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Determination of normal frequencies and modes of liquid sloshing in reservoir with variable bottom

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A problem about normal oscillations of ideal homogeneous liquid, which partially fills a reservoir with vertical walls, but with variable (inclined) bottom, is under consideration. A numerical implementation of the Ritz method was developed, and by partial example, which corresponds to a plane inclined bottom, the results of numerical realization are shown. We determined frequencies and normal modes of liquid oscillations as well as errors of satisfaction of the non-flowing boundary condition at the bottom. The results showed that the wave profiles for different directions of liquid oscillations differ. The magnitude of errors makes it possible to use the determined normal modes as coordinate functions for solving the nonlinear problem of liquid sloshing in such types of reservoirs.

Keywords: *liquid oscillations, liquid free surface, reservoir with inclined bottom, normal frequencies and modes, errors of satisfying of boundary conditions.*

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1. Introduction

A problem about determination of normal frequencies and modes of liquid with a free surface in a rectangular reservoir with inclined bottom is under consideration. A peculiarity of the mentioned problem is determined by non-symmetry of the domain of liquid. A such kind of problems practically was not considered before. The objective of the present investigation is the development of problem statement for determination of normal frequencies and modes of liquid oscillations in reservoirs with inclined bottom, determination of coordinate functions and optimal method for problem numerical solving for providing solution of high accuracy, which can be used for investigation of nonlinear problems of dynamics. Similar types of reservoirs are used in systems of mineral processing, on investigation of waves in coastal closed water basins.

2. Object of study

It is known [1, 2] that a linear problem about free oscillations of liquid with a free surface is described by the following boundary value problem

$$\Delta\varphi = 0 \quad \text{in} \quad \tau_0; \qquad \frac{\partial\varphi}{\partial \mathrm{n}} = \begin{cases} 0 & \text{on} \quad \Sigma_0, \\ \lambda\varphi & \text{on} \quad S_0, \end{cases} \tag{1}$$

or its variational analog

$$\delta I = 0, \tag{2}$$

where

$$I = \int\limits_{\tau_0} (\nabla\varphi)^2 \, d\tau - \lambda \int\limits_{S_0} \varphi^2 ds.$$

In relations (1) and (2) φ is the velocity potential, τ_0 is domain occupied by liquid in unperturbed state, S_0. Is a free surface of liquid in unperturbed state, Σ_0 is moisten surface of liquid in unperturbed state, λ is eigenvalue. This problem in operator form represents positively definite self-adjoint problem, for which it is natural to use the Ritz method [1, 2]. After introduction of the system of coordinate functions in numerical sense the problem is reduced to the generalized algebraic symmetric eigenvalue problem

$$Ax - \lambda Bx = 0, \tag{3}$$

where

$$A_{ij} = \int\limits_{\tau_0} \nabla\psi_i \nabla\psi_j d\tau; \quad B_{ij} = \int\limits_{S_0} \psi_i \psi_j ds. \tag{4}$$

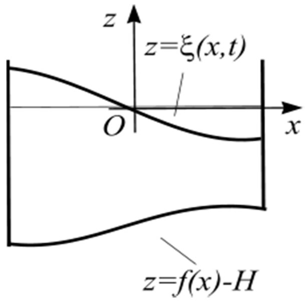

Fig. 1. General scheme of the object.

The coordinate system was selected such that a free surface of liquid in undisturbed state coincides with plane Oxy, origin of the coordinate system is in the center of undisturbed free surface, axis Ox is aimed right-hand, and axis Oz is aimed upward. Let us denote mean depth of liquid in reservoir as H, $z = \xi(x,t)$ is equation of level of a free surface of liquid. Bottom of reservoir is specified by the relation

$$z = f(x) - H, \tag{5}$$

where $f(x)$ is a given function. We assume that reservoir width is $2a$. General scheme of the object is shown in Fig. 1.

