Universal mathematical model of asynchronous machine as an element microgrid in smart grid

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An universal mathematical model of an induction machine (IM) has been constructed, taking into account the saturation of the main magnetic circuit of its magnetic core and the active power losses. The proposed approach to IM modeling expands the abilities of MATLAB / Simulink environment for analysis of the electric power supply systems (microgrid in the smart grid) with the nonlinear elements and dynamic load in abnormal, asymmetric and fault regimes.

1. Introduction

Intensive developing of microgrid in smart grid technologies along with the traditional use of induction machines expands the scope of their application for power generation (asynchronous generators), conversion of single-phase voltage into three-phase voltage system and as part of multi-motor units in technological facilities and etc.

Mathematical modeling of their modes and processes plays an important role in the research and design of units and devices with IM [1]. Overwhelming majority of such objects research can be performed in MATLAB / Simulink environment. Most of researches are aimed to analyze and optimize symmetric modes and processes of electric drives. Thereby studies used the equations formed in the rotating orthogonal d-q Park axes or the stationary α-β Clark axes without proper consideration of the influence of schematic, mode and parametric features of smart grid or microgrid elements on IM functioning [2–7]. Such modes occur using the capacitor braking, semiconductor converters or valve elements (diodes, thyristors), asymmetric load of asynchronous generators and etc. Thus to study the processes of asynchronous machine it is necessary to take into account adequately the saturation of the main magnetic circuit, the effect of current displacement in the rotor windings, active power losses in the magnetic core, the influence of stator elements saturation on the inductances and mutual leakage inductances.

The paper presents a new approach to the development of the universal mathematical model of AM as a part of smart grid and microgrid in MATLAB / Simulink environment. The obtained results are based on the theory of the electromechanical energy conversion [1] and developed the mathematical models of AM given in [7–9]. The universal asynchronous machine model will significantly expand the research possibilities of modes and processes in the smart grid and microgrid, especially in the analysis of their modes and processes under abnormal, asymmetric and emergency states.

2. Universal mathematical model of asynchronous machine

Mathematical model of stator and movable rotor circuit electric voltages equilibrium of three-phase IM with star connected stator windings and isolated neutral developed in the phase physical coordinates in matrix-vector form presented in [7–9] is:

$$\boldsymbol{v}_s = \frac{d\boldsymbol{\lambda}_s}{dt} + \boldsymbol{R}_s \boldsymbol{i}_s, \tag{1}$$

$$\boldsymbol{v}_r = \frac{d\boldsymbol{\lambda}_r}{dt} + \boldsymbol{R}_r \boldsymbol{i}_r, \tag{2}$$

where $\boldsymbol{v}_s = (v_{sa}, v_{sb}, v_{sc})^t$, $\boldsymbol{v}_r = (v_{ra}, v_{rb}, v_{rc})^t$