensated charge, $m_d > 0$ is a parameter that is determined by the electroneutrality condition:

$$\int_{-\infty}^{\infty} dz (F_1(z) - F_{ion}(z)) = 0. \quad (7)$$

The unary distribution function $F_1(z)$ was calculated in the research [17]. Knowing $F_1(z)$ it is possible to write the intensity of induced electrostatic field as follows

$$E(z) = -\frac{d\varphi(z)}{dz} = -4\pi en \int_{-\infty}^z dz' (n(z') - n(z)). \quad (8)$$

Here $n(z)$ is defined according to the condition $F_1(z) = en(z)$.

$\varphi(\mathbf{r})$ and $E(\mathbf{r})$ were calculated using the results obtained in [17] for the unary function $F_1(z)$. Results of these calculations in accordance with Gell-Mann-Brueckner parameter r_s [14] are presented in Figs. 4 and 5.

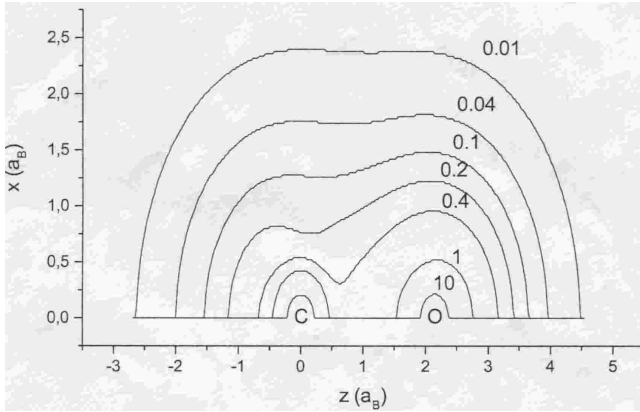


Fig. 2. Electronic density of CO molecule obtained by using Firefly.

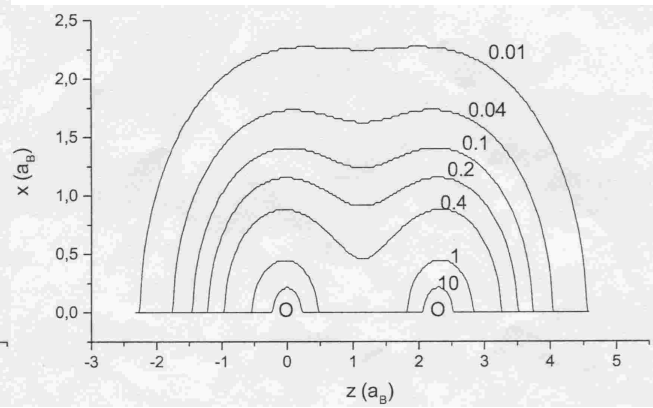


Fig. 3. Electronic density of O_2 molecule obtained by using Firefly.

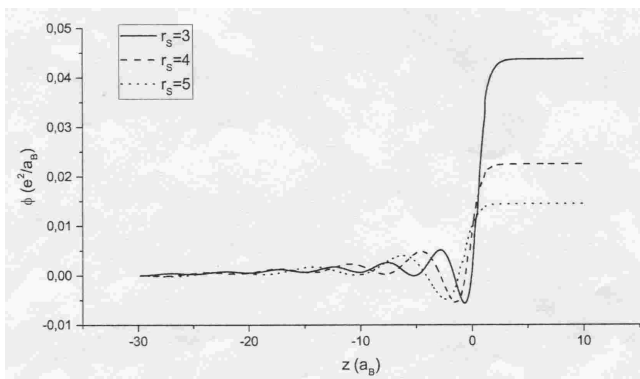


Fig. 4. Electrostatic potential $\varphi(z)$ for different values of r_s .

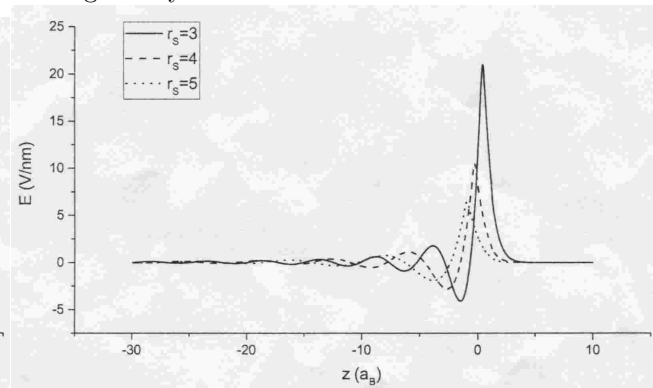


Fig. 5. Intensity $E(z)$ of field for different values of r_s .