3. Method for problem solving

For solving the mentioned problem we select the system of harmonic coordinate functions, which consists of four sets of functions:

$$\psi_i = \frac{\sin\left(\frac{(2i-1)\pi}{2a}x\right)\cosh\left(\frac{(2i-1)\pi}{2a}(z+H)\right)}{\frac{(2i-1)\pi}{2a}\sinh\left(\frac{(2i-1)\pi}{2a}H\right)}, \quad \psi_j = \frac{\cos\left(\frac{j\pi}{a}x\right)\cosh\left(\frac{j\pi}{a}(z+H)\right)}{\frac{j\pi}{a}\sinh\left(\frac{j\pi}{a}H\right)},$$

$$\psi_k = \frac{\sin\left(\frac{(2k-1)\pi}{2a}x\right)\cosh\left(\frac{(2k-1)\pi}{2a}z\right)}{\frac{(2k-1)\pi}{2a}\sinh\left(\frac{(2k-1)\pi}{2a}H\right)}, \quad \psi_l = \frac{\cos\left(\frac{l\pi}{a}x\right)\cosh\left(\frac{l\pi}{a}z\right)}{\frac{l\pi}{a}\sinh\left(\frac{l\pi}{a}H\right)}.$$

The first and second set of functions hold nonflowing condition through the plane $z = -H$. The number of functions, which enter the first and second sets are equal to d_a and d_s correspondingly. The third and fourth sets of functions hold nonlowing condition through the plane $z = 0$. The number of functions, which enter the third and fourth sets, are equal correspondingly to p_a and p_s. Here functions from the first and third sets are antisymmetric, and functions from the second and fourth sets are symmetric. All selected coordinate functions hold nonflowing condition through planes $x = -a$ and $x = a$, i.e., vertical walls of the reservoir.

For numerical integration of quadratures (4) we used the Gaussian elimination method. Here the number of intervals of intervals with fixed quantity of partition points was selected depending on the number of coordinate functions. For solving of the generalized symmetric positively defined algebraic eigenvalue problem (3) we used the Jacobi method of rotations.

4. Analysis of numerical results

Criterion of verification of efficiency of the applied methods and correctness of the selected way of determination of coordinate functions for solving of the mentioned problem is realization of the nonflowing condition through inclined bottom

$$\frac{\partial \varphi}{\partial n} = 0 \quad \text{for} \quad z = -H + f(x). \tag{6}$$

As the result of verification of this condition we determined optimal quantity of coordinate functions: $d_a = 30$, $d_s = 10$, $p_a = 30$, $p_s = 10$. The number of integration intervals was selected as $m = 60$ from the point of view of sufficient accuracy and computation rate. For numerical examples it was accepted $a = 1\,\text{m}$, $f(x) = -0.15\,x$, $H = 0.5\,\text{m}$. For the given quantity of coordinate functions and integration intervals errors of realization of the condition (6) are sufficiently small. We adduce errors of realization of nonflowing condition (6) at different points of reservoir bottom for the first five normal modes of oscillations and their frequencies. Here relative error is determined according to the formula $\delta = \left.\frac{\partial \varphi}{\partial n}\right|_{S_d} / \max\left(\left.\frac{\partial \varphi}{\partial n}\right|_{S_0} \right)$, where S_d is reservoir bottom ($z = -H + f(x)$), S_0 is a free surface of liquid ($z = 0$). Here δ_1 is error at the left corner point, δ_2 is error at the point of bottom with abscissa $x = -0.9$, δ_3 is error at the right corner point, λ is frequency parameter, N is number of mode.

Table 1.

N	δ_1	δ_2	δ_3	λ
1	0.0279	−0.008	0.0007	1.0143
2	0.0224	−0.0066	−0.0005	2.8513
3	−0.0172	0.0051	−0.0003	4.6031
4	0.0127	−0.0039	−0.0002	6.2448
5	−0.0091	0.0029	−0.0001	7.8409

It is seen from Table 1 that maximal error on bottom for the first five normal modes of oscillations are about 0.01. The obtained accuracy shows that the determined normal modes suit for usage as coordinate functions on investigation of nonlinear problem of dynamics of liquid sloshing.

Profiles of oscillations of a free surface of liquid for the first three normal modes are shown in Fig. 2.

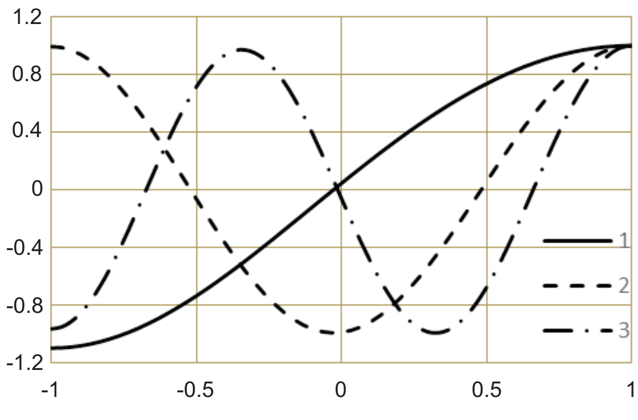

Fig. 2. Profiles of waves for the first three normal modes.

