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Stability under stochastic perturbation of solutions of mathematical models of information spreading process with external control

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In this paper mathematical model of spreading any number of information types with external influences is considered. The model takes the form of n (number of information channels) non-linear Ito stochastic differential equations. Conditions for asymptotic stability in quadratic average in first-approximation of the special points are considered for general stationary model and special case with non-stationary parameters. The results of example are presented for the special case of the base model with stationary parameters.

Keywords: *mathematical model of information spreading process, stochastic stability, asymptotic stability in quadratic average, "white" noise.*

1. Introduction

In the last century opportunities of information for influence on members of social communities are begun to research. Scientists from different countries (USA, Great Britain, Russia and other) define the information spreading process as leading factor of internal and external socio-political processes. Since then work about information spreading process, its features, the possibility of using the process in own interests are showed up.

These researches have caused to need mathematical models of information spreading process one of the way to solve this problem is systems of ordinary differential equations.

Consider the basic model of spreading of any number of information messages in the society.

Let there be a certain social community of N people, potentially subject n information flows. At time moment $t_0 = 0$ community falls under influence one of the N information channels. $x_k(t)$ are numbers of people who have accepted the information from the k-th information flow $(k = \overline{1,n})$. This value depends on external influences (media etc.) and interpersonal communication (internal influences).

There are b_k, $k = \overline{1,n}$ – the intensity of communication, u_k are external influences. In this case we get

$$\dot{x}_k(t) = b_k(t)x_k(t)\left(N - \sum_{i=1}^{n} x_i(t)\right) + u_k(t), \quad x_k(0) = x_{0k}, \quad k = \overline{1,n}. \tag{1}$$

Works [1–3] are devoted to analysis of solution oh this model (1) with stationary parameters and the special type of $u_k(t)$. Properties of solution of the system with the special type of $u_k(t)$ are analyzed in work [4]. Conditions of existence of range of first-approximation stability of the solutions are considered in [5]. Problems of building estimations of parameters of model (1) are investigated in [6–10].

2. Stochastic stability in the first approximation of general model of information confrontation

Consider the case where the parameters are non-stationary and the external influences are simulated us

$$u_k(t) = \sum_{i=1}^{n} a_{ki} x_i(t) + c_k, \quad k = \overline{1, n}.$$

Then model (1) has the following form:

$$\dot{x}_k(t) = b_k x_k(t) \left(N - \sum_{i=1}^{n} x_i(t) \right) + \sum_{i=1}^{n} a_{ki} x_i(t) + c_k, \quad x_k(0) = x_{0k}, \quad k = \overline{1, n}. \tag{2}$$

We assume that the parameters of the intensity of communication b_k, $k = \overline{1, n}$ are exposed disturbing influence. Them the model can be written as the system of Ito stochastic differential equations

$$dx_k(t) = \left[b_k x_k(t) \left(N - \sum_{i=1}^{n} x_i(t) \right) + \sum_{i=1}^{n} a_{ki} x_i(t) + c_k \right] dt$$

$$+ g_k x_k(t) \left(N - \sum_{i=1}^{n} x_i(t) \right) dw_k(t), \quad k = \overline{1, n}, \tag{3}$$

with the initial conditions

$$x_k(0) = x_{0k}, \quad k = \overline{1, n},$$

where $w_k(t)$ are Wiener processes, $dx_k(t)$ and dw_k are stochastic differentials of processes $x_k(t)$ and $w_k(t)$, $k = \overline{1, n}$ (in the sense of Ito).

