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Effect of surface tension on the antiplane deformation of bimaterial with a thin interface microinclusion

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Within the framework of the concept of micromechanics, a method for taking into account the effect of surface energy for a thin interface micro-inclusion in the bimaterial under conditions of longitudinal shear has been proposed. The possibility of non-ideal contact between inclusion and matrix is provided, in particular, tension contact. This significantly extends the scope of applicability of the results. A generalized model of a thin inclusion with arbitrary elastic mechanical properties was built. Based on the application of the theory of functions of a complex variable and the jump function method, the stress field in the vicinity of the inclusion during its interaction with the screw dislocation was calculated. Several effects have been identified that can be used to optimize the energy parameters of the problem.

Keywords: *micro-inhomogeneities, bimaterial, surface tension, nonperfect contact, jump functions, dislocation.*

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1. Introduction

The use of nanocomposites with specific properties in engineering and technology has significantly shifted the interest from the study of objects at the macro-level ($10^0 - 10^{-1}$ m) and micro-level ($10^{-3} - 10^{-6}$ m) to the nano-level (10^{-9} m) [1–4]. The main difficulty in modeling nanostructures in comparison with the macro-level requires the construction of complex constitutional laws [2]. Such modeling is mainly based on the concept of "representative volume element" (RVE) [2–4] to clarify the classical continuum model by certain related relations characterizing the presence and basic properties of structural heterogeneities. In particular, taking into account the influence of the surface mechanics of the structural boundaries of the composite components becomes more noticeable on a nanoscale. One of the ways of such accounting was empirical when a scaling law [5–7] with its scale was developed based on various studies of the effect of surface tension and surface energy [8–13]. This allowed for an approximate consideration of changes in basic non-dimensional mechanical properties depending on the characteristic size of the nanoscale. A considerable amount of work has been devoted to the consideration of surface stresses and energy [5, 8–10, 12–15]. In particular, most of them use Eshelby's theory of ellipsoidal heterogeneity [6, 7, 11, 16, 17]. However, the phenomenon of "polynomial conservatism", according to which if the infinity of the stress field is a polynomial of some order, then inside the inclusion the stress field is characterized by polynomials of the same order, essentially limits the analysis of the stress-strain state (SSS) inside the inhomogeneity. Several works are devoted to the analysis of the influence of surface stresses on cracked medium [18]. The proposed general theory is suitable for any type of loads (on infinity, concentrated forces, moments, dipoles, and dislocations, the inclusion is located not in a homogeneous matrix, and lies on the boundary of two different media, it may have an arbitrarily variable small thickness and physically nonlinear mechanical properties). This study is designed to investigate, within the framework of the micromechanics concept, the effect of surface stresses in a thin interphase linear elastic inclusion and its vicinity, whose SSS is formed by force factors and dislocations.

2. Formulation of the problem

Let us consider the antiplane deformation of the structure consisting of a cross-section of two half-spaces with an elastic constant G_1, G_2 by a plane xOy perpendicular to the direction z of its longitudinal shear. The flat cross-sections of the half-spaces form two half-places S_k $(k = 1, 2)$ and the interface between them corresponds to the abscissa axis $L \sim x$. There is a thin inclusion $2h$ $(h \ll a)$ with orthotropic mechanical properties G_y^{in}, G_x^{in} along the segment (Fig. 1).

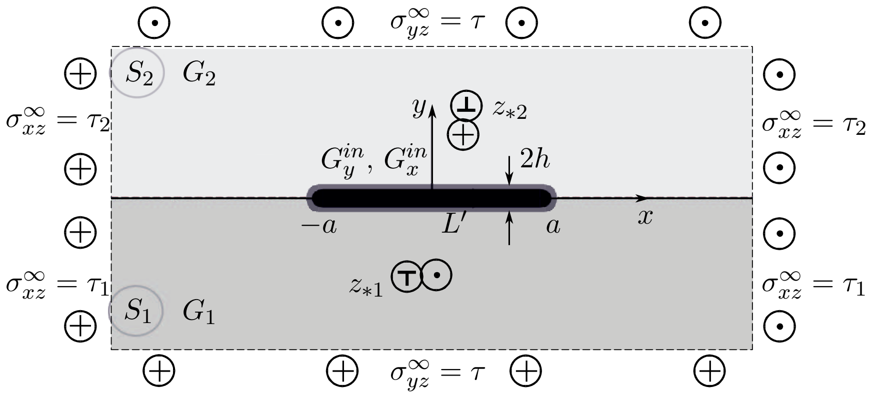

Fig. 1. Geometry and loading scheme of the problem.

