MATHEMATICAL MODELING AND COMPUTING, Vol. 8, No. 3, pp. 432–443 (2021)

On the existence, uniqueness and computational analysis of a fractional order spatial model for the squirrel population dynamics under the Atangana–Baleanu–Caputo operator

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(Received 25 April 2021; Accepted 19 May 2021)

In this paper, we examine the fractional order analysis of a diffusion competition spatial model describing the interactions between the externally introduced grey and local red squirrel under the Atangana–Baleanu–Caputo (ABC) sense. Also, we establish the existence and uniqueness analysis of the fractional order spatial model of the squirrel population dynamics, while the numerical computation of the fractional order spatial model is carried out using the two dimensional Fractional Order Differential Transform Method (FODTM). Simulations of the variables of the model reveal that as the system evolves, the grey squirrels increase in density with increase in time, while the red squirrels decrease in density with increase in time. Also the simulations show that the FODTM is efficient and convergent with low computational cost.

Keywords: *Atangana–Baleanu–Caputo (ABC), diffusion competition model, Fractional order Differential Transform Method (FODTM).*

2010 MSC: 92B05, 93D05 **DOI:** 10.23939/mmc2021.03.432

1. Introduction

Mathematical models have long been used to depict species interactions in ecological systems. In 1926, Lotka and Volterra [1], derived mathematical models to describe dynamics of species and later Kermack and McKendrik [2], Anderson and May [3] proposed compartmental models divided into Susceptible-Infected-Recovered (SIR) population model to describe the interactions between man and its environment [4]. Between the 18th to 21st centuries, the grey squirrel named, *Sciurus carolinensis* was released from various places in Britain, where the grey squirrel has spread immensely in England, Wales and Scotland [5–7]. About the same time, the indigenous red squirrel named, *"Sciurus Vulgerns"* disappeared from these communities. An hypothesis was proposed, which led to the disappearance of red squirrel; Firstly, there is competition with the grey squirrel, secondly, environmental changes reduces the red squirrel population independent of the grey squirrel and thirdly, squirrel flu disease transmitted from the grey squirrel to the local red squirrels led to their extinction [8–13]. Reasonably, an interaction between the two species through indirect or direct competition for resources and space acted in favor of grey squirrel to drive out red squirrels [14, 15]. In recent times, fractional derivatives and integrals have been applied to real life problems, which gives better results than the classical order, because it produces better and accurate results due to the memory effect and ability to recall past information [16–18]. Several fractional operators like Caputo [19], Caputo–Fabrizio [20], Riemann–Lioville and host of others have been used to analyze nonlinear models, but the fractional operator of interest in this study is the Atangana–Baleanu–Caputo (ABC) operator. ABC operator arises from the fact that Caputo and Fabrizio proposed a fractional order derivative based on exponential function to solve problems of singular kernel. They also showed that their derivative was appropriate for some groups of physical problems. Furthermore, some issues were shown against this derivative as the kernel was non-singular and non-local which showed that the integral associate is not a fractional operator. In order to solve the issue of non-sigular and non-local kernel, Atangana and Baleanu derived two fractional derivatives in the sense of Caputo and Riemann–Liouville. In their results, the derivatives now have fractional integral as anti-derivative of their operators. Therefore, since the nonlinear dynamics and crossover effect of several physical and biological phenomena cannot be explained appropriately with the classical order derivative because of its singular kernel, a generalized Mittag–Leffler function as non local and non singular kernel is explored by Atangana and Baleanu [21, 22].

In addition, obtaining the exact or approximate solution of several linear and nonlinear classical or fractional order models using analytical or semi-analytical techniques is of interest to many authors. The numerical method of interest in this work is the Differential Transform Method (DTM). The DTM was first examined and studied by Zhou [23], where he solved linear and nonlinear problems involving electric circuit analysis. Several authors have applied DTM to one or two dimensional ordinary or partial differential equations describing real phenomena [24–29]. Iteratively, this method is based on Taylor's series expansion which establish an analytic solution in form of a polynomial. Merits of DTM includes low computational cost and no discretization, while the demerit of DTM is that, it results to a truncated series solution that does not reveal the actual behavior of the problem but forms a good approximation in a very small neighborhood [30–32]. Motivated by the work on spatial modeling of squirrel dynamics, fractional calculus and application of semi analytical DTM to solve nonlinear models, this work examines and extend the work of Okubo *et al.,* [33]. In their work, a diffusion competition mathematical model describing the competition between grey and local red squirrel species in great Britain is formulated, given by

$$\begin{cases} \dfrac{\partial S_1}{\partial T} = D_1 \nabla^2 S_1 + a_1 S_1 (1 - b_1 S_1 - c_1 S_2), \\ \dfrac{\partial S_2}{\partial T} = D_2 \nabla^2 S_2 + a_2 S_2 (1 - b_2 S_2 - c_2 S_1), \end{cases} \tag{1}$$

where, for $i = 1, 2$, a_i are the net birth rates of the red and grey squirrels respectively. $\frac{1}{b_i}$ denote the carrying capacities, c_i denote competition coefficients and D_i represent diffusion coefficients which are all nonnegative. In order to investigate the possibility of traveling waves of grey squirrels invasion which drives out the reds. Eq. (1) is non-dimensionalized by setting

$$\begin{cases} \theta_i = b_i S_i, \quad i = 1, 2, \quad t = a_1 T, \quad x = \left(\dfrac{a_1}{D_1}\right)^{\frac{1}{2}} X, \\ \gamma_1 = \dfrac{c_1}{b_2}, \quad \gamma_2 = \dfrac{c_2}{b_1}, \quad k_o = \dfrac{D_2}{D_1}, \alpha_1 = \dfrac{a_1}{a_2}. \end{cases} \tag{2}$$

The quantities in Eq. (2) denote the non-dimensional population densities at dimensionless time t and spatial coordinate x. Also, $k_o = \frac{D_2}{D_1}$ is defined to be the ratio of diffusion of red squirrel to the grey squirrels, where $\alpha_1 = \frac{a_1}{a_2}$ denote the ratio of red squirrel to grey squirrel growth rate. All these assumptions results to

$$\begin{cases} \dfrac{\partial \theta_1}{\partial T} = D_1 \nabla^2 \theta_1 + \alpha_1 \theta_1 (1 - b_1 \theta_1 - \gamma_1 \theta_2), \\ \dfrac{\partial \theta_2}{\partial T} = D_2 \nabla^2 \theta_2 + \alpha_2 \theta_2 (1 - b_2 \theta_2 - \gamma_2 \theta_1). \end{cases} \tag{3}$$