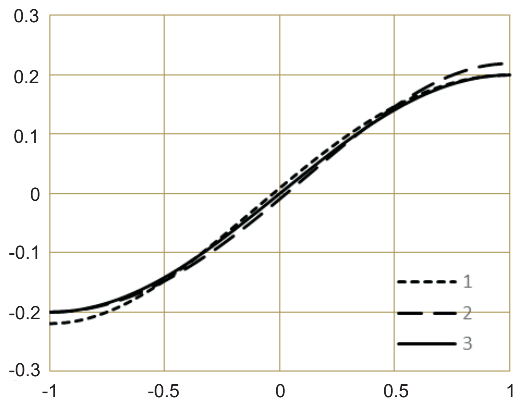

Fig. 3. Comparison of wave profiles.

It is seen from figure that wave profiles on a free surface of liquid qualitatively coincide with the first three normal modes of oscillations of a free surface of liquid in the case of rectangular parallelepiped with horizontal bottom, however, it is seen from table that frequencies fir this cases are different.

Deeper analysis testifies that all three normal modes of a free surface of liquid have deviation from symmetry, which represents difference between wave profiles in reservoir with inclined bottom and the corresponding reservoir with symmetric bottom.

In Fig. 3, the curve 1 corresponds to direct wave of the first normal mode in reservoir with inclined bottom, the curve 2 corresponds to inverse wave mirrored relative to the axis Oz in reservoir with inclined bottom and the curve 3 corresponds to the direct wave in the reservoir with horizontal bottom. Here it is seen that wave profiles, which correspond to the curves 1 and 2, differ from each other, which shows the difference of normal modes for the reservoir inclined bottom from the reservoir with horizontal bottom, in which the corresponding profiles coincide. It is also seen that the profile of the direct wave in the reservoir with inclined bottom differs from the profile of the direct wave in the reservoir with horizontal bottom, especially near reservoir walls.

5. Conclusions

The problem of determination of normal frequencies and modes of oscillations of ideal homogeneous incompressible liquid in reservoir with vertical walls and bottom, which differs from horizontal, was considered. For problem solving we used the Ritz method. We select the system of harmonic coordinate functions, which exactly hold boundary conditions on vertical walls and approximately on bottom and a free surface. Implementation of the method was done for particular case of reservoir with inclined flat bottom. It was shown that the method makes it possible to determine with high accuracy eigenfreqencies, and the determined normal modes with high accuracy hold boundary condition on the bottom. Here the obtained accuracy enables usage of normal modes as coordinate functions for investigation of problems of nonlinear dynamics of reservoirs with liquid with a free surface.

Analysis of normal modes shows that qualitatively they are sufficiently close to normal modes of liquid oscillations in reservoir with horizontal bottom. However, in the case of reservoir with inclined bottom the profiles of waves on a free surface of liquid for direct and inverse waves have certain difference, caused by non-symmetry of the domain, occupied by liquid. It was also ascertained that with the increase of liquid depth, the effects, caused by the influence of bottom inclination, decrease.

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Визначення власних частот і форм коливань рідини в резервуарі зі змінним дном

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Розглянуто задачу про власні коливання ідеальної однорідної рідини, що частково заповнює резервуар з вертикальними стінками і змінним (нахиленим) дном. Розвинено чисельну реалізацію на основі методу Рітца і для частинного випадку плоского нахиленого дна показано результати чисельної реалізації. Визначено частоти і власні форми коливань рідини, а також похибки задовільнення умови неперетікання на дні. Результати свідчать, що профілі хвиль для різних напрямків коливань рідини відрізняються. Величини похибок дозволяють використати визначенні форми коливань як координатні функції під час розв'язання нелінійної задачі про коливання рідини в резервуарі.

Ключові слова: *коливання рідини, вільна поверхня рідини, резервуар з нахиленим дном, нормальні частоти і форми, похибки задоволення граничних умов.*

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