The magnitude and direction of action of external force factors implementing longitudinal shift of the array change quasistatically and provide for the presence of evenly distributed at infinity stresses $\sigma_{yz}^{\infty} = \tau$, $\sigma_{xzk}^{\infty} = \tau_k$, concentrated forces of intensity Q_k, screw dislocations with Burgers vector component b_k at points $z_{*k} \in S_k$ $(k = 1, 2)$. To ensure straightness, the material interface at infinity must meet the condition $\tau_2 G_1 = \tau_1 G_2$. We consider the ideal contact between the half-space along the line $L'' = L/L'$

$$w_1(x, +0) = w_1(x, -0), \quad \sigma_{yz2}(x, +0) = \sigma_{yz1}(x, -0), \quad x \in L'' \tag{1}$$

and between the inclusion banks and the matrix along L' accept the conditions of contact with additional tension:

$$w^{in}(x, \pm h) = w(x, \pm h), \quad \sigma_{yz}^{in}(x, \pm h) = \sigma_{yzk}(x, \pm h) - T_k, \quad k = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix}, \tag{2}$$

where σ_{yz}, w are the components of stress tensor and displacement vector, additional surface tenses T_k may be dependent on both the SSS intensity and the material properties of the bulk and inclusion. The upper index "in" indicates the SSS components in the inclusion area. Hereinafter the "+" sign corresponds to the value of $k = 2$ and "−" sign corresponds to the value of $k = 1$.

3. Thin inclusion model

We consider that the presence of a thin inclusion in the bulk is modeled by a SSS perturbation — a jump of the components of the stress f_3 and displacement f_6 vectors on L' [19–24]:

$$\sigma_{rz}(x, y) = \sigma_{rz}^0 + \hat{\sigma}_{rz}(x, y), \quad r = \{x, y\}, \\ w(x, y) = w^0(x, y) + \hat{w}(x, y), \tag{3}$$

$$\left[\sigma_{yz}\right]_h \cong \sigma_{yz}^- - \sigma_{yz}^+ = f_3(x), \\ \left[\frac{\partial w}{\partial x}\right]_h = \frac{\partial w^-}{\partial x} - \frac{\partial w^+}{\partial x} = \left[\frac{\sigma_{xz}}{G}\right]_h \equiv \frac{\sigma_{xz}^-}{G_1} - \frac{\sigma_{xz}^+}{G_2} = f_6(x), \quad x \in L'; \tag{4}$$

$f_3 = f_6(x) = 0$, if $x \in L'$. Hereinafter it is denoted: $[\varphi]_h = \varphi(x, -h) - \varphi(x, +h)$, $\langle\varphi\rangle_h = \varphi(x, -h) + \varphi(x, +h)$; the upper indexes "+" and "−" correspond to the limit values of functions at the upper and lower edges of the line; the values marked with the index “0” at the top, characterize the corresponding values in a bulk without inhomogeneities at the corresponding external load (homogeneous solution) [24], and the values marked with the symbol “$\wedge$” at the top are the perturbations of the main SSS field.

The mathematical model of a thin inclusion and layers of thin coverage is presented as the so-called interaction conditions [19, 24], which are equivalent to the conditions of nonideal contact between the matrix surfaces adjacent to the inclusion.

The main relations for any inclusion material are equilibrium conditions

$$\frac{\partial \sigma_{xz}^{in}}{\partial x} + \frac{\partial \sigma_{yz}^{in}}{\partial y} + \rho F^{in} = 0, \tag{5}$$

constitutive dependence of deformations on stresses of the kind

$$\sigma_{xz}^{in} = G_x^{in} \frac{\partial w^{in}}{\partial x}, \quad \sigma_{yz}^{in} = G_y^{in} \frac{\partial w^{in}}{\partial y} \tag{6}$$

and thinwallness ratio

$$\frac{\partial w^{in}}{\partial y}(x,h) + \frac{\partial w^{in}}{\partial y}(x,-h) \cong \frac{w^{in}(x,h) - w^{in}(x,-h)}{h} = -\frac{[w^{in}]_h}{h} \tag{7}$$

Then, taking into account (5)–(7), the model of a thin orthotropic inclusion is described quite accurately by two equations:

$$\frac{G_x^{in}}{2} \left\langle \frac{\partial w^{in}}{\partial x} \right\rangle_h - \sigma_{xz}^{in}(-a) - \frac{1}{2h} \int_{-a}^{x} \left[\sigma_{yz}^{in}\right]_h (\xi) d\xi + F_{aver}^{in}(x,h) = 0, \tag{8}$$

$$-\frac{\left[w^{in}\right]_h}{h} = \frac{\langle \sigma_{yz}^{in} \rangle_h}{G_y^{in}}, \tag{9}$$

where $F_{aver}^{in}(x,h) = \frac{\rho}{2h} \int_{-h}^{h} \int_{-a}^{h} F^{in}(\xi, y) d\xi dy$ ρ is the density of the inclusion material and F^{in}. Substitution of boundary conditions (2) taking into account (3) in the model (8), (9) allows to obtain a system of constitutive equations for solving the problem:

$$\int_{-a}^{x} f_3(\xi) d\xi = -N_{xz}(-a) + \omega_x^{in} \left\langle \frac{\partial w}{\partial x} \right\rangle_h + (x+a)(T_1 - T_2) + 2hF_{aver}^{in}(x,h), \tag{10}$$

$$\int_{-a}^{x} f_6(\xi) d\xi = -[w]_h(-a) + h \left\langle \frac{\sigma_{yzk}}{G_k} \right\rangle_h - \omega_y^{in} \left\{ \langle \sigma_{yzk} \rangle_h - T_1 - T_2 \right\}. \tag{11}$$

Here $\omega_x^{in} = hG_x^{in}$, $\omega_y^{in} = h/G_y^{in}$, $N_{xz}(-a,x) = 2h\sigma_{xz}^{in}(-a)$.