In view of Eqs. (1)–(3) and motivated by the importance of fractional calulus, Eq. (3) is changed from the classical to a integer order spatial model system given by

$$\begin{cases} \dfrac{\partial^\tau \theta_1}{dt} = \dfrac{\partial^2 \theta_1}{dx^2} + \theta_1 (1 - \theta_1 - \gamma_1 \theta_2), \\ \dfrac{\partial^\gamma \theta_2}{dt} = k_o \dfrac{\partial^2 \theta_2}{dx^2} + \alpha_1 \theta_2 (1 - \theta_2 - \gamma_2 \theta_1). \end{cases} \tag{4}$$

The existence and uniqueness of Eq. (4) is analyzed under the ABC sense in Section 2, while Section 3 involves obtaining the approximate solutions of Eq. (4) using the FODTM. Finally, Section 4 discusses the numerical simulations and conclusion of the work.

2. Preliminaries, existence and uniqueness criteria

2.1. Preliminaries

In recent times, Atangana and Baleanu [21,22] proposed new fractional differential operators based on Mittag–Leffler law, known as ABC-fractional derivative and integral operators.

Definition 1 (Refs. [21, 22]). *The ABC fractional derivative operator is given by*

$$ {}^{ABC}D^{\alpha}_{\tau}\psi(\tau) = \frac{B(\alpha)}{(1-\alpha)} \int_c^{\tau} \psi'(s) E_{\sigma}\Big[\frac{(-\alpha(\tau-s)^{\alpha})}{(1-\sigma)}\Big] ds, \tag{5}$$

where $B(\alpha)$ satisfy the property $B(0) = B(1) = 1$.

Definition 2 (Refs. [21, 22]). *The ABC fractional integral operator is given by*

$$ {}^{ABC}I^{\alpha}_{\tau}\psi(\tau) = \frac{1-\alpha}{B(\alpha)}\psi(\tau) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_c^{\tau} \psi(s)(\tau-s)^{\alpha-1} ds. \tag{6}$$

Applying the ABC fractional order operator to Eq. (4) yields

$$\begin{cases} {}^{ABC}D^{\tau}_{t}\theta_1(t) = \nabla^2\theta_1(t) + \theta_1(t)(1-\theta_1(t)-\gamma_1\theta_2(t)), \\ {}^{ABC}D^{\gamma}_{t}\theta_2(t) = k_o\nabla^2\theta_2(t) + \alpha_1\theta_1(t)(1-\theta_2(t)-\gamma_2\theta_1(t)). \end{cases} \tag{7}$$

Subject to the boundary conditions $\theta_1(x,t) = 1$, $\theta_2(x,t) = 0$, $\theta_2(x,t) = 1$ and $\theta_1(x,t) = 0$. Hereafter, we shall be referring to Eq. (7) in subsequent sectional analysis.

2.2. Existence analysis of the fractional order spatial model

Here, the fixed point technique for the existence of solutions of ABC fractional order spatial model in Eq. (7) is utilized by applying ABC-fractional integral operator in Definition 2, to obtain

$$\begin{cases} \theta_1(t)-\theta_1(0) = \frac{1-\tau}{B(\tau)}\left(\nabla^2\theta_1(t) + \theta_1(t)\big(1-\theta_1(t)-\gamma_1\theta_2(t)\big)\right) \\ \qquad + \frac{\tau}{B(\tau)\Gamma(\tau)}\int_0^t (t-\alpha)^{\tau-1}\left(\nabla^2\theta_1(t) + \theta_1(t)\big(1-\theta_1(t)-\gamma_1\theta_2(t)\big)\right) d\alpha, \\ \theta_2(t)-\theta_2(0) = \frac{1-\gamma}{B(\gamma)}\left(k_o\nabla^2\theta_2(t) + \alpha\theta_1(t)\big(1-\theta_2(t)-\gamma_2\theta_1(t)\big)\right) \\ \qquad + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\int_0^t (t-\alpha)^{\gamma-1}\left(k_o\nabla^2\theta_2(t) + \alpha_1\theta_1(t)\big(1-\theta_2(t)-\gamma_2\theta_1(t)\big)\right) d\alpha. \end{cases} \tag{8}$$

We assume that

$$K_1(t,\theta_1) = \nabla^2\theta_1(t) + \theta_1(t)\big(1-\theta_1(t)-\gamma_1\theta_2(t)\big) \tag{9}$$

and

$$K_2(t,\theta_2) = k_o\nabla^2\theta_2(t) + \alpha_1\theta_1(t)\big(1-\theta_2(t)-\gamma_2\theta_1(t)\big). \tag{10}$$

Let

$$p(t) = \big(1-\theta_1(t)-\gamma_1\theta_2(t)\big) \tag{11}$$

and

$$q(t) = \alpha_1\big(1-\theta_2(t)-\gamma_2\theta_1(t)\big). \tag{12}$$

— (A_1) For continuous functions f, g and $\theta_1, \theta_2, \theta_{1_1}, \theta_{2_1} \in L[0,1]$, there exist some constants $\delta_1, \delta_2 > 0$, such that the following holds true;

$$\begin{cases} \|f(t,\theta_1) - f(t,\theta_{1_1})\| \leqslant \delta_1 \|\theta_1 - \theta_{1_1}\|, \\ \|g(t,\theta_2) - f(t,\theta_{2_1})\| \leqslant \delta_2 \|\theta_2 - \theta_{2_1}\|. \end{cases} \tag{13}$$

— (A_2) For continuous functions $\gamma_2(t), \gamma_1(t), \theta_1(t), \theta_{1_1}(t), \theta_2(t), \theta_{2_1}(t)$, there exist some constants $\delta_3, \delta_4, \delta_5, \delta_6 > 0$, such that the following holds true:

$$\begin{cases} \|p(t).\nabla\theta_1(t) - p(t).\nabla\theta_{1_1}(t)\| \leqslant \delta_3 \|p(t)\| \|\theta_1 - \theta_{1_1}\|, \\ \|q(t).\nabla\theta_2(t) - q(t).\nabla\theta_{2_1}(t)\| \leqslant \delta_4 \|q(t)\| \|\theta_2 - \theta_{2_1}\|, \\ \|\nabla\theta_1(t) - \nabla\theta_{1_1}\| \leqslant \delta_5 \|\theta_1 - \theta_{1_1}\|, \\ \|\nabla\theta_2(t) - \nabla\theta_{2_1}\| \leqslant \delta_6 \|\theta_2 - \theta_{2_1}\|. \end{cases} \tag{14}$$