4. Calculation of flue gas and oxygen molecules energy near the catalyst surface

Let us go back to the expression for the energy of diatomic molecule in an electric field (4):

$$W - W_0 = \frac{1}{2} \int_V \varphi(z) \rho^e(\mathbf{r}) d\mathbf{r} - \frac{1}{2} e (Q_1 \varphi(z_1) + Q_2 \varphi(z_2)) - d_{0z} E(z_d) - \varepsilon_0 \alpha E^2(z_d),$$

In the first term, let us make a transformation from coordinate system of catalyst to coordinate system of the diatomic molecule (Fig. 1). Let $\mathbf{r} = (x, y, z)$ be coordinates in the catalyst coordinate system, $\mathbf{r}' = (x', y', z')$ are coordinates in the molecule coordinate system. These coordinate systems are related by the correlations:

$$\mathbf{r}' = \mathbf{r}_0 - A\mathbf{r} \quad (9)$$

where $\mathbf{r}'_0 = (x'_0, y'_0, z'_0)$ is a coordinate origin of molecule in catalyst coordinates, A is a rotation matrix, which contains direction cosines between the axes.

$$A = \begin{pmatrix} \cos_{xx} & \cos_{xy} & \cos_{xz} \\ \cos_{yx} & \cos_{yy} & \cos_{yz} \\ \cos_{zx} & \cos_{zy} & \cos_{zz} \end{pmatrix}. \quad (10)$$

Whereas these relations are linear then inverse relations can be determined identically. Let J be Jacobian matrix of the transformation from \mathbf{r} to \mathbf{r}' . Since a transformation is a rotation and does not

depend on coordinates then the determinant $|J|$ is a constant. Hence we have that

$$\frac{1}{2} \int_V \varphi(\mathbf{r}) \rho^e(\mathbf{r}') d\mathbf{r} = \frac{1}{2} |J| \int_V \varphi(z(\mathbf{r}')) \rho^e(\mathbf{r}') d\mathbf{r}'. \quad (11)$$

Note, since in the explored model only the angle β of the slope of the axis z plays a role then one of the axes x or y is going to be unchangeable. Thus, the matrix (10) can be expressed as follows (for the unchangeable axis x)

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & -\sin \beta \\ 0 & \sin \beta & \cos \beta \end{pmatrix}.$$

The determinant of this matrix is equal to 1. Nondimensionalize the expression (11). Let

$$\varphi = \varphi^* \frac{e}{a_B}, \quad \rho^e = \rho^* \frac{e}{a_B^3}, \quad d\mathbf{r}' = a_B^3 d\mathbf{r}^*. \quad (12)$$

Then, including that $|J| = 1$,

$$\frac{1}{2} \int_V \varphi(z(\mathbf{r}')) \rho^e(\mathbf{r}') d\mathbf{r}' = \frac{1}{2} \frac{e^2}{a_B} \int_V \varphi^*(z^*(\mathbf{r}^*)) \rho^*(\mathbf{r}^*) d\mathbf{r}^*. \quad (13)$$

In a similar way to (11), let us nondimensionalize the second term related to nucleus:

$$\frac{1}{2} e (Q_1 \varphi(z_1) + Q_2 \varphi(z_2)) = \frac{1}{2} \frac{e^2}{a_B} (Q_1 \varphi^*(z_1^*) + Q_2 \varphi^*(z_2^*)). \quad (14)$$

Let us analyze a contribution related to the presence of a dipole moment in a molecule. The dipole moment is directed towards the field by OZ axis of catalyst coordinate system. Then

$$d_{0z} = d_0 \cos \beta,$$

where d_0 is the value of the dipole moment of the molecule (for CO molecule experimental value of d_0 is equal to 0.112 Debye, for O_2 molecule it is equal to 0). Nondimensionalizing the last term in (4), we obtain

$$W - W_0 = \frac{e^2}{a_B} \left[\frac{1}{2} \int_V \varphi^*(z^*(\mathbf{r}^*)) \rho^*(\mathbf{r}^*) d\mathbf{r}^* + \frac{1}{2} (Q_1 \varphi^*(z_1^*) + Q_2 \varphi^*(z_2^*)) - 0.00155 (d_0^* \cos \beta E(z_d^*) + 0.0026 \alpha^* E^*(z_d^*)) \right]. \quad (15)$$