Find the special point $(x_{10}, \ldots, x_{n0})$ for deterministic case of the system (3) from conditions

$$\begin{cases} L - \sum_{i=1}^{n} x_{i0} = 0, \\ \sum_{i=1}^{n} a_{ki} x_{i0} + c_k = 0, \quad k = \overline{1, n}. \end{cases} \tag{4}$$

Let us rewrite the system of linear equations (4) in a matrix form

$$\begin{pmatrix} 1 & 1 & \ldots & 1 \\ a_{11} & a_{12} & \ldots & a_{1n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix} \begin{pmatrix} x_{10} \\ \ldots \\ x_{n0} \end{pmatrix} = \begin{pmatrix} L \\ -c_1 \\ \ldots \\ -c_n \end{pmatrix}. \tag{5}$$

The sufficient condition of non-negative elements of the vector $(x_{10}, \ldots, x_{n0})$ is the existence of the matrix

$$\begin{pmatrix} 1 & 1 & \ldots & 1 \\ a_{11} & a_{12} & \ldots & a_{1n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix}^{-1}. \tag{6}$$

On the basis of the Kronecker-Capelli theorem the linear system (5) has the unique solution if and only if the next condition is fulfilled

$$\text{rang} \begin{pmatrix} 1 & 1 & \ldots & 1 \\ a_{11} & a_{12} & \ldots & a_{1n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix} = \text{rang} \begin{pmatrix} 1 & 1 & \ldots & 1 & L \\ a_{11} & a_{12} & \ldots & a_{1n} & -c_1 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} & -c_n \end{pmatrix} = n. \quad (7)$$

Let us consider the linear approximation of the system (3) in neighbourhood of the special point $(x_{10}, \ldots, x_{n0})$

$$dx_k(t) = \left[b_k \left(N - \sum_{i=1}^{n} x_{i0} \right) (x_k(t) - x_{k0}) + \sum_{i=1}^{n} (a_{ki} - b_k x_{k0})(x_i(t) - x_{i0}) \right] dt$$
$$+ g_k \left[\left(N - \sum_{i=1}^{n} x_{i0} \right) (x_k(t) - x_{k0}) - x_{k0} \sum_{i=1}^{n} (x_i(t) - x_{i0}) \right] dw_k(t), \quad k = \overline{1, n}. \quad (8)$$

Taking into account (4) from the system (8) we get

$$dx_k(t) = \sum_{i=1}^{n} (a_{ki} - b_k x_{k0})(x_i(t) - x_{i0})dt - g_k x_{k0} \sum_{i=1}^{n} (x_i(t) - x_{i0})dw_k(t), \quad k = \overline{1, n}.$$

After replacement $\overline{x}_k(t) = x_k(t) - x_{k0}, k = \overline{1, n}$ we get the system of linear Ito stochastic differential equations

$$d\overline{x}_k(t) = \sum_{i=1}^{n} (a_{ki} - b_k x_{k0})\overline{x}_i(t)dt - g_k x_{k0} \sum_{i=1}^{n} \overline{x}_i(t)dw_k(t), \quad k = \overline{1, n}. \quad (9)$$

Let $\overline{x}(t) = (\overline{x}_1(t), \ldots, \overline{x}_n(t))^{\mathrm{T}}$, $w(t) = (w_1(t), \ldots, w_n(t))^{\mathrm{T}}$, T is the symbol of transpose than we can write the system (9) in matrix form

$$d\overline{x}(t) = A\overline{x}(t)dt + \sum_{i=1}^{n} \overline{x}_i(t)Bdw(t), \quad (10)$$

where

$$A = \begin{pmatrix} a_{11} - b_1 x_{10} & \ldots & a_{1n} - b_1 x_{10} \\ \ldots & \ldots & \ldots \\ a_{n1} - b_n x_{n0} & \ldots & a_{nn} - b_n x_{n0} \end{pmatrix}, \quad B = \begin{pmatrix} -g_1 x_{10} & \ldots & 0 \\ \ldots & \ldots & \ldots \\ 0 & \ldots & -g_n x_{n0} \end{pmatrix}.$$

Theorem 1. *Let the condition of existence matrix (6) and condition (7) for system (3) are satisfied. The solutions of the system (3) are asymptotic stable in quadratic average in the first approximation in the area of special point $(x_{10}, \ldots, x_{n0})$ if and only if maximal eigenvalue of the matrix C_n have negative real parts, where*

$$C_n = A + A^{\mathrm{T}} + \text{Sp}\, B^{\mathrm{T}} B E_n,$$

and

$$E_n = \begin{pmatrix} 1 & \ldots & 1 \\ \ldots & \ldots & \ldots \\ 1 & \ldots & 1 \end{pmatrix}$$

is matrix of dimension $n \times n$.