Theorem 1 (Refs. [21, 22]). *Assume that (A_1) and (A_2) are satisfied. Then, for $\|k_o(t)\| \leqslant m_1$, $\|p(t)\| \leqslant m_2$, $\|q(t)\| \leqslant m_3$, the functions $K_1(t,\theta_1), K_2(t,\theta_2)$, defined in Eqs. (9) and (10), respectively satisfy the Lipschitz conditions and are contractions provided $m_1\delta_6 + m_3\delta_4 + \delta_1 < 1$ and $m_3\delta_3 + \delta_5 + \delta_2 < 1$.*

Proof. In order to show that $K_1(t,\theta_1)$ satisfies the Lipschitz condition, consider

$$\begin{aligned} &\|K_1(t,\theta_1) - K_1(t,\theta_{1_1})\| \\ &\quad = \left\|\nabla^2\theta_1(t) + \theta_1(t)(1 - \theta_1(t) - \gamma_1\theta_2(t)) - (\nabla^2\theta_{1_1}(t) + \theta_{1_1}(t)(1 - \theta_{1_1}(t) - \gamma_1\theta_{2_1}(t))\right\| \\ &\quad \leqslant \left\|\nabla^2\right\| \|\theta_1(t) - \theta_{1_1}(t)\| + \|p(t)\cdot\theta_1(t) - p(t)\cdot\theta_{1_1}(t)\| \\ &\quad \leqslant m_1\delta_6\|\theta_1 - \theta_{1_1}\| + m_3\delta_4\|\theta_1 - \theta_{1_1}\| + \delta_1\|\theta_1 - \theta_{1_1}\| \\ &\quad = (m_1\delta_6 + m_3\delta_4 + \delta_1)\|\theta_1 - \theta_{1_1}\|, \end{aligned} \tag{15}$$

and

$$\begin{aligned} &\|K_2(t,\theta_2) - K_2(t,\theta_{2_1})\| \\ &\quad = \left\|k_o\nabla^2\theta_2(t) + \theta_2(t)(1 - \theta_2(t) - \gamma_1\theta_2(t)) - (\nabla^2\theta_{2_1}(t) + \theta_{2_1}(t)(1 - \theta_{2_1}(t) - \gamma_1\theta_{1_1}(t))\right\| \\ &\quad \leqslant \left\|\nabla^2\right\| \|\theta_2(t) - \theta_{2_1}(t)\| + \|p(t)\cdot\theta_1(t) - p(t)\cdot\theta_{2_1}(t)\| \\ &\quad \leqslant m_3\delta_3\|\theta_2 - \theta_{2_1}\| + \delta_5\|\theta_1 - \theta_{2_1}\| + \delta_2\|\theta_1 - \theta_{2_1}\| \\ &\quad = (m_3\delta_3 + \delta_5 + \delta_2)\|\theta_2 - \theta_{2_1}\|. \end{aligned} \tag{16}$$

Using Eqs. (15) and (16), the functions $K_1(t,\theta_1)$ and $K_2(t,\theta_2)$ satisfies the Lipschitz condition and they are contractions with $\beta_1 = m_1\delta_6 + m_3\delta_4 + \delta_1 < 1$ and $\beta_2 = m_3\delta_3 + \delta_5 + \delta_2 < 1$. This completes the proof. ■

Furthermore, using Eqs. (9) and (10) and Eq. (8), we obtain

$$\begin{cases} \theta_1(t) = \theta_1(0) + \dfrac{1-\tau}{B(\tau)} K_1(t,\theta_1(t)) + \dfrac{\tau}{B(\tau)\Gamma(\tau)} \displaystyle\int_0^t (t-\alpha)^{\tau-1} \big(K_1(\alpha,\theta_1)\big) d\alpha, \\ \theta_2(t) = \theta_2(0) + \dfrac{1-\gamma}{B(\gamma)} K_2(t,\theta_2(t)) + \dfrac{\gamma}{B(\gamma)\Gamma(\gamma)} \displaystyle\int_0^t (t-\alpha)^{\gamma-1} \big(K_2(\alpha,\theta_2)\big) d\alpha. \end{cases} \tag{17}$$

From Eq. (17), the following recursive formulas are defined as:

$$\begin{cases} \theta_{1_n}(t) = \dfrac{1-\tau}{B(\tau)} K_1(t,\theta_{1_{(n-1)}}(t)) + \dfrac{\tau}{B(\tau)\Gamma(\tau)} \displaystyle\int_0^t (t-\alpha)^{\tau-1} \big(K_1(\alpha,\theta_{1_{(n-1)}})\big) d\alpha, \\ \theta_{2_n}(t) = \dfrac{1-\gamma}{B(\gamma)} K_2(t,\theta_{2_{(n-1)}}(t)) + \dfrac{\gamma}{B(\gamma)\Gamma(\gamma)} \displaystyle\int_0^t (t-\alpha)^{\gamma-1} \big(K_2(\alpha,\theta_{2_{(n-1)}})\big) d\alpha \end{cases} \tag{18}$$

with the conditions $\theta_{1_0}(t) = \theta_1(0)$ and $\theta_{2_0}(t) = \theta_2(0)$. Furthermore, the following differences are considered.

$$\begin{cases}\phi^{n+1}(t) = (\theta_{1_{n+1}} - \theta_{1_n})(t) = \dfrac{1-\tau}{B(\tau)}\big(K_1(t,\theta_{1n}(t)) - K_1(t,\theta_{1_{n-1}}(t))\big)\\ \qquad + \dfrac{\tau}{B(\tau)\Gamma(\tau)}\displaystyle\int_0^t (t-\alpha)^{\tau-1}\big(K_1(\alpha,\theta_{1_n}(\alpha)) - K_1(\alpha,\theta_{1_{n-1}}(\alpha))\big)d\alpha,\\ \psi^{n+1}(t) = (\theta_{2_{n+1}} - \theta_{2_n})(t) = \dfrac{1-\gamma}{B(\gamma)}\big(K_2(t,\theta_{2n}(t)) - K_2(t,\theta_{2_{n-1}}(t))\big)\\ \qquad + \dfrac{\gamma}{B(\gamma)\Gamma(\gamma)}\displaystyle\int_0^t (t-\alpha)^{\gamma-1}\big(K_2(\alpha,\theta_{2_n}(\alpha)) - K_2(\alpha,\theta_{2_{n-1}}(\alpha))\big)d\alpha.\end{cases} \tag{19}$$