The adequacy of the model (10)–(11) is verified by particular cases:

$A)$ $h \to 0$, $\left[\frac{\partial w}{\partial x}_{0,h}\right] \to 0$, $[w]_{0,h} \to 0$;

$B)$ $G_y^{in} \to 0$: $\int_{-a}^{x} [\sigma_{yz}]_h(\xi) d\xi + N_{xz}(-a) - (x+a)(T_1 - T_2) - 2hF_{aver}^{in}(x,h) = 0$;

$C)$ $G_y^{in} \to \infty$: $\int_{-a}^{x} (\xi) d\xi + w(-a) - h \left\langle \frac{\sigma_{yz}^0}{G_k} \right\rangle_h = 0$ or $[w]_h \to 0$;

$D)$ $G_y^{in} \to \infty$: $\left\langle \frac{\partial w}{\partial x} \right\rangle \to 0$, $[w]_h \to 0$;

$E)$ $G_y^{in} \to 0$: $\langle \sigma_{yzk} \rangle_h \to T_1 + T_2$;

$F)$ $G_y^{in} \to G_k$: $[w]_h = \frac{h}{G_y^{in}}(T_1 + T_2)$.

Also, the constructed model generalizes the known models of “surface layer” [8,10,14,25] for the case of an open contour, as well as, as a special case, the model of interphase crack with surface tension [18].

4. Construction of integral equations using the jump function method

Applying the method [19–24] to solve the problem, one can obtain dependencies according to which the stress tensor components and derivatives of displacement vectors inside the infinite plane as well as on the line are equal to

$$
\begin{gathered}
\sigma_{yzk}^{\pm}(x) = mp_k f_3(x) - C g_6(x) + \sigma_{yzk}^{0\pm}(x),\\
\sigma_{xzk}^{\pm}(x) = mC f_6(x) + p_k g_3(x) + \sigma_{xzk}^{0\pm}(x),\\
\frac{\partial w^{\pm}}{\partial y}(x) = mpf_3(x) - p_{3-k} g_6(x) + \frac{\sigma_{yzk}^{0\pm}(x)}{G_k},\\
\frac{\partial w^{\pm}}{\partial x}(x) = mp_{3-k} f_3(x) - pg_6(x) + \frac{\sigma_{xzk}^{0\pm}(x)}{G_k},\\
\sigma_{yz}(\varsigma) + i\sigma_{xz}(\varsigma) = \sigma_{yz}^{0}(\varsigma) + i\sigma_{xz}^{0}(\varsigma) + ip_k g_3(\varsigma) - C g_6(\varsigma) \quad k = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix},\\
(\varsigma = x + iy \in S_k; \quad r = 3, 6; \quad k = 1, 2)
\end{gathered}
\tag{12}
$$

where

$$
g_r(\varsigma) \equiv \frac{1}{\pi} \int_{L'} \frac{f_r(x)dx}{x - \varsigma}, \quad s_r(x) \equiv \int_{-a}^{x} f_r(x)dx, \quad C = G_{3-k}p_k, \quad p_k = pG_k, \quad p = \frac{1}{G_1 + G_2}.
$$

By substituting (12) in the model (10)–(11) we obtain a system of singular integral equations (SSIE).

$$
\begin{cases}
\alpha_1 f_6(x) + \beta_1 g_3(x) - \delta_1 s_3(x) = F_3(x),\\
\alpha_2 f_3(x) + \beta_2 g_6(x) - \delta_2 s_6(x) = F_6(x),
\end{cases}
\tag{13}
$$

$$
\begin{gathered}
\alpha_1 = p_2 - p_1, \quad \beta_1 = 2p, \quad \gamma_1 = a/\omega_x^{in},\\
F_3(x) = \gamma_1 \Big\{ N_{xz}(-a) - (x + a)(T_1 - T_2) - 2hF_{aver}^{in}(x, h) \Big\} - \left\langle \frac{\sigma_{xzk}^{0}}{G_k} \right\rangle_h,\\
\alpha_2 = p_2 - p_1, \quad \beta_1 = 2C - G_y^{in}, \quad \gamma_2 = a/\omega_y^{in},\\
F_6(x) = \gamma_2 \left\{ [w](-a) - h\left\langle \frac{\sigma_{yzk}^{0}}{G_k} \right\rangle_h \right\} + \langle \sigma_{yzk}^{0} \rangle_h - T_1 - T_2,
\end{gathered}
$$

with additional conditions of balance

$$
\int_{-a}^{a} f_3(\xi)d\xi = N_{xz}(a) - N_{xz}(-a) + 2h\rho F_{aver}^{in}(a, h) + 2a(T_1 - T_2),
$$

$$
\int_{-a}^{a} f_6(\xi)d\xi = [w]_h(a) - [w]_h(-a).
\tag{14}
$$

If besides, the materials of the matrix are identical, the SSIE (13) is separated into two independent equations

$$
\begin{cases}
\beta_1 g_3(x) - \delta_1 s_3(x) = F_3(x),\\
\beta_2 g_6(x) - \delta_2 s_6(x) = F_6(x).
\end{cases}
\tag{15}
$$