Further we minimize the expression for energy (15) for different values of the internuclear distance $|\mathbf{R}_1 - \mathbf{R}_2|$ and for the angle β . In Figs. 6–11, there are represented numerical results of investigations of catalyst electric field influence on energy of flue gas and oxygen molecules in the distances $3a_B, a_B, 0$ with the minimal possible distance of m_d and $m_d - 0.5a_B$ (condition (7)) and the angles $\beta = 0^\circ, 45^\circ, 90^\circ$. The Gell-Mann-Brueckner parameter is chosen specific to platinum ($r_s = 4$). Below the surface $z = m_d$, a positive charge lays and a molecule with respect to strong Coulomb repulsion in our model cannot enter. For $r_s = 3$ $m_d = 0.341a_B$; for $r_s = 4$ $m_d = -0.3a_B$; for $r_s = 5$ $m_d = -0.98a_B$.

The results of numerical investigation of the proposed model show the following. For the O_2 molecule, the equilibrium interatomic distance $R_0^{O_2} = |\mathbf{R}_1 - \mathbf{R}_2|$ in the presence of electric field near the catalyst surface changes and for all the distances of the O_2 molecule charge center from the catalyst

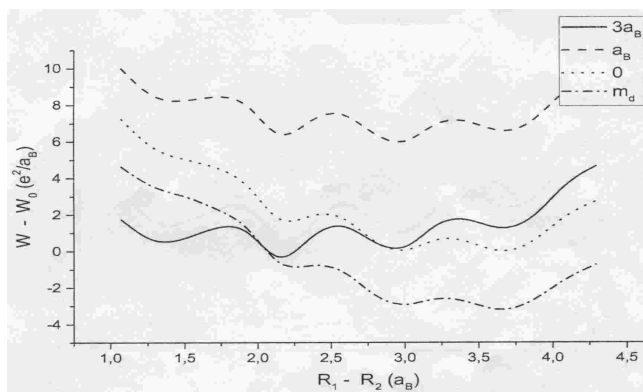


Fig. 6. Dependence of the energy (15) on internuclear distance of CO for different distances to the surface for the angle of $\beta = 0^\circ$.

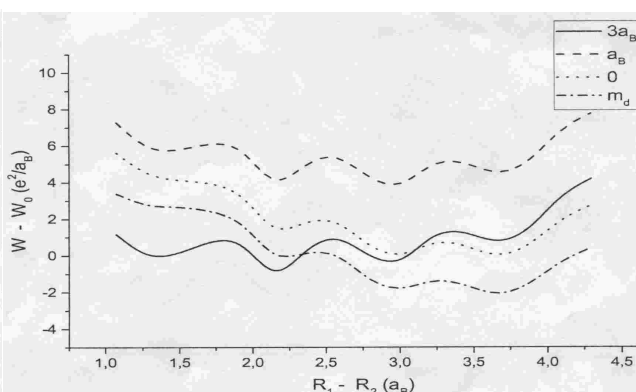


Fig. 7. Dependence of the energy (15) on internuclear distance of CO for different distances to the surface for the angle of $\beta = 45^\circ$.

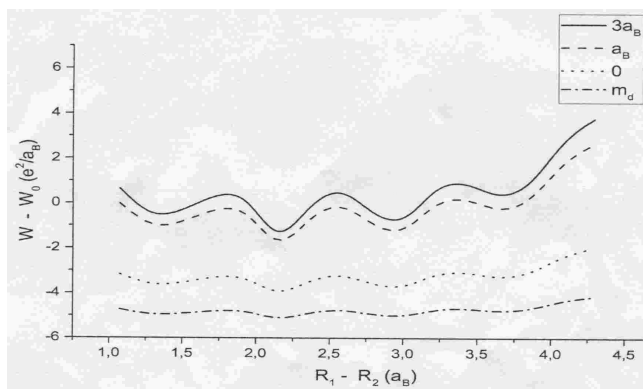


Fig. 8. Dependence of the energy (15) on internuclear distance of CO for different distances to the surface for the angle of $\beta = 90^\circ$.