Proof. By definition the zero solutions $\overline{x}(t) \equiv 0$ of the system (9) is asymptotic stability in quadratic average if the next consider is fulfilled

$$E\left(\overline{x}(t), \overline{x}(t)\right) \underset{t \to \infty}{\longrightarrow} 0,$$

where E the symbol of expected value.

On the basis of the Ito's lemma we obtain system of differential equations in matrix form

$$\frac{dE\left(\overline{x}(t), \overline{x}(t)\right)}{dt} = 2E\left(A\overline{x}(t), \overline{x}(t)\right) + 2\frac{1}{2}E\left[\left(\sum_{i=1}^{n} \overline{x}_i(t)\right)^2 \operatorname{Sp} B^{\mathrm{T}}B\right], \tag{11}$$

with initial conditions

$$E\left(\overline{x}(0), \overline{x}(0)\right) = \sum_{i=1}^{n} (x_{0k} - x_{k0})^2.$$

Equation (11) we can be represented in the form

$$\frac{dE\left(\overline{x}(t), \overline{x}(t)\right)}{dt} = E\left(\left[A + A^{\mathrm{T}} + \operatorname{Sp} B^{\mathrm{T}}B E_n\right]\overline{x}(t), \overline{x}(t)\right). \tag{12}$$

Here is an estimate for equation (12)

$$\lambda_{\min}(C_n)E\left(\overline{x}(t), \overline{x}(t)\right) \leqslant \frac{dE\left(\overline{x}(t), \overline{x}(t)\right)}{dt} \leqslant \lambda_{\max}(C_n)E\left(\overline{x}(t), \overline{x}(t)\right),$$

where $\lambda_{\min}(C_n)$ and $\lambda_{\max}(C_n)$ are minimal and maximal eigenvalues of the matrix C_n.

On the basis of Gronwall-Bellman inequality we obtain

$$\exp\left\{\int_0^t \lambda_{\min}(C_n)\right\} E\left(\overline{x}(0), \overline{0}(t)\right) \leqslant E\left(\overline{x}(t), \overline{x}(t)\right) \leqslant \exp\left\{\int_0^t \lambda_{\max}(C_n)\right\} E\left(\overline{x}(0), \overline{0}(t)\right).$$

Because the parameters of the system (3) are stationary we have inequalities

$$\exp\left\{\lambda_{\min}(C_n)t\right\} \sum_{i=1}^{n}(x_{0k} - x_{k0})^2 \leqslant E\left(\overline{x}(t), \overline{x}(t)\right) \leqslant \exp\left\{\lambda_{\max}(C_n)t\right\} \sum_{i=1}^{n}(x_{0k} - x_{k0})^2. \tag{13}$$

On the basis (13) the expression

$$E\left(\overline{x}(t), \overline{x}(t)\right) \underset{t \to \infty}{\longrightarrow} 0$$

is fulfilled if $\lambda_{\max}(C_n)$ have negative real parts. Thus the zero solution $\overline{x}(t) \equiv 0$ of the system (10) is asymptotic stable in quadratic average. The solutions of system (3) are asymptotic stable in quadratic average in the first approximation in the area of special point $(x_{10}, \ldots, x_{n0})$ if maximal eigenvalues of the matrix C_n have negative real parts. ∎

3. Stochastic stability in the first approximation of special case of model of information confrontation

Consider the special case of the system (1)

$$\dot{x}_k(t) = (a_k(t) + b_k(t)x_k(t))\left(N - \sum_{i=1}^{n} x_i(t)\right) + \gamma_k(t)(x_k(t) - m_k N), \quad x_k(0) = x_{0k}, \quad k = \overline{1, n},$$

where $m_k \geqslant 0$, $\sum_{i=1}^{n} m_i = 1$.