By considering the norms

$$\begin{aligned}\|\phi^{n+1}(t)\| &= \|(\theta_{1_{n+1}} - \theta_{1_n})\|\\ &= \bigg\|\frac{1-\tau}{B(\tau)}\big(K_1(t,\theta_{1_n}(t)) - K_1(t,\theta_{1_{n-1}}(t))\big)\\ &\quad + \frac{\tau}{B(\tau)\Gamma(\tau)}\int_0^t (t-\alpha)^{\tau-1}\big(K_1(\alpha,\theta_{1_n}(\alpha)) - K_1(\alpha,\theta_{1_{n-1}}(\alpha))\big)d\alpha\bigg\|\\ &\leqslant \bigg\|\frac{1-\tau}{B(\tau)}\big(K_1(t,\theta_{1_n}(t)) - K_1(t,\theta_{1_{n-1}}(t))\big)\\ &\quad + \frac{\tau}{B(\tau)\Gamma(\tau)}\int_0^t (t-\alpha)^{\tau-1}\big(K_1(\alpha,\theta_{1_n}(\alpha)) - K_1(\alpha,\theta_{1_{(n-1)}}(\alpha))\big)d\alpha\bigg\|,\end{aligned} \tag{20}$$

and

$$\begin{aligned}\|\psi^{n+1}(t)\| &= \|(\theta_{2_{n+1}} - \theta_{2_n})(t)\|\\ &= \bigg\|\frac{1-\gamma}{B(\gamma)}K_2(t,\theta_{2_{n-1}}(t)) + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\int_0^t (t-\alpha)^{\gamma-1}\big(K_2(\alpha,\theta_{2_{n(\alpha)}}) - K_2(\alpha,\theta_{2_{(n-1)}}(\alpha))\big)d\alpha\bigg\|\\ &\leqslant \bigg\|\frac{1-\gamma}{B(\gamma)}\big(K_1(\alpha,\theta_{2_n}(\alpha)) - K_2(t,\theta_{2_{(n-1)}}(t))\big)\\ &\quad + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\int_0^t (t-\alpha)^{\gamma-1}\big(K_2(\alpha,\theta_{2_n}(\alpha)) - K_2(\alpha,\theta_{2_{n-1}}(\alpha))\big)d\alpha\bigg\|.\end{aligned} \tag{21}$$

Eqs. (20) and (21), proved the existence of solution for the fractional order spatial model in Eq. (7) in the subsequent Theorem 2.

Theorem 2 (Refs. [21, 22]). *The fractional order spatial model system in Eq.* (7) *has a solution provided that the following holds true:*

$$\beta = \max\{m_1\delta_6 + m_3\delta_4 + \delta_1,\ m_3\delta_3 + \delta_5 + \delta_2\} < 1. \tag{22}$$

Proof. Assume the functions $F_n(t) = \theta_{1_{n+1}}(t) - \theta_1(t) + \theta_1(0), G_n(t) = \theta_{2_{n+1}}(t) - \theta_2(t) + \theta_2(0)$. Then, by Eqs.(20) and (21), we obtain

$$\begin{aligned}\|F_n(t)\| &= \bigg\|\frac{1-\tau}{B(\tau)}\big(K_1(t,\theta_{1_n}(t)) - K_1(t,\theta_1(t))\big)\\ &\quad + \frac{\tau}{B(\tau)\Gamma(\tau)}\int_0^t (t-\alpha)^{\tau-1}\big(K_1(\alpha,\theta_{1_n}(\alpha)) - K_1(\alpha,\theta_1(\alpha))\big)d\alpha\bigg\|,\\ &\leqslant \frac{1-\tau}{B(\tau)}\big\|K_1(\alpha,\theta_{1_n}(\alpha)) - K_1(\alpha,\theta_1(t))\big)\big\|\\ &\quad + \frac{\tau}{B(\tau)\Gamma(\tau)}\int_0^t (t-\alpha)^{\tau-1}\big\|\big(K_1(\alpha,\theta_{1_n}(\alpha)) - K_1(\alpha,\theta_1(\alpha))\big)d\alpha\big\|\\ &\leqslant \left[\frac{1-\tau}{B(\tau)}\|\theta_{1_n} - \theta_1\| + \frac{\|\theta_{1_n} - \theta_1\|}{B(\tau)\Gamma(\tau)}\right](m_1\delta_6 + m_3\delta_4 + \delta_1)\end{aligned} \tag{23}$$

$$\leqslant \left[\frac{1-\tau}{B(\tau)} + \frac{1}{B(\tau)\Gamma(\tau)}\right]^n \|\theta_{1_n} - \theta_1\|\beta^n \to 0, \quad \text{as} \quad n \to \infty. \tag{24}$$

Similarly, we have

$$\begin{aligned}
\|G_n(t)\| &= \left\| \frac{1-\gamma}{B(\gamma)}(K_2(t,\theta_{1_n}(t)) - (K_2(t,\theta_1(t))) \right. \\
&\quad \left. + \frac{\gamma}{B(\gamma)\Gamma(\gamma)} \int_0^t (t-\alpha)^{\gamma-1}(K_2(\alpha,\theta_{2_n}(\alpha)) - K_2(\alpha,\theta_2(\alpha)))d\alpha \right\| \\
&\leqslant \frac{1-\gamma}{B(\gamma)} \|K_2(\alpha,\theta_{2_n}(\alpha)) - K_1(\alpha,\theta_2(t))\| \\
&\quad + \frac{\gamma}{B(\gamma)\Gamma(\gamma)} \int_0^t (t-\alpha)^{\gamma-1}\|K_1(\alpha,\theta_{1_n}(\alpha)) - K_1(\alpha,\theta_1(\alpha))d\alpha\| \\
&\leqslant \left[\frac{1-\gamma}{B(\gamma)}\|\theta_{2_n} - \theta_2\| + \frac{\|\theta_{1_n} - \theta_1\|}{B(\gamma)\Gamma(\gamma)}\right](m_3\delta_3 + \delta_5 + \delta_2) \\
&\leqslant \left[\frac{1-\gamma}{B(\gamma)} + \frac{1}{B(\gamma)\Gamma(\gamma)}\right]^n \|\theta_{2_n} - \theta_2\|\beta^n \to 0, \quad \text{as} \quad n \to \infty.
\end{aligned} \tag{25}$$