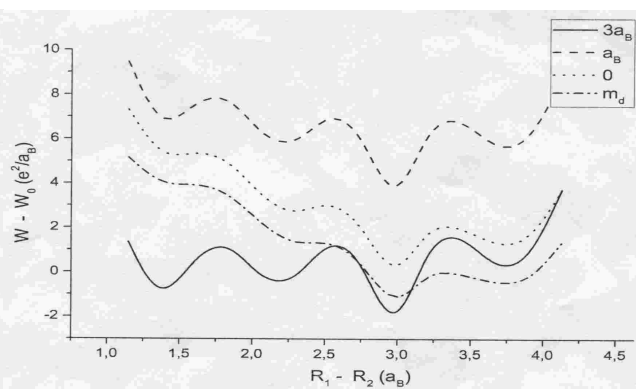


Fig. 9. Dependence of the energy (15) on internuclear distance of O_2 for different distances to the surface for the angle of $\beta = 0^\circ$.

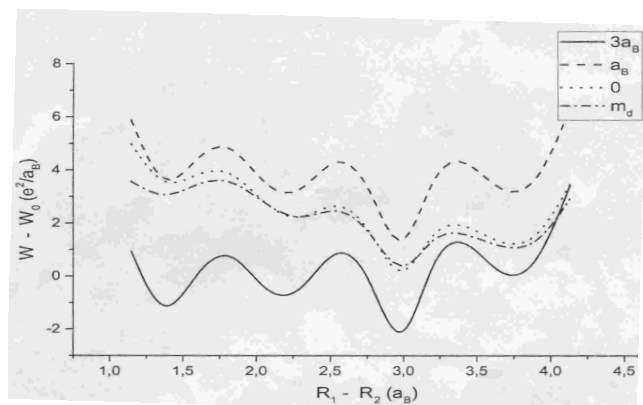


Fig. 10. Dependence of the energy (15) on internuclear distance of CO for different distances to the surface for the angle of $\beta = 45^\circ$.

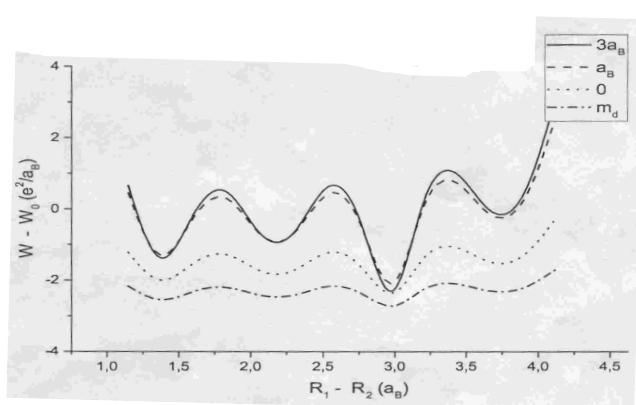


Fig. 11. Dependence of the energy (15) on internuclear distance of CO for different distances to the surface for the angle of $\beta = 90^\circ$.

surface $R_0^{O_2} = 2.22943a_B$. It is 130% of the equilibrium distance $R_0^{O_2}$ when the electric field of catalyst is absent: $R_0^{O_2} = 2.147a_B$.

The energy change $\Delta E = E_{\min}(E \neq 0) - \varepsilon_{\min}(E = 0)$ depends on the angle β and is the following:

$$\begin{aligned}\beta = 0^\circ: \Delta E &= -1\frac{e^2}{2a_B}, & R_0^{O_2} &= 2.2942, \\ \beta = 45^\circ: \Delta E &= -0.5\frac{e^2}{2a_B}, & R_0^{O_2} &= 2.2942, \\ \beta = 90^\circ: \Delta E &= -3\frac{e^2}{a_B}, & R_0^{O_2} &= 2.2942.\end{aligned}$$

These results testify that the shown dipole moment \mathbf{d}_{O_2} of the molecule O_2 when it is approaching the catalyst surface becomes parallel to the surface and the molecule becomes O_2 asymmetric.

For the CO molecule, the location of energy minimum (R_0^{CO}) depends on the angle β of the dipole moment and on the distance r_0 of the CO molecule charge center from the surface of catalyst, namely

Table 1. Dependence of CO molecule energy minimum location on model parameters.

| r_0/β | $0^\circ \div 50^\circ$ | $50^\circ \div 70^\circ$ | $70^\circ \div 90^\circ$ |
|--------------|-------------------------|--------------------------|--------------------------|
| $1 \div 3$ | 2.147 | 2.147 | 2.147 |
| $0 \div m_d$ | 3.650 | 3.00 | 2.147 |

This table demonstrates that when the CO molecule approaches the catalyst surface, it does not tend to decay into atoms (in contrast to O_2 molecule).