And the parameters $b_k(t)$, $k = \overline{1,n}$ are exposed disturbing influence, thus we obtain the system of Ito stochastic differential equations

$$dx_k(t) = \left[(a_k(t) + b_k(t)x_k(t))\left(N - \sum_{i=1}^{n} x_i(t)\right) + \gamma_k(t)(x_k(t) - m_kN)\right]dt$$

$$+ g_k(t)x_k(t)\left(N - \sum_{i=1}^{n} x_i(t)\right)dw_k(t), \quad k = \overline{1,n}, \quad (14)$$

with the initial conditions

$$x_k(0) = x_{0k}, \quad k = \overline{1,n}.$$

The special point of system (14) is $(m_1N, \ldots, m_nN)$. After linear approximation in neighbourhood of the special point and replacement $\overline{x}_k(t) = x_k(t) - m_kN$, $k = \overline{1,n}$ we get system of linear Ito stochastic differential equations in matrix form

$$d\overline{x}(t) = A(t)\overline{x}(t)dt + \sum_{i=1}^{n} \overline{x}_i(t)B(t)dw(t), \quad (15)$$

where

$$A(t) = \begin{pmatrix} \gamma_1(t) - a_1(t) - b_1(t)m_1N & \ldots & -a_1(t) - b_1(t)m_1N \\ \ldots & \ldots & \ldots \\ -a_n(t) - b_n(t)m_nN & \ldots & \gamma_n(t) - a_n(t) - b_n(t)m_nN \end{pmatrix},$$

$$B(t) = \begin{pmatrix} -g_1(t)m_1N & \ldots & 0 \\ \ldots & \ldots & \ldots \\ 0 & \ldots & -g_n(t)m_nN \end{pmatrix}.$$

Theorem 2. *The solutions of the system (14) are asymptotic stable in quadratic average in the first approximation in the area of special point $(m_1N, \ldots, m_nN)$ if and only if condition $\frac{1}{t}\int_0^t \lambda_{\max}(D_n(\tau))d\tau \leqslant -c$, $c > 0$, $\forall t$ is satisfied, where $\lambda_{\max}(D_n(t))$ is maximal eigenvalue of the matrix $D_n(t)$,*

$$D_n(t) = \frac{1}{2}\left[A(t) + A^T(t) + \operatorname{Sp} B(t)^T B(t)E_n + A^*(t) + (A^T(t))^* + \left(\operatorname{Sp} B^T(t)B(t)E_n\right)^*\right],$$

where $A^*(t)$ is conjugate transpose of matrix $A(t)$.

Proof. We need to find conditions of asymptotic stability in quadratic average of the zero solutions $\overline{x}(t) \equiv 0$ of the system (15).

On the basis of the Ito's lemma we obtain system of differential equations in matrix form

$$\frac{dE\left(\overline{x}(t), \overline{x}(t)\right)}{dt} = E\left(\left[A(t) + A^T(t) + \operatorname{Sp} B^T(t)B(t)E_n\right]\overline{x}(t), \overline{x}(t)\right), \quad (16)$$

with initial conditions

$$E\left(\overline{x}(0), \overline{x}(0)\right) = \sum_{i=1}^{n}(x_{0k} - m_kN)^2.$$

In basis of Vazhevsky inequality [11] we obtain estimate for equation (16)

$$\exp\left\{\int_0^t \lambda_{\min}(D_n(t))\right\}E\left(\overline{x}(0), \overline{0}(t)\right) \leqslant E\left(\overline{x}(t), \overline{x}(t)\right) \leqslant \exp\left\{\int_0^t \lambda_{\max}(D_n(t))\right\}E\left(\overline{x}(0), \overline{0}(t)\right).$$

Thus the zero solution $\overline{x}(t) \equiv 0$ of the system (15) is asymptotic stable in quadratic average if condition $\frac{1}{t} \int_0^t \lambda_{\max}(D_n(\tau))d\tau \leqslant -c$, $c > 0$, $\forall t$ is satisfied. The solutions of system (14) are asymptotic stable in quadratic average in the first approximation in the area of special point $(m_1 N, \ldots, m_n N)$ if condition $\frac{1}{t} \int_0^t \lambda_{\max}(D_n(\tau))d\tau \leqslant -c$, $c > 0$, $\forall t$ is satisfied. ∎