Thus, Eqs. (23) and (25) implies that the functions $F_n(t), G_n(t) \to 0$, as $n \to \infty$ for $\beta < 1$, which shows that $\lim_{n\to\infty} \theta_{1_{n+1}} = \theta_1$ and $\lim_{n\to\infty} \theta_{2_{n+1}} = \theta_2$. Consequently, the solutions of the fractional order spatial model in Eq. (7) exist. ■

2.3. Uniqueness of solutions of fractional order spatial model

This section is devoted to the uniqueness analysis of solutions of fractional order spatial model in Eq. (7) which is based on the assumptions $(A_1), (A_2)$.

Theorem 3 (Refs. [21,22]). *Assume that $(A_1), (A_2)$ are satisfied and*

$$\wedge = \min\left\{1 - \left[\frac{1-\tau}{B(\tau)} + \frac{1}{B(\tau)\Gamma(\tau)}\right](m_1\delta_6 + m_3\delta_4 + \delta_1), 1 - \left[\frac{1-\gamma}{B(\gamma)} + \frac{1}{B(\gamma)\Gamma(\gamma)}\right](m_3\delta_3 + \delta_5 + \delta_2)\right\}, \tag{26}$$

then the fractional order spatial model in Eq.(7) has a unique solution.

Proof. For the uniqueness of solution of the fractional order spatial model Eq. (7), we assume contrarily for the proof. That is, let there exist some solution $(\theta_1,\ \theta_2,\ \theta_{1_1},\ \theta_{2_1})$ satisfying the integral system given by

$$\begin{cases}
\theta_{1_1}(t) = \theta_{1_1}(0) + \dfrac{1-\tau}{B(\tau)}\Big(K_1(t,\theta_{1_1}(t)) + \dfrac{\tau}{B(\tau)\Gamma(\tau)} \displaystyle\int_0^t (t-\alpha)^{\tau-1}(K_1(\alpha,\theta_{1_1}))\Big)d\alpha \\
\theta_{2_1}(t) = \theta_{2_1}(0) + \dfrac{1-\gamma}{B(\gamma)}\Big(K_2(t,\theta_{2_1}(t)) + \dfrac{\gamma}{B(\gamma)\Gamma(\gamma)} \displaystyle\int_0^t (t-\alpha)^{\gamma-1}(K_2(\alpha,\theta_{2_1}))\Big)d\alpha.
\end{cases} \tag{27}$$

For the fractional order spatial model in Eq. (7), we have $\theta_{1_1}(0) = \theta_{2_1}(0) = 0$. Consider

$$\begin{aligned}
\|\theta_1(t) - \theta_{1_1}(t)\| &= \frac{1-\tau}{B(\tau)}\|K_1(t,\theta_1(t)) - K_1(t,\theta_{1_1}(t))\| \\
&\quad + \frac{\tau}{B(\tau)\Gamma(\tau)} \int_0^t (t-\alpha)^{\tau-1}\|K_1(\alpha,\theta_1(t)) - K_1(\alpha,\theta_{1_1}(t))\|d\alpha \\
&\leqslant \left[\frac{1-\tau}{B(\tau)} + \frac{1}{B(\tau)\Gamma(\tau)}\right] \|\theta_1 - \theta_{1_1}\|\big(m_1\delta_6 + m_3\delta_4 + \delta_1\big),
\end{aligned} \tag{28}$$

which implies that

$$\|\theta_1(t) - \theta_{1_1}(t)\| \left(1 - \left[\frac{1-\beta}{B(\beta)} + \frac{1}{B(\beta)\Gamma(\beta)}\right](m_1\delta_6 + m_3\delta_4 + \delta_1)\right) \leqslant 0. \tag{29}$$

Similarly we obtain

$$\begin{aligned}\|\theta_2(t) - \theta_{2_1}(t)\| &= \frac{1-\gamma}{B(\gamma)}\big\|K_2(t,\theta_2(t)) - K_2(t,\theta_{2_1}(t))\big\| \\ &\quad + \frac{\gamma}{B(\gamma)\Gamma(\gamma)}\int_0^t (t-\alpha)^{\gamma-1}\big\|K_2(\alpha,\theta_2(t)) - K_2(\alpha,\theta_{2_1}(t))\big\|d\alpha \\ &\leqslant \left[\frac{1-\gamma}{B(\gamma)} + \frac{1}{B(\gamma)\Gamma(\gamma)}\right]\|\theta_2 - \theta_{2_1}\|\big(m_2\delta_3 + \delta_4 + \delta_2\big),\end{aligned} \tag{30}$$

which implies that

$$\|\theta_2(t) - \theta_{2_1}(t)\| \left(1 - \left[\frac{1-\gamma}{B(\gamma)} + \frac{1}{B(\gamma)\Gamma(\gamma)}\right]\big(m_3\delta_3 + \delta_5 + \delta_2\big)\right) \leqslant 0. \tag{31}$$

By the condition in Eq. (26), and Eqs. (27)–(31) we obtain $\|\theta_1 - \theta_{1_1}\| \to 0$ as well $\|\theta_2 - \theta_{2_1}\| \to 0$. Consequently, $\theta_1(t) = \theta_{1_1}(t)$ and $\theta_2(t) = \theta_{2_1}(t)$. Thus, the solution of the fractional order spatial model in Eq. (7) under the ABC sense is unique. ■

3. The two dimensional fractional order differential transform method (FODTM)

In order to apply the two-dimensional FODTM to the fractional order spatial model in Eq. (7), a function of two variables, $u(r,s)$ is considered and suppose that it can be represented as a product of two single variable functions, i.e., $u(r,s) = f(r)g(s)$, based on the properties of a generalized two-dimensional FODTM [32], the function $u(r,s)$ can be represented as

$$u(r,s) = \sum_{k=0}^{\infty} F_\tau(k)(r-r_0)^{k\tau} \sum_{h=0}^{\infty} G_\gamma(h)(s-s_0)^{h\gamma} = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} U_{\tau,\gamma}(k,h)(r-r_0)^{k\tau}(s-s_0)^{h\gamma}, \tag{32}$$

where $0 < \alpha, \beta \leqslant 1$, $U_{\tau,\gamma}(k,h) = F_\tau(k)G_\gamma(h)$ is called the spectrum of $u(r,s)$. The generalized two-dimensional FODTM of the function $u(r,s)$ is given by

$$U_{\tau,\gamma}(k,h) = \frac{1}{(\Gamma(\tau k+1)\Gamma(\gamma h+1))}\left[(D_r^\tau)^k (D_s^{\gamma h}) u(r,s)\right]_{(r_0,s_0)}. \tag{33}$$

Hence, the generalized two-dimensional FODTM properties are tabulated below.

Table 1. Basic operational properties of a two dimensional FODTM.

Basic Function	Transform Function
$u(r,s) = v(r,s) \pm w(r,s)$	$U_{\tau,\gamma}(k,h) = V_{\tau,\gamma}(k,h)(r,s) \pm W_{\tau,\gamma}(k,h)(r,s)$
$u(r,s) = av(r,s)$	$U_{\tau,\gamma}(k,h) = aV_{\tau,\gamma}(k,h)$
$u(r,s) = v(r,s)w(r,s)$	$U_{\tau,\gamma}(k,h) = \sum_{a=0}^{r}\sum_{b=0}^{s} V_{\tau,\gamma}(a,h-b)W_{\tau,\gamma}(k-a,b)$
$u(r,s) = (r-r_0)^{n\tau}(s-s_0)^{m\gamma}$	$U_{\tau,\gamma}(k,h) = \delta(k-n)\delta(h-m)$
$u(r,s) = D_r^\tau v(r,s)$	$U_{\tau,\gamma}(k,h) = \frac{\Gamma(\tau(k+1)+1)}{(\Gamma(\tau k+1))} V_{\tau,\gamma}(k+1,h)$
$u(r,s) = \frac{\partial v(r,s)}{\partial r}$	$U_{\tau,\gamma}(k,h) = (k+1)V_{\tau,\gamma}(k+1,h)$

Applying the two-dimensional FODTM to Eq. (7), we obtain

$$\theta_{1_{1,\tau}}(k,s+1)\frac{\Gamma(\tau k+1)}{\Gamma(\tau(k+1)+1)}=[(k+1)(k+2)\theta_{1_{1,\tau}}(k+2,s)+\theta_1(k,s)(1-\theta_1(k,s)-\gamma_1\theta_2(k,s))], \quad (34)$$

so that

$$\theta_{1_{1,\tau}}(k,s+1)=\frac{\Gamma(\tau(k+1)+1)}{\Gamma(\tau k+1)}[(k+1)(k+2)\theta_{1_{1\tau}}(k+2,s)+\theta_1(k,s)(1-\theta_1(k,s)-\gamma_1\theta_2(k,s))], \quad (35)$$

and

$$\theta_{2_{1,\gamma}}(k,s+1)\frac{\Gamma(\gamma k+1)}{\Gamma(\gamma(k+1)+1)}=[k_0(k+1)(k+2)\theta_{2_{1,\gamma}}(k+2,s)+\alpha_1\theta_1(k,s)(1-\theta_2(k,s)-\gamma_2\theta_1(k,s))], \quad (36)$$

so that

$$\theta_{2_{1,\gamma}}(k,s+1)=\frac{\Gamma(\gamma(k+1)+1)}{\Gamma(\gamma k+1)}[k_0(k+1)(k+2)\theta_{2_{1,\gamma}}(k+2,s)+\alpha_1\theta_1(k,s)(1-\theta_1(k,s)-\gamma_2\theta_1(k,s))]. \quad (37)$$

Solving Eqs. (35) and (37) and transforming the boundary conditions in Eq. (8), we obtain

$$\begin{cases}\theta_1(0)=\theta_1(1)=0,\\ \theta_1(2)=\dfrac{\Gamma}{\Gamma(\tau+1)}(2!),\\ \theta_1(3)=\dfrac{\Gamma(\tau+1)}{\Gamma(\tau+2)}(3!),\\ \theta_1(4)=\dfrac{\Gamma(\tau+2)}{\Gamma(\tau+3)}\left(13\dfrac{\Gamma}{\Gamma(\tau+1)}(2!)-\left(\dfrac{\Gamma}{\Gamma(\tau+1)}(2!)\right)^2-\gamma_1\dfrac{\Gamma}{\Gamma(\tau+1)}(2!)\dfrac{\Gamma}{\Gamma(\gamma+1)}(2!k_0)\right),\\ \theta_1(5)=\dfrac{\Gamma(\tau+3)}{\Gamma(\tau+4)}\left(21\dfrac{\Gamma(\tau+1)}{\Gamma(\tau+2)}(3!)-\left(\dfrac{\Gamma(\tau+1)}{\Gamma(\tau+2)}(3!)\right)^2-\gamma_1\dfrac{\Gamma(\tau+1)}{\Gamma(\tau+2)}(3!)\dfrac{\Gamma(\gamma+1)}{\Gamma(\gamma+2)}(3!k_0)\right),\end{cases} \quad (38)$$

and

$$\begin{cases}\theta_2(0)=\theta_2(1)=0,\\ \theta_2(2)=\dfrac{\Gamma}{\Gamma(\gamma+1)}(2!k_0),\\ \theta_2(3)=\dfrac{\Gamma(\gamma+1)}{\Gamma(\gamma+2)}(3!k_0),\\ \theta_2(4)=\dfrac{\Gamma(\gamma+2)}{\Gamma(\gamma+3)}\Bigg(12(k_0+\alpha_1)\dfrac{\Gamma}{\Gamma(\gamma+1)}(2!k_0)+\alpha_1\left(\dfrac{\Gamma}{\Gamma(\gamma+1)}(2!k_0)\right)^2\\ \qquad -\gamma_2\alpha_1\dfrac{\Gamma}{\Gamma(\gamma+1)}(2!k_0)\dfrac{\Gamma}{\Gamma(\tau+1)}(2!)\Bigg),\\ \theta_2(5)=\dfrac{\Gamma(\gamma+3)}{\Gamma(\gamma+4)}\Bigg(12(k_0+\alpha_1)\dfrac{\Gamma(\gamma+1)}{\Gamma(\gamma+2)}(3!k_0)+\alpha_1\left(\dfrac{\Gamma(\gamma+1)}{\Gamma(\gamma+2)}(3!k_0)\right)^2\\ \qquad +\gamma_2\alpha_1\dfrac{\Gamma(\tau+1)}{\Gamma(\tau+2)}(3!)\dfrac{\Gamma(\gamma+1)}{\Gamma(\gamma+2)}(3!k_0)\Bigg).\end{cases} \quad (39)$$

Furthermore, substituting Eqs. (38) and (39) into Eq. (7) after some simplification and re-arrangement, the closed form solution of Eq. (7) is given by;

$$\begin{aligned}\theta_1(x,t)&=1+t+\left(\frac{\Gamma}{\Gamma(\tau+1)}(2!)\right)t^2+\left(\frac{\Gamma(\tau+1)}{\Gamma(\tau+2)}(3!)\right)t^3\\ &+\left(\frac{\Gamma(\tau+2)}{\Gamma(\tau+3)}\left(13\frac{\Gamma}{\Gamma(\tau+1)}(2!)-\left(\frac{\Gamma}{\Gamma(\tau+1)}(2!)\right)^2-\gamma_1\frac{\gamma}{\Gamma(\tau+1)}(2!)\frac{\Gamma}{\Gamma(\gamma+1)}(2!k_0)\right)t^4\right.\end{aligned} \quad (40)$$

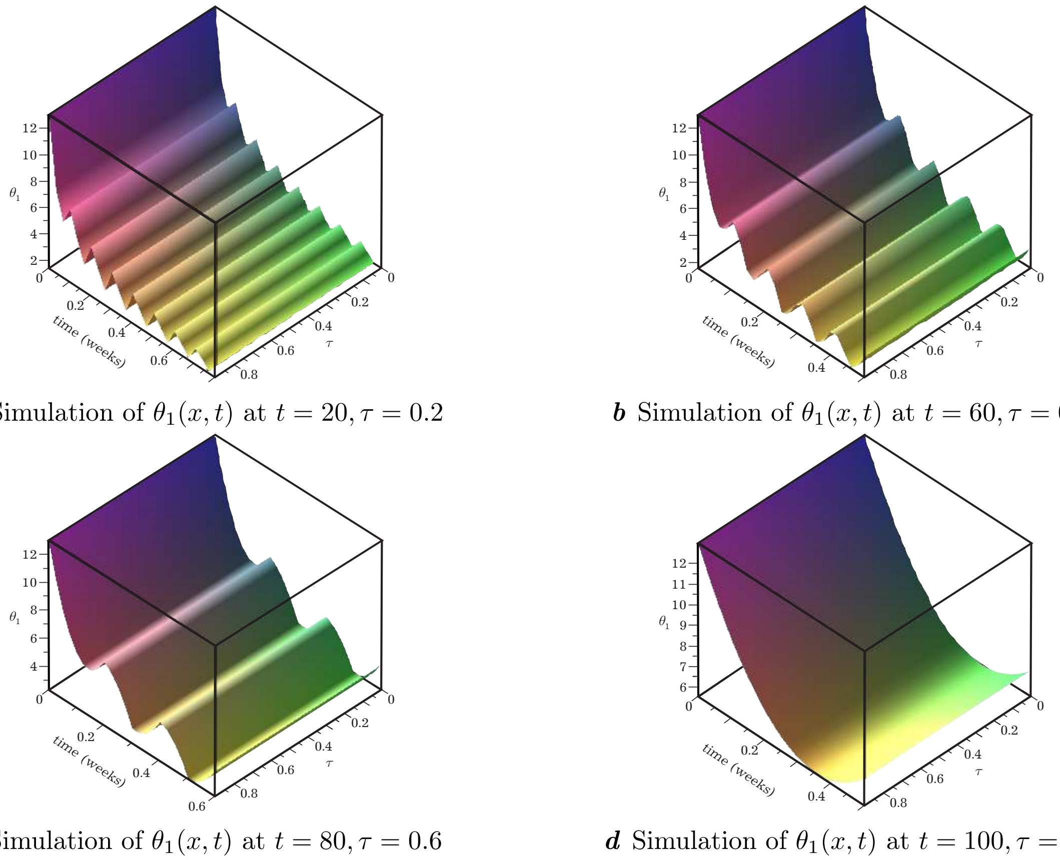
$$+\frac{\Gamma(\tau+3)}{\Gamma(\tau+4)}\left(21\frac{\Gamma(\tau+1)}{\Gamma(\tau+2)}(3!)-\frac{\Gamma(\tau+1)}{\Gamma(\tau+2)}(3!)\right)^2-\gamma_1\frac{\Gamma(\tau+1)}{\Gamma(\tau+2)}(3!)\frac{\Gamma(\gamma+1)}{\Gamma(\gamma+2)}(3!k_0)\bigg)t^5+\ldots,$$

and

$$\theta_2(x,t)=1+t+\frac{\Gamma}{\Gamma(\gamma+1)}(2!k_0)t^2+\left(\frac{\Gamma(\gamma+1)}{\Gamma(\gamma+2)}(3!k_0)\right)t^3$$
$$+\frac{\Gamma(\gamma+2)}{\Gamma(\gamma+3)}\left(12(k_0+\alpha_1)\frac{\Gamma}{\Gamma(\gamma+1)}(2!k_0)+\alpha_1\left(\frac{\Gamma}{\Gamma(\gamma+1)}(2!k_0)\right)^2-\gamma_2\alpha_1\frac{\Gamma}{\Gamma(\gamma+1)}(2!k_0)\frac{\Gamma}{\Gamma(\tau+1)}(2!)\right)t^4$$
$$+\left(\frac{\Gamma(\gamma+3)}{\Gamma(\gamma+4)}\left(12(k_0+\alpha_1)\frac{\Gamma(\gamma+1)}{\Gamma(\gamma+2)}(3!k_0)+\alpha_1\frac{\Gamma(\gamma+1)}{\Gamma(\gamma+2)}(3!k_0)\right)^2\right.$$
$$\left.+\gamma_2\alpha_1\frac{\Gamma(\tau+1)}{\Gamma(\tau+2)}(3!)\frac{\Gamma(\gamma+1)}{\Gamma(\gamma+2)}(3!k_0)\right)t^5+\ldots. \quad (41)$$

4. Numerical simulations

Having obtained the closed form solutions in Eqs. (40) and (41) we adopt the following parameter values; $\gamma_1=0.2$, $\gamma_2=1.5$, $\alpha_1=\frac{0.82}{0.61}$, $k_o=1$, $D_1=D_2=10^4$, $a_1=0.82$, $a_2=0.61$ as seen in the work of Okubo et al., [33]. Figures $1a$–$1d$ displays the spreading wave of the density of the red squirrel population. As the system evolves, the red squirrels gradually decrease in density as time t increases ($t=20$ weeks) at different fractional orders ($\tau=0.2-0.8$) due to the action of the externally introduced grey squirrels in driving out the red squirrels.



a Simulation of $\theta_1(x,t)$ at $t=20, \tau=0.2$

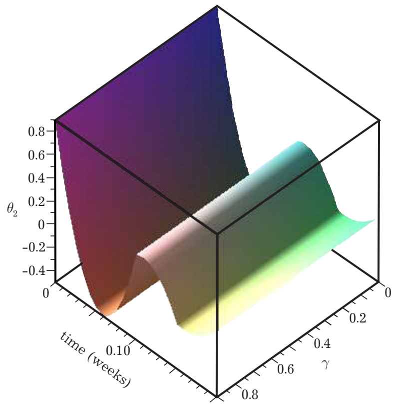
b Simulation of $\theta_1(x,t)$ at $t=60, \tau=0.4$

c Simulation of $\theta_1(x,t)$ at $t=80, \tau=0.6$

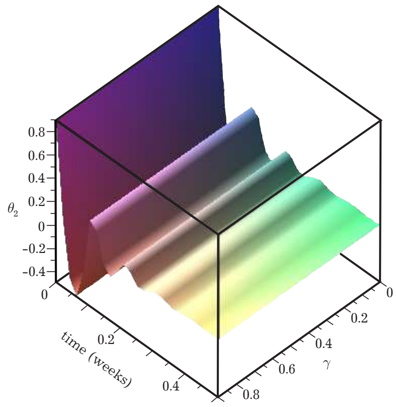
d Simulation of $\theta_1(x,t)$ at $t=100, \tau=0.8$

Fig. 1. Model simulations of the population of the red squirrels $\theta_1(x,t)$ using FODTM

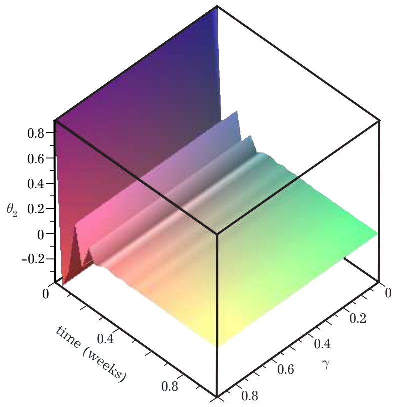
Figures $2a$–$2d$ displays the spreading wave of the density of the grey squirrel population. As the system evolves, the grey squirrels gradually increase in density as time t increases ($t = 20$ weeks) at different fractional order values ($\gamma = 0.2 - 0.8$) due to the action of the externally introduced grey squirrels in driving out the red squirrels.



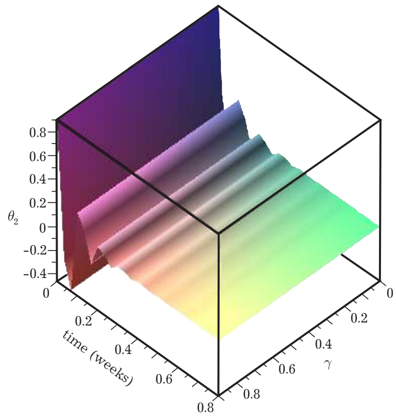
a Simulation of $\theta_2(x,t)$ at $t = 20, \tau = 0.2$



b Simulation of $\theta_2(x,t)$ at $t = 20, \tau = 0.4$



c Simulation of $\theta_2(x,t)$ at $t = 20, \tau = 0.6$



d Simulation of $\theta_2(x,t)$ at $t = 20, \tau = 0.8$

Fig. 2. Model simulations of the population of the grey squirrels $\theta_2(x,t)$ using FODTM

5. Conclusions

In this work, we studied the diffusion competition model describing the interactions between invading grey squirrels and local red squirrels in Great Britain. Since this model is a classical order diffusion model, we extend it by changing the classical order to an integer order, studied under the ABC sense. Further, we established the existence and uniqueness results of the fractional order spatial model. We performed the numerical simulations by solving the fractional order spatial model using the two dimensional FODTM. Our results reveal that the population of the grey squirrel increases faster as the system evolves and drives out the red squirrels, leading to their decrease as time increases at different fractional order values. This shows the effectiveness of FODTM with low computational cost and effectiveness. In order to curtail this, the grey squirrel population needs to be controlled and the red squirrel population needs to be preserved through the application of optimal control theory. Also, other fractional operators like the Grunwald Letnikov, Caputo–Fabrizio, Atangana bi-order operators etc., can be applied to the model.

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Про існування, єдиність та обчислювальний аналіз просторової моделі дробового порядку для динаміки популяції білок за оператором Атангана–Балеану–Капуто

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У цій роботі досліджується аналіз дробового порядку просторової моделі дифузійної конкуренції, що описує взаємодію між введеною іззовні сірою та місцевою рудою білкою в розумінні Атангана–Балеану–Капуто (АБК). Також встановлено існування та аналіз єдиності просторової моделі дробового порядку динаміки популяції білки, тоді як чисельний розрахунок просторової моделі дробового порядку проводиться за допомогою двовимірного методу диференціального перетворення дробового порядку (МДПДП). Моделювання змінних моделі показує, що популяція сірих білок збільшується зі збільшенням часу, тоді як популяція червоних білок зменшується. Також моделювання показують, що МДПДП є ефективним та конвергентним із низькими обчислювальними витратами.

Ключові слова: *Атангана–Балеану–Капуто (АБК), модель дифузійної конкуренції, метод диференціального перетворення дробового порядку (МДПДП).*