5. Conclusions

A simple mathematical model of diatomic molecule (CO or O_2) behavior description near the Pt-catalyst surface is suggested and investigated. It is shown that a near-surface electric field has a significant influence on the change (or appearance) of dipole moment of molecules and changes the equilibrium interatomic distance. The changes of these characteristics for the molecules CO and O_2 are evaluated.

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- [1] Kostrobij P. P., Tokarchuk M. V., Markovich B. M., Ignatjuk V. V., Gnativ B. V. Reakcijno-difuzijni procesy v sistemah "metal-gaz". Lviv, Lviv Polytechnic National University (2009), (in Ukrainian).
 - [2] Richardson J. T. Principles of Catalyst Development, Springer Science+Business Media, LLC (1992).
 - [3] Rideal E. K. Concepts in Catalysis, Academic Press (1968).
 - [4] Baxter R. J., Hu P. Insight into why the Langmuir-Hinshelwood mechanism is generally preferred. J. Chem. Phys. **116**, No.11, 4379–4381 (2002).
 - [5] Zhang C., Hu P., Alavi A. A general mechanism for CO oxidation mechanism on close-packed transition metal surfaces. J. Am. Chem. Soc. **121**, 7931–7932 (1999).
 - [6] Zhdanov V. P., Kasemo B. Simulation of CO adsorption and oxidation on Pt(110). Phys. Rev. B. **114**, No.12, 5351–5357 (2001).
 - [7] Reuter K., Frenkel D., Scheffler M. The steady-state of heterogeneous catalysis, studied by first-principles statistical mechanics. Preprint cond-mat. 0408080 v.1 (2004).
 - [8] Mryglod I. M., Bzovska I. S. Effect of adsorbed impurities on catalytic oxidation of CO: a lattice-gas model. Ukr. J. Phys. **52**, No.5, 466–471 (2007).
 - [9] Ziff R. M., Gulari E., Barshad Y. Kinetic phase transitions in an irreversible surface reaction model. Phys. Rev. Letters. **56**, No.24, 2553–2556 (1986).

- [10] Johanek V., Laurin M., Grant A. W., Kasemo B., Henry C. R., Libuda J. Fluctuations and bistabilities on catalyst nanoparticles. *Science*. **304**, 1639–1644 (2004).
- [11] Kostrobij P. P., Markovych B. M., Suchorski Y. Revisiting local electric fields on a closed-packed metal surfaces: theory versus experiments. *Solid state phenomena*. **128**, 219–224 (2007).
- [12] Kostrobij P. P. Doctoral Thesis. Reaction-diffusion processes in systems with a “metal–gas” interface: the quantum statistical description. Lviv (2009), (in Ukrainian).
- [13] Kostrobij P. P., Markovych B. M. Statistical theory of the spacebounded systems of charged fermi-particles: I. The functional integration method and effective potentials. *Journ. of Phys. Stud.* **7**, 195–206 (2003) (in Ukrainian).
- [14] Vakarchuk I. O. *Kvantova mehanika*. Lviv, Lviv National University (2004), (in Ukrainian).
- [15] Kostrobij P. P., Markovych B. M. Investigation of the Influence of External Electric Field on the Electron Density of Semi-bounded Metal. *Ukr. J. Phys.* **52**, No.2, 167–171 (2007).
- [16] Fudzinaga C. *Metod molekuljarnih orbitalej*. Moskva, Mir (1983), (in Russian).
- [17] Kostrobij P. P., Markovych B. M. Statistical theory of the spacebounded systems of charged fermi-particles: II. Distribution functions. *Journ. of Phys. Stud.* **7**, 298–312 (2003) (in Ukrainian).
- [18] Granovsky Alex A. Firefly version 8, [www http://classic.chem.msu.su/gran/firefly/index.html](http://classic.chem.msu.su/gran/firefly/index.html).

Моделювання впливу поверхні Pt-катализатора на характеристики молекул кисню і окису вуглецю

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Розглянуто просту математичну модель впливу поверхні Pt-катализатора на поведінку молекул чадного газу та кисню. Показано, що електричне поле, локалізоване в приповерхневому шарі Pt-катализатора, призводить до значної зміни дипольного моменту та рівноважної міжатомної відстані.

Ключові слова: *двоатомні молекули в електричному полі, явище каталізу, пів-обмежений метал*

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