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Stress-optimal modes of convective heating and electromagnetic radiation within infrared frequency range of the shells of revolution

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The numerical-analytical method of solving the problem of determining the stress-optimal modes of heating by convection and by heat sources due to electromagnetic radiation in the infrared range for piecewise-homogeneous shells of revolution is presented.

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In the present work, a numerical analytical method is proposed for determining optimal (in terms of stress) conditions for convective and volume heating (particularly by infrared radiation) of piecewise-homogeneous shells of revolution under given constraints for temperature and stresses.

The shell consists of the n homogeneous segments of the constant thickness $2h$ occupying the domain Ω. It is expedient to introduce mixed orthogonal coordinate system $\{\alpha_j(j = 1, 2), \gamma\}$, the coordinate axes of which $\alpha_j = \text{const}$ are lines of principal curvature, while γ is the normal to the median surface. Heating of the shell occurs on account of internal heat sources (volume heating) and convective heat transfer with the external atmosphere, the temperature of which is the control function.

The constrains on the temperature and temperature stress are specified in the form of domains of permissible variation

$$T_p^{\pm} \leqslant T_k^{\pm} \leqslant T_0^{\pm}; \quad T_{00}^{\pm} \leqslant \dot{T}_k^{\pm} \leqslant T_{**}^{\pm}; \quad T_s^{*\pm} \leqslant T_{s,k}^{\pm} \leqslant T_{s0}^{**\pm}; \tag{1}$$

$$\sigma_{0i,k}^{\pm} \leqslant \sigma_{i,k}^{\pm} \leqslant \sigma_{*i,k}^{\pm}, \tag{2}$$

where the plus and minus signs in the superscripts correspond to quantities at the external and internal surfaces, respectively; a prime denotes the time derivative; $T_k^{\pm}$, $T_{s,k}^{\pm}$ are the temperatures of the k-th shell, the interval, and external atmosphere at the surfaces $\gamma = \pm h$, respectively; $\sigma_{0i,k}^{\pm} \leqslant 0$, $\sigma_{*i,k}^{\pm} \geqslant 0$; $\sigma_{i,k}$ is the normal stress ($i = 1$ and $i = 2$ correspond to meridional and to annular stresses, respectively); $k = \overline{1, n}$ denotes the domains of homogeneity.

The shell is to be heated from the constant initial temperature T_p to the maximum temperature T_0 at the surface $\gamma = h$, and then cooled to the temperature T_* ($T_* \leqslant T_0$), with observing the conditions (1), (2).

Additional conditions of the form

$$F_1(\alpha_{1*}, \alpha_{2*}, \gamma_*, T_k, T_0, \dot{T}_k, T_*, t_0, t_*) \leqslant 0 \tag{3}$$

reflect the technological features of the heat treatment; α_{j*}, γ_* are the coordinates of a fixed shell cross section; t_0, t_* are the minimum times to reach the maximum T_0 and final T_* temperatures of the external shell surface, respectively. In particular, such conditions include the well-known target conditions for the shell temperature of a nonferrous TV picture tube in the thermal conditions of degasification (specification of the required initial, maximum, and final temperature at the external shell surface) Ref. [1].

The optimal condition is assumed to correspond to minimization of the functional of maximum normal stress

$$I = \max(\sigma_{i,k}(\alpha_1, \alpha_2, \gamma, t)), \quad \alpha_1, \alpha_2, \gamma \in \Omega, \quad 0 \leqslant t \leqslant t_* \tag{4}$$

characterizing the strength of the given glass shell Ref. [2].

The process of solving of defined optimization problem is based on principle of stage-by-stage parametrical optimization. On solving the direct problem, i.e. on finding temperature and stress in convective heating, the conditional extremum of functional Eq. (4) is found with the use of the method of local variations Ref. [3] in stationary space of control function. Note that solution of the direct problem can be found on the basis of any thermomechanical theory. The theory of thermoelasticity of shells with temperature-dependent thermal expansion coefficient is chosen here; this theory is often used to describe the mechanical behavior of glass shells Ref. [4].

The temperature field in the shell is then described by the heat conduction equation

$$\frac{\partial^2 T_k}{\partial \gamma^2} + p_k^2 T_k = -\frac{Q_{*k}}{\lambda_k}, \tag{5}$$

with the initial conductions

$$T_k(\alpha_1, \alpha_2, \gamma, 0) = T_0(\alpha_1, \alpha_2, \gamma) \equiv \text{const}, \tag{6}$$

and the conductions of convective heat transfer with the external atmosphere

$$\begin{aligned} &\frac{\partial T_k}{\partial \gamma} + H_k^+(T_k - T_{s,k}^+) = 0 \quad \text{when} \quad \gamma = h, \\ &\frac{\partial T_k}{\partial \gamma} + H_k^-(T_k - T_{s,k}^-) = 0 \quad \text{when} \quad \gamma = -h, \end{aligned} \tag{7}$$

where

$$p_k^2 = \Delta - \frac{1}{a_k}\frac{\partial}{\partial t}, \quad \Delta = \frac{1}{A_1 A_2}\left[\frac{\partial}{\partial \alpha_1}\left[\frac{A_2}{A_1}\frac{\partial}{\partial \alpha_1}\right] + \frac{\partial}{\partial \alpha_2}\left[\frac{A_1}{A_2}\frac{\partial}{\partial \alpha_2}\right]\right],$$

A_1, A_2 are the coefficient of the first quadratic form of the median surface; $Q_{*k}\,(\alpha_1, \alpha_2, \gamma, t)$ is the density of heat sources; a_k, λ_k are the thermal conductivity and thermal diffusivity; H_k^+, H_k^- are the relative heat-transfer coefficients with the lateral surfaces $\gamma = \pm h$.

In addition to the boundary conditions corresponding to convective heat transfer, a closed shell can also be in more complex conditions of heat transfer with the internal medium, which can be described in general form by the following functional relation

$$F_2(T_k, T_{s,k}, H_k^-, \lambda_k, \ldots) = 0 \tag{8}$$

between the temperature of the internal medium and the shell temperature and also the thermophysics characteristics of the material and the medium.

At the contact surface of different sections of the shell, the conditions of ideal thermal and mechanical contact are assumed Ref. [5]. The mechanical boundary conditions can by formulated in terms of both stress and displacement Ref. [5].

For approximate solution of the heat-conduction problem, the temperature distribution over the thickness coordinate γ is represented as the m-th order polynomial

$$T_k(\alpha_1, \alpha_2, \gamma, t) = \sum_{i=1}^{m} b_{i-1,k}(\alpha_1, \alpha_2, t)\gamma^{i-1}. \tag{9}$$

The function $b_{i-1,k}(\alpha_1, \alpha_2, t)$ is expressed in terms of the main characteristics of the temperature field over the shell thickness Ref. [6]

$$T_{p,k} = \frac{2p-1}{2h^p}\int_{-h}^{h} T_k \gamma^{p-1} d\gamma \tag{10}$$

and the specified boundary conditions (7). The equations for determining the p ($p = m-1$) mean that characteristics $T_{p,k}$ are obtained by multiplication of Eq. (5) by γ^{p-1} and integration with respect to this coordinate, taking into account Eq. (10). Assuming a cubic temperature distribution over the shell thickness, the following system of equations is obtained for $T_{1,k}$ and $T_{2,k}$

$$\begin{aligned}
&\left(\Delta - \frac{1}{a_k}\frac{\partial}{\partial t}\right) T_{1,k} - 2R_{1,k}T_{1,k} - 2R_{2,k}T_{2,k} = -W_{1,k} - 3\left(R_{4,k}T^{+}_{s,k} + R_{5,k}T^{-}_{s,k}\right),\\
&\left(\Delta - \frac{1}{a_k}\frac{\partial}{\partial t}\right) T_{2,k} + 6R_{3,k}T_{2,k} - 6R_{2,k}T_{1,k} = -W_{2,k} - 15\left(R_{7,k}T^{+}_{s,k} - R_{6,k}T^{-}_{s,k}\right),
\end{aligned} \tag{11}$$

where

$$R_{1,k} = 3\frac{3(Bi^{+}_k + Bi^{-}_k) + Bi^{+}_k Bi^{-}_k}{h^2 R_{8,k}}, \qquad R_{2,k} = \frac{15}{2}\frac{Bi^{+}_k - Bi^{-}_k}{h^2 R_{8,k}},$$

$$R_{3,k} = -5\frac{2(Bi^{+}_k + Bi^{-}_k) + Bi^{+}_k Bi^{-}_k + 3}{h^2 R_{8,k}}, \qquad R_{4,k} = \frac{(6 + Bi^{-}_k)Bi^{+}_k}{h^2 R_{8,k}},$$

$$R_{5,k} = \frac{(6 + Bi^{+}_k)Bi^{-}_k}{h^2 R_{8,k}}, \qquad R_{6,k} = \frac{(3 + Bi^{+}_k)Bi^{-}_k}{h^2 R_{8,k}},$$

$$R_{7,k} = \frac{(6 + Bi^{-}_k)Bi^{+}_k}{h^2 R_{8,k}}, \qquad R_{8,k} = 36 + 9(Bi^{+}_k + Bi^{-}_k) + 2Bi^{+}_k Bi^{-}_k,$$

$$W_k = \lambda_k^{-1} Q_{*k}(\alpha_1, \alpha_2, \gamma, t); \qquad W_{1,k} = \frac{1}{2h}\int_{-h}^{h} W_k(\alpha_1, \alpha_2, \gamma, t)d\gamma,$$

$$W_{2,k} = \frac{3}{2h^2}\int_{-h}^{h} \gamma W_k(\alpha_1, \alpha_2, \gamma, t)d\gamma,$$

$Bi^{\pm}_k \equiv H^{\pm}_k h$ are Biot coefficients.

The coefficients $b_{i-1,k}$ in Eq. (9) are determined from the system of equations obtained by substituting Eq. (9) into Eqs. (7) and (10), and take the form

$$\begin{aligned}
b_{0,k} &= \left[1 + \frac{h^2}{3}R_{1,k}\right] T_{1,k} + \frac{h^2}{3}R_{2,k}T_{2,k} - \frac{h^2}{2}(R_{4,k}T^{+}_{s,k} + R_{5,k}T^{-}_{s,k}),\\
b_{1,k} &= \frac{3h}{5}R_{2,k}T_{1,k} + \left[\frac{1}{h} - \frac{3h}{5}R_{3,k}\right] T_{2,k} - \frac{5}{2h}(R_{7,k}T^{+}_{s,k} - R_{6,k}T^{-}_{s,k}),\\
b_{2,k} &= -R_{1,k}T_{1,k} - R_{2,k}T_{2,k} + \frac{3}{2}(R_{4,k}T^{+}_{s,k} + R_{5,k}T^{-}_{s,k}),\\
b_{3,k} &= -\frac{1}{h}R_{2,k}T_{1,k} + \frac{1}{h}R_3 T_{2,k} + \frac{5}{2h}(R_{7,k}T^{+}_{s,k} - R_{6,k}T^{-}_{s,k}).
\end{aligned} \tag{12}$$

Initial and ideal contact heat conditions in terms of $T_{1,k}$, $T_{2,k}$ are as follows

$$T_{1,k} = T_p, \quad T_{2,k} = 0, \quad \frac{\partial T_{1,k}}{\partial t} = 0, \quad \frac{\partial T_{2,k}}{\partial t} = 0, \quad \text{for} \quad t = 0,$$

$$\begin{aligned} &T_{1,k} = T_{1,k+1}, \quad T_{2,k} = T_{2,k+1}, \\ &\lambda_k \frac{\partial T_{1,k}}{\partial \xi} = \lambda_{k+1} \frac{\partial T_{1,k+1}}{\partial \xi}, \quad \lambda_k \frac{\partial T_{2,k}}{\partial \xi} = \lambda_{k+1} \frac{\partial T_{2,k+1}}{\partial \xi}, \end{aligned} \tag{13}$$

where $\boldsymbol{\xi}$ is the vector normal to contact surface.

Constitutive correlations are formulated both in the coordinates $\{\alpha_1, \gamma\}$ and in canonical coordinates $\{s, \gamma\}$. Grid method for determination of integral characteristics at unconditionally stable difference scheme is involved to find the solution of thermal conductivity equation.

Discrete values of integral characteristics are used for determination, according to known correlations, the temperature in grid points.

As an example, consider the optimal (in terms of the stress) conditions of uniform heating of the external atmosphere with specified heat sources for a piecewise-homogeneous cylindrical shell that consists of three different parts. The internal shell surface is heat-insulated, and the control function is the time-varying temperature of the external shell surface $T^+(t)$ satisfying conditions of the form

$$T^+(0) = T_p; \quad T^+(t_*) = T_0; \quad \frac{dT^+(t)}{dt} \leqslant 10^\circ\text{C/min}, \quad \frac{dT^+(t_*)}{dt} = 0, \tag{14}$$

which represent a particular case when the first two conditions of (13) correspond to Eq. (1) and the second two conditions correspond to Eq. (3). Here t_* is the time to heat the surface from the initial temperature to the maximum temperature T_0. Heat sources of the constant density ($Q_{*k} = 10^5\,\text{W/m}^3$) act in the shell.

Numerical investigation are conducted for the shell of the radius $R = 0.25\,\text{m}$ and the thickness $2h = 0.014\,\text{m}$ made of glass with the following characteristics

$$E_1 = 65.4\,\text{GPa}, \quad E_2 = 75.6\,\text{GPa}, \quad E_3 = 63.3\,\text{GPa};$$

$$\lambda_1 = 1.63\,\text{Wt/(m K)}, \quad \lambda_2 = 0.065\,\text{Wt/(m K)}, \quad \lambda_3 = 0.74\,\text{Wt/(m K)};$$

$$C_1 = 795\,\text{J/(kg K)}, \quad C_2 = 239\,\text{J/(kg K)}, \quad C_3 = 736\,\text{J/(kg K)},$$

$$\rho_1 = 2560\,\text{kg/m}^3, \quad \rho_2 = 4080\,\text{kg/m}^3, \quad \rho_3 = 2800\,\text{kg/m}^3;$$

$$\nu_1 = \nu_2 = \nu_3 = 0.215.$$

The temperature dependence of the coefficients of linear thermal expansion is shown in Fig. 1.

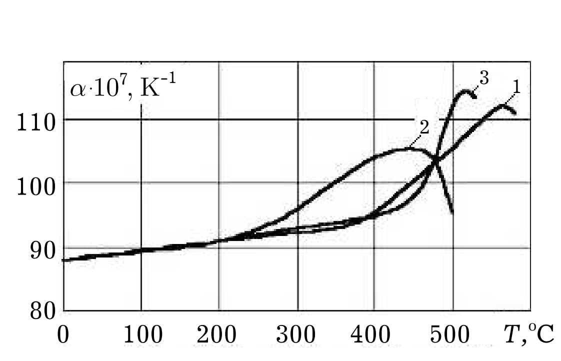

Fig. 1. Temperature dependence of linear temperature expansions coefficients.

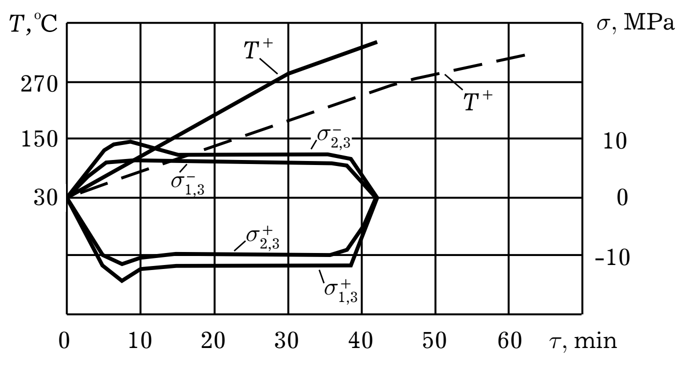

Fig. 2. Time dependence of optimum temperature and stresses.

The optimal variation in time of the control function — the temperature $T^+(t)$corresponding to the annular and meridional temperature stress at the external and internal shell surfaces in cross section $x = 0.001$ (chosen as a result of analysis of the maximum temperature stress) is shown in Fig. 2.

Then, the discrete values of the key functions, forces, moments, and stresses for each point s_n, t_m are determined on the basis of structure of the general solutions of mechanics equations for shells of the given type Refs. [4, 5, 7] and of piecewise-linear approximation of the temperature dependence of the thermal-expansion coefficient.

Outlined optimization method consists of two iterative processes: variation process for the control function $T_{s,k} \equiv \{f_k(t_i)\}$ at discrete time instants with fixed variation step δ and process of dividing of this step until its given value is attained. In this process, the choice of zero approximation for the control function, which convergence of the iterative process depends upon, is important. To construct such an approximation, the iterative algorithm is developed. This algorithm is founded on the usage as an initial control function, the optimal (in terms of the stress) convective heating of homogeneous spherical shell, with its following correction.

In many cases, piecewise-homogeneous glass shells are made of materials with similar thermophysical characteristics. For such shells, with heat sources that do not depend on the coordinate s, effective target conditions of uniform temperature of the external atmosphere (depending only on the time t) can be constructed. In this case, the temperature variation along the meridional coordinate $s$is slight and can be neglected. Consequently, it is assumed that the temperature in each part of the shell is a function of the thickness γ and time t, and the displacement is a function of the time and the coordinates γ, s. Then governing equations system is one of first-order differential equations with constant coefficients. Note that its solution may be found more effectively (in terms of computer time and memory requirements) by the least-squares method Ref. [8, 9] with a finite-element approximation of $T_s(t)$, rather than by the difference method outlined above. This significantly simplifies the procedure for numerical determination of the thermos-stressed state parameters of a piecewise-homogeneous shell (the procedure for solving the direct problem) in the optimization algorithm. In heating from the initial to the maximum temperature, the greatest tensile temperature stress is the annular stress that arises at the internal shell surface. The optimal heating conditions are calculated such that the permissible tensile stress for all stress components is the same. The dashed curve corresponds to the optimal heating conditions in the absence of heat sources and with the same constraints for the tensile stress at the internal surface. As it is evident, the use of additional heating in this case permits the reduction in the heating period to 20 min with the same maximum heating temperature.

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Оптимальний за напруженнями режим нагріву конвективним способом та електромагнітним випромінюванням інфрачервоного діапазону частот оболонок обертання

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Запропоновано числово-аналітичний метод знаходження оптимальних за напруженнями режимів нагріву конвективним способом та джерелами тепла, створюваних електромагнітним випромінюванням інфрачервоного діпазону частот кусково-однорідних оболонок обертання.

Ключові слова: *оболонки, конвективне та випромінювальне нагрівання, оптимальні за напруженнями режими.*

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