To assess the impact of the inclusion on SSS, it is convenient to analyze some characteristics, in particular, energy and stress intensity factors. The expression for the inclusion deformation energy is

$$
W^d = \int_{L'} \big(\sigma_{yz2}(x, h)w(x, h) - \sigma_{yz1}(x, -h)w(x, -h)\big)dx,
\tag{16}
$$

and it can be somewhat simplified for the case of soft inclusion.

$$W^d = W_0^d + W_L^d, \quad W^d = -\frac{1}{2}\int_{L'} \langle \sigma_{yzk} \rangle_h(x)[w]_h(x)dx, \tag{17}$$

where

$$W_L^d = -\frac{1}{2}(T_2 + T_1)\int_{L'} [w]_h(x)dx, \quad W_0^d = -\frac{1}{2}\int_{L'} \langle \sigma_{yz}^{in} \rangle_h(x)[w]_h(x)dx,$$

where W_0^d is deformation energy of the direct inclusion, W^d is deformation energy in the matrix, W_L^d is deformation energy at the inclusion-matrix boundary. And if (11) is taken into account, the expressions for the energy can be written as

$$W_0^d = \frac{\omega_y^{in}}{2}\int_{L'} \left\langle \sigma_{yz}^{in} \right\rangle_h^2(x)dx, \quad W_L^d = \frac{\omega_y^{in}}{2}(T_2 + T_1)\int_{L'} \left\langle \sigma_{yz}^{in} \right\rangle_h(x)dx, \tag{18}$$

The generalized stress intensity factors (SIF) are introduced into consideration by the expression [24]

$$K_{31} + iK_{32} = \lim_{r \to 0(\theta=0)} \sqrt{2\pi r}(\sigma_{yz} + i\sigma_{xz}) \tag{19}$$

It is also possible to use the above method to determine the forces acting on a dislocation with Burgers vector b in a point z_{*k} by the Peach–Koehler formulas [2, 24],

$$F_x(z_{*k}) = b\hat{\sigma}_{yz}(z_{*k}), \quad F_y(z_{*k}) = -b\hat{\sigma}_{xz}(z_{*k}) \tag{20}$$

The method [24, 26] can be applied to the SSIE solution (13)–(14). As a result of the application of this technique SSIE (15) is reduced to a system of linear algebraic equations to the unknown coefficients of the decomposition of the jump functions $f_r(x)$ in series of orthogonal polynomials of Jacobi or Chebyshev.

5. Numerical analysis

For an illustration of the research technique, we will make a detailed analysis of the problem solution for a particular case of elastic characteristics equality of half-spaces ($G_1 = G_2 = G$, $T_1 = T_2 = T$) and the presence of b intensity dislocation at a point $z_2 = x_2 + iy_2$. Thereinafter, the calculations were carried out for dimensionless values

$$\tilde{b} = b/\pi a, \quad \tilde{x}_2 = x_2/a, \quad \tilde{y}_2 = y_2/a, \quad \tilde{T}_2 = \tilde{T}_1 = T/G, \quad \tilde{G}_{in} = G^{in}/G, \quad \tilde{\sigma}_{yz} = \sigma_{yz}/G,$$
$$\tilde{W}^d = \tilde{W}^d/a^2G, \quad \tilde{W}_{in}^d/a^2G, \quad \tilde{K}_{31} = K_{31}/G\sqrt{\pi a}, \quad \tilde{K}_{32} = K_{32}/G\sqrt{\pi a},$$
$$\tilde{F}_x = F_x/a\pi G, \quad \tilde{F}_y = F_y/a\pi G.$$

In this case, the forces of additional surface tension were taken both constant and dependent on the elastic properties of the inclusion in the form $\tilde{T}_2 = \tilde{T}_1 = k_T(G_{in})^\alpha$.

Figures 2, 3 illustrate the effect of surface tension on the energy of deformation $\tilde{W}^d$ of the matrix and the inclusion $\tilde{W}_{in}^d$ in the range of changes in coordinates $(\tilde{x}_2, \tilde{y}_2)$ of the point of application of the dislocation for a soft inclusion. The main tendencies for strain energy change are as follows: 1) in general, the appearance of additional surface tension reduces the strain energy intensity and stress fields in the vicinity of the soft inclusion; 2) rapid decrease of the dislocation effect with its distance from the inclusion axis to the SSS field and, consequently, the strain energy; 3) the presence of a local energy extremum when the dislocation point $\tilde{x}_2$ is shifted approximately to $1.5a$ along the inclusion axis (Fig. 3).

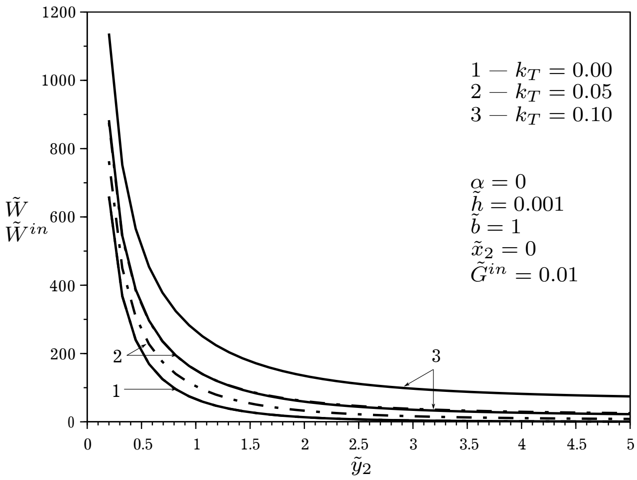

Fig. 2. Effect of coefficient on the energy of deformation $\tilde{W}^d$ of the matrix (solid curve) and the inclusion $\tilde{W}^d_{in}$ (dashed-dotted curve) when the dislocation is distanced from the axis of the soft inclusion.

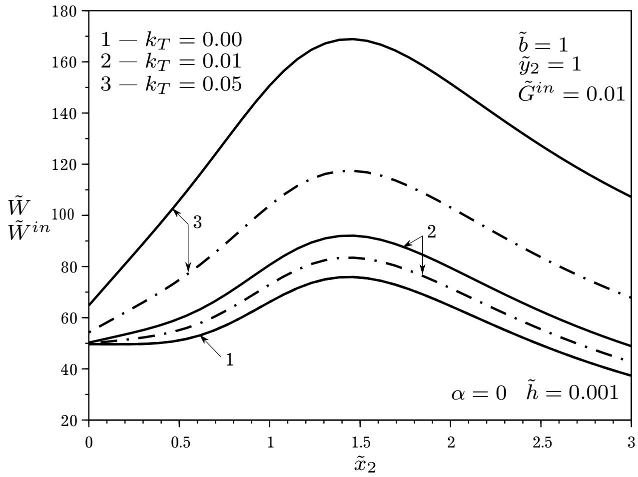

Fig. 3. Effect of coefficient k_T on the energy of deformation of the matrix $\tilde{W}^d$ (solid curve) and the inclusion $\tilde{W}^d_{in}$ (dashed-dotted curve) when the point of application of the dislocation is shifted along the axis of the soft inclusion.

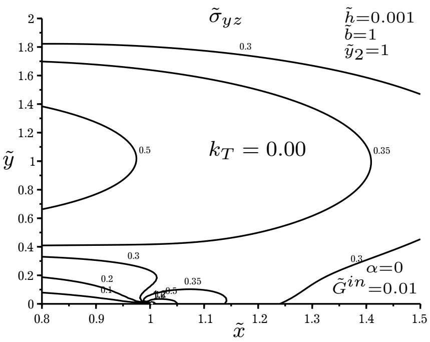

Fig. 4. Change of the stress field $\tilde{\sigma}_{yz}$ in the vicinity of the inclusion tip under no surface tension.

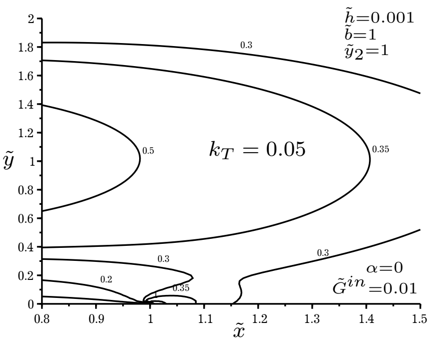

Fig. 5. Change of the stress field $\tilde{\sigma}_{yz}$ in the vicinity of the switch-on tip when a tension appears.

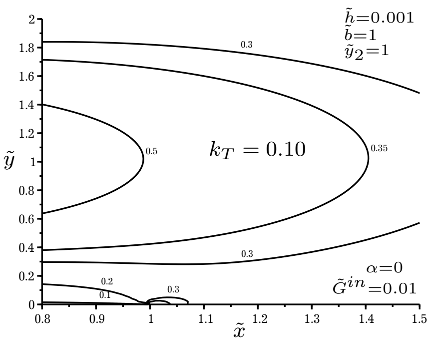

Fig. 6. Change of the stress field $\tilde{\sigma}_{yz}$ in the vicinity of the switch-on tip when a tension appears.

The effect of tension on the stress field $\tilde{\sigma}_{yz}$ in the vicinity of the switch-on is shown in Figs. 4–8. The appearance of a surface tension reduces the intensity of a field $\tilde{\sigma}_{yz}$ especially appreciably in a soft range of $\tilde{G}_{in}$ (Figs. 4–6). It was found out that the surface tension significantly affects only by $\tilde{K}_{31}$ reducing it while $\tilde{K}_{32}$ practically does not change (Figs. 7, 8). At the same time, the distance from the inclusion of a dislocation location point perpendicular to its axis is expected to reduce $\tilde{K}_{31}$, while there is an extremum $\tilde{K}_{32}$ at an approximate height a. Changing the location point of a dislocation along the inclusion axis significantly affects both SIFs, decreasing $\tilde{K}_{31}$ to an extremum at a distance of about half a length of the inclusion length from its tip and increasing $\tilde{K}_{32}$ to an extremum above the inclusion tip (Fig. 8).

The forces acting on a dislocation (Figs. 9–12) significantly depend on the location application coordinates and mechanical properties of the inclusion. Thus, regardless of the stiffness, there is a local extremum $\tilde{F}_x$, $\tilde{F}_y$ when the dislocation is located approximately above the inclusion tip (Figs. 11, 12). At the same time, the inclusion has almost no effect on dislocation already at an altitude of approximately (Figs. 9, 10). The effect of tension on $\tilde{F}_x$, $\tilde{F}_y$ is noticeable only for a soft inclusion. All calculations were performed in the Scilab system.

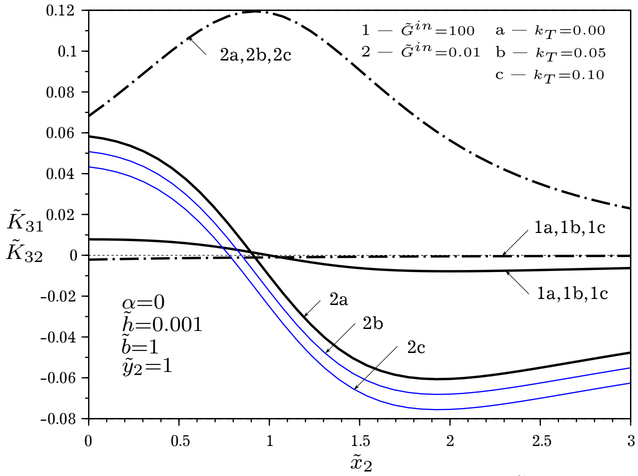

Fig. 7. Effect of coefficient on the SIFs $\tilde{K}_{31}$ (solid curve) and $\tilde{K}_{32}$ (dashed-dotted curve) when the dislocation is distanced from the inclusion axis.

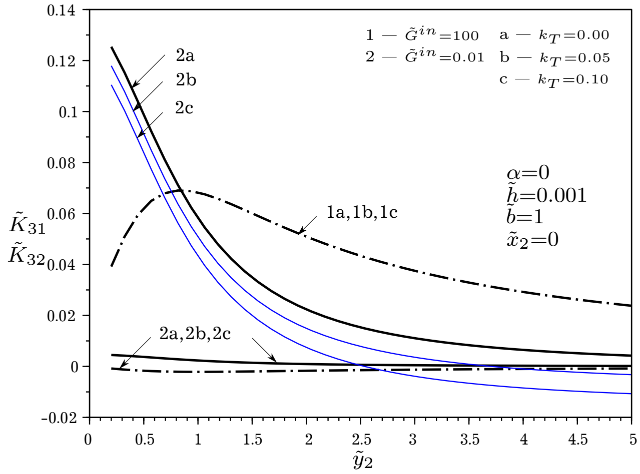

Fig. 8. Effect of coefficient k_T on the SIFs $\tilde{K}_{31}$ (solid curve) and $\tilde{K}_{32}$ (dashed-dotted curve) when the point of application of the dislocation is shifted along the axis of the inclusion.

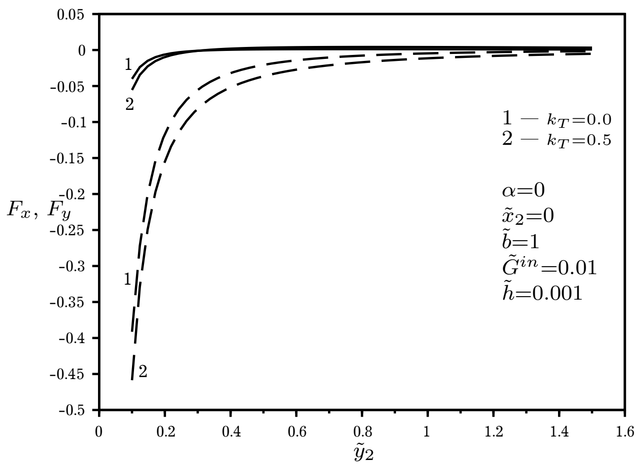

Fig. 9. Effect of coefficient k_T on forces $\tilde{F}_x$ (solid curve), $\tilde{F}_y$ (dashed curve), acting on the dislocation at its distance from the soft inclusion axis.

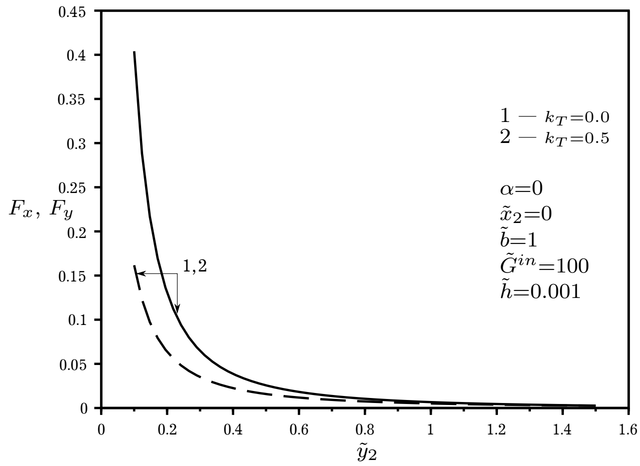

Fig. 10. Effect of coefficient k_T on forces $\tilde{F}_x$ (solid curve), $\tilde{F}_y$ (dashed curve), acting on the dislocation at its distance from the rigid inclusion axis.

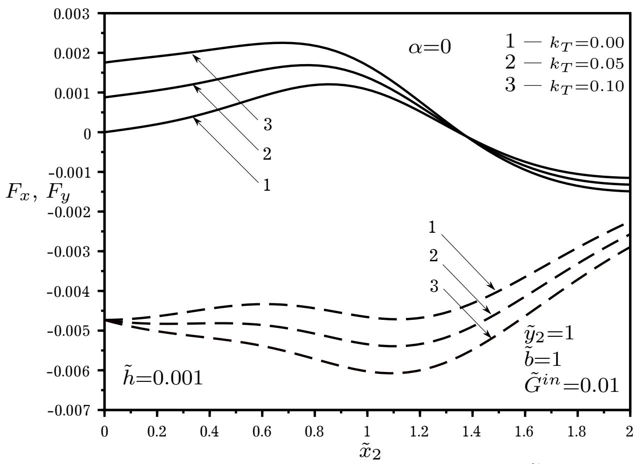

Fig. 11. Effect of coefficient k_T on forces $\tilde{F}_x$ (solid curve), $\tilde{F}_y$ (dashed curve), acting on the dislocation when the point of application of the dislocation is shifted along the axis of the soft inclusion.

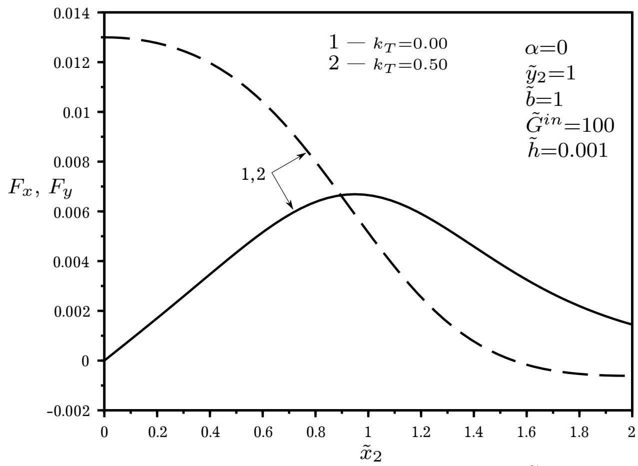

Fig. 12. Effect of coefficient k_T on forces $\tilde{F}_x$ (solid curve), $\tilde{F}_y$ (dashed curve), acting on the dislocation when the point of application of the dislocation is shifted along the axis of the rigid inclusion.

6. Conclusions

The proposed rather simple and mathematically correct methodology allowed us to construct a model of a deformable thin linear interphase inclusion introduced with tension into the matrix. Some effects from the presence of surface tension on SSS in the matrix and inclusion in the presence of dislocation have been revealed. In particular, the appearance of surface tension reduces the intensity of deformation energy and stress fields in the vicinity of the inclusion. There are certain combinations of mechanical inclusion parameters and dislocation location, when the extremes of strain energy, stress intensity coefficients, and forces acting on the dislocation are well expressed.

The above effects can be used for SSS optimization in the problem under consideration.

[1] Kizler P., Uhlmann D., Schmauder S. Linking nanoscale and macroscale: calculation of the change in crack growth resistance of steels with different states of Cu precipitation using a modification of stress-strain curves owing to dislocation theory. Nuclear Engineering and Design. **196** (2), 175–183 (2000).

[2] Kizler P., Uhlmann D., Schmauder S. Linking nanoscale and macroscale: calculation of the change in crack growth resistance of steels with different states of Cu precipitation using a modification of stress-strain curves owing to dislocation theory. Naukova dumka, Kyiv (1978), (in Ukrainian).

[3] Mura T. Micromechanics of Defects in Solids. Springer, Dordrecht (1987).

[4] Nemat-Nasser S., Hori M. Micromechanics: overall properties of heterogeneous materials. Elsevier, Amsterdam (1999).

[5] Sharma P., Ganti S., Bhate N. Effect of surfaces on the size-dependent elastic state of nano-inhomogeneities. Appl. Phys. Lett. **82** (4), 535–537 (2003).

[6] Duan H. L., Wang J., Huang Z. P., Karihaloo B. L. Eshelby formalism for nano-inhomogeneities. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences. **461** (2062), 3335–3353 (2005).

[7] Wang J., Karihaloo B. L., Duan H. L. Nano-mechanics or how to extend continuum mechanics to nano-scale. J. Bulletin of the Polish academy of sciences. Technical sciences. **55** (2), 133–140 (2007).

[8] Gurtin M. E., Murdoch A. I. Surface Stress in Solids. International Journal of Solids and Structures. **14** (6), 431–440 (1978).

[9] Hrytsyna O. Determination of solids surface energy. Physico-mathematical modelling and information technologies. **17**, 43–54 (2013), (in Ukrainian).

[10] Kim C. I., Schiavone P., Ru C. Q. The Effects of Surface Elasticity on Mode-III Interface Crack. Archives of Mechanics. **63** (3), 267–286 (2011).

[11] Kushch V. I., Shmegera S. V., Buryachenko V. A. Interacting elliptic inclusions by the method of complex potentials. Interacting elliptic inclusions by the method of complex potentials. **42** (20), 5491–5512 (2005).

[12] Povstenko Yu. Z. Theoretical investigation of phenomena caused by heterogeneous surface-tension in solids. Journal of the Mechanics and Physics of Solids. **41** (9), 1499–1514 (1993).

[13] Steigmann D. J., Ogden R. W. Elastic surface — substrate interactions. Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences. **455** (1982), 437–474 (1999).

[14] Gurtin M. E., Murdoch A. I. A continuum theory of elastic material surfaces. Archive for Rational Mechanics and Analysis. **57**, 291–323 (1975).

[15] Sharma P., Ganti S. Size-Dependent Eshelby's Tensor for embedded nano-inclusions incorporating surface/interface energies. J. Appl. Mech. **71** (5), 663–671 (2004).

[16] Eshelby J. D. The determination of the elastic field of an ellipsoidal inclusion and related problems. Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences. **241** (1226), 376–396 (1957).

[17] Wang J., Duan H. L., Huang Z. P., Karihaloo B. L. A scaling law for properties of nano-structured materials. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences. **462** (2069), 1355–1363 (2006).

[18] Wang Xu, Schiavone P. A mode III interface crack with surface strain gradient elasticity. Journal of integral equations and applications. **28**, 123–148 (2016).

[19] Sulim G. T., Piskozub J. Z. Thermoelastic equilibrium of piecewise homogeneous solids with thin inclusions. Journal of Engineering Mathematics. Special Issue Thermomechanics. **61**, 315–337 (2008).

[20] Sulym H., Piskozub L., Piskozub Y., Pasternak I. Antiplane deformation of a bimaterial containing an interfacial crack with the account of friction. 2. Repeating and Cyclic loadingv. Acta Mechanica et Automatica. **9** (3), 178–184 (2015).

[21] Sulym H., Piskozub L., Piskozub Y., Pasternak I. Antiplane deformation of a bimaterial containing an interfacial crack with the account of friction. I. Single loading. Acta Mechanica et Automatica. **9** (2), 115–121 (2015).

[22] Sulym H. et al. Longitudinal shear of a bimaterial with frictional sliding contact in the interfacial crack Journal of Theoretical and Applied Mechanics. **54** 529 (2015).

[23] Sulym H. T., Piskozub I. Z. Nonlinear deformation of a thin interface inclusion. Materials Science. **53**, 600–608 (2018).

[24] Sulym H. T. Bases of the mathematical theory of thermoelastic equilibrium of deformable solids with thin inclusions. Research and Publishing center of NTSh, L'viv (2007), (in Ukrainian).

[25] Benveniste Y., Miloh T. Imperfect soft and stiff interfaces in two-dimensional elasticity. Mechanics of Materials. **33** (6), 309–323 (2001).

[26] Piskozub I. Z., Sulym H. T. Asymptotic of stresses in the vicinity of a thin elastic interphase inclusion. Materials Science. **32**, 421–432 (196).

Вплив поверхневих напружень на антиплоске деформування біматеріалу з тонким міжфазним мікровключенням

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У межах концепції мікромеханіки запропоновано методику врахування впливу поверхневих напружень для тонкого міжфазного мікровключення у біматеріалі за умов поздовжнього зсуву. При цьому передбачено можливість неідеального контакту між включенням та матрицею, зокрема контакту з натягом. Це значно розширює сферу застосовності результатів. Побудовано узагальнену модель тонкого включення з довільними пружними механічними властивостями. На основі застосування теорії функції комплексної змінної та методу функцій стрибка проведено розрахунок поля напружень в околі включення при його взаємодії з гвинтовою дислокацією. Виявлено ряд ефектів, які можуть бути використані для оптимізації енергетичних параметрів задачі.

Ключові слова: *мікронеоднорідності, біматеріал, поверхневі напруження, неідеальний контакт, функції стрибка, дислокації.*