4. Numerical experiment

We obtain the special case of the system (14) when $n = 2$ and parameters of model are stationary thus we obtain the system of Ito stochastic differential equations

$$dx_k(t) = [(a_k + b_k x_k(t))(N - x_1(t) - x_2(t)) + \gamma_k(x_k(t) - m_k N)] dt$$
$$+ g_k x_k(t)(N - x_1(t) - x_2(t)) dw_k(t), \quad m_1 + m_2 = 1, \quad m_k \geqslant 0, \quad k = 1, 2. \quad (17)$$

After linear approximation in neighbourhood of the special point $(m_1 N, m_2 N)$ and replacement $\overline{x}_k(t) = x_k(t) - m_k N$, $k = \overline{1,2}$ we get system of linear Ito stochastic differential equations in matrix form

$$d\overline{x}(t) = A\overline{x}(t)dt + (\overline{x}_1 + \overline{x}_2)Bdw(t),$$

where

$$A = \begin{pmatrix} \gamma_1 - a_1 - b_1 m_1 N & a_1 - b_1 m_1 N \\ a_2 - b_2 m_2 N & \gamma_2 - a_2 - b_2 m_2 N \end{pmatrix}, \quad B = \begin{pmatrix} -g_1 m_1 N & 0 \\ 0 & -g_2 m_2 N \end{pmatrix}.$$

On the basis of Theorem 1 the solutions of the system (17) are asymptotic stable in quadratic average in the first approximation in the area of special point $(m_1 N, m_2 N)$ if and only if all eigenvalues of the matrix C_2 have negative real parts, where

$$C_2 = \begin{pmatrix} 2(\gamma_1 - a_1 - b_1 m_1 N) + \mathrm{Sp}\, B^{\mathrm{T}} B & \mathrm{Sp}\, B^{\mathrm{T}} B - a_1 - b_1 m_1 N - a_2 - b_2 m_2 N \\ \mathrm{Sp}\, B^{\mathrm{T}} B - a_1 - b_1 m_1 N - a_2 - b_2 m_2 N & 2(\gamma_2 - a_2 - b_2 m_2 N) + \mathrm{Sp}\, B^{\mathrm{T}} B \end{pmatrix},$$

$$\mathrm{Sp}\, B^{\mathrm{T}} B = (g_1 m_1 N)^2 + (g_2 m_2 N)^2.$$

Thus the eigenvalues of the matrix C_2 have negative real parts if the next conditions are fulfilled

$$\begin{cases} \gamma_1 + \gamma_2 - a_1 - b_1 m_1 N - a_2 - b_2 m_2 N + \mathrm{Sp}\, B^{\mathrm{T}} B < 0, \\ 4(\gamma_1 - a_1 - b_1 m_1 N)(\gamma_2 - a_2 - b_2 m_2 N) - (a_1 + b_1 m_1 N + a_2 + b_2 m_2 N)^2 + 2\, \mathrm{Sp}\, B^{\mathrm{T}} B(\gamma_1 + \gamma_2) > 0. \end{cases}$$

As an example, we present the results of computer modeling of the dynamics of spreading two type of information.

Let there be a certain social community of $N = 100$ people, potentially subject information flow. And we analysis the mathematical model on the time interval $(0, 10)$.

Let parameters of the model satisfy $a_1 = 0.4$, $b_1 = 0.003$, $a_2 = 0.2$, $b_2 = 0.004$, $\gamma_1 = -0.1$, $\gamma_2 = -0.4$, $g_1 = 0.001$, $g_2 = 0.008$, $m_1 = 0.3$, $m_2 = 0.7$ and initial conditions are $x_1(0) = 40$, $x_2(0) = 20$.

Then the system (17) takes the form

$$\begin{cases} dx_1(t) = [(0.4 + 0.003 x_1(t))(100 - x_1(t) - x_2(t)) - 0.1(x_1(t) - 30)] dt \\ \qquad\qquad + 0.001 x_1(t)(100 - x_1(t) - x_2(t)) dw_1(t), \\ dx_2(t) = [(0.2 + 0.004 x_2(t))(100 - x_1(t) - x_2(t)) - 0.4(x_2(t) - 70)] dt \\ \qquad\qquad + 0.008 x_1(t)(100 - x_1(t) - x_2(t)) dw_2(t), \\ \qquad\qquad m_1 + m_2 = 1, \quad m_k \geqslant 0, \quad k = 1, 2. \end{cases} \quad (18)$$

The solutions of the system (18) are asymptotic stable in quadratic average in the first approximation in the area of special point $(30, 70)$ because the conditions are fulfilled

$$\begin{cases} \gamma_1 + \gamma_2 - a_1 - b_1 m_1 N - a_2 - b_2 m_2 N + \operatorname{Sp} B^{\mathrm{T}} B = -1.156 < 0, \\ 4(\gamma_1 - a_1 - b_1 m_1 N)(\gamma_2 - a_2 - b_2 m_2 N) - (a_1 + b_1 m_1 N + a_2 + b_2 m_2 N)^2 \\ \qquad + 2 \operatorname{Sp} B^{\mathrm{T}} B (\gamma_1 + \gamma_2) = 0.821 > 0. \end{cases}$$

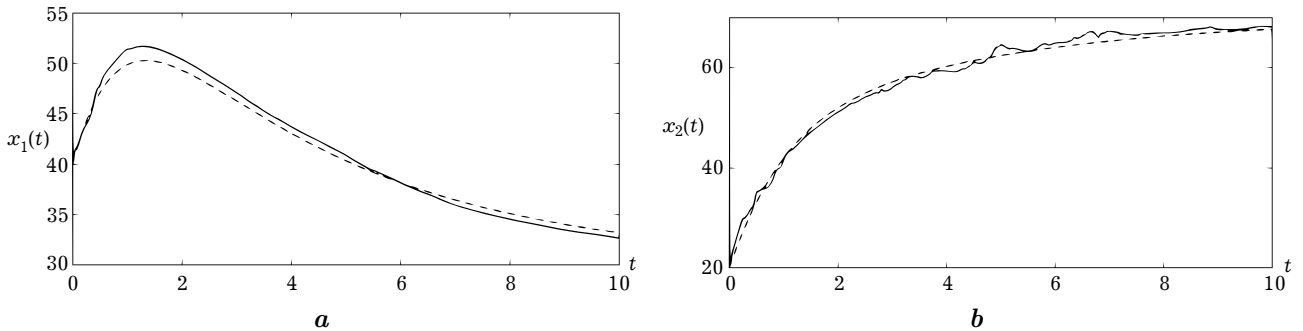Results of numerical experiment for system (18) are presented in Fig. 1.

Fig. 1. The dynamics of the mathematical model (18); a) dynamic of first equation of the model (18) where solid curve is for $x_1(t)$, dotted curve is for dynamics of spreading first type of information without disturbing influence; b) dynamic of second equation of the model (18) where full line is $x_2(t)$, dotted curve is for dynamics of spreading second type of information without disturbing influence.

5. Conclusions

We have considered the mathematical models of information spreading process with a disturbing influence to the parameters of the system. The analysis is conducted using the first approximation in neighbourhood of the special point. This approach makes it possible to have conditions of asymptotic stable in quadratic average in the area of special point. The numerical experiment demonstrates the practical meaning of the offered results. These results can be used to choosing strategy, to select values of parameters (characteristic of actions) and to achieve desirable results.

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Стійкість під час стохастичних збурень розв'язків у математичних моделях розповсюдження інформації зі зовнішніми впливами

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Наведено загальну схему аналізу стохастичної стійкості за першим наближенням в околі точок стійкості моделі розповсюдження довільної кількості типів інформації на прикладах узагальненої моделі з стаціонарними параметрами та моделі з нестаціонарними параметрами та спеціальним представленням зовнішнього впливу. Результати числового експерименту демонструють практичні можливості цієї схеми. Отримані результати дали змогу визначати для параметрів моделі допустимі області, значення з яких будуть гарантувати асимптотичну стійкість у середньоквадратичному за першим наближенням в околі стаціонарних точок.

Ключові слова: *математична модель поширення інформації, стохастична стійкість, асимптотична стійкість у середньоквадратичному, "білий" шум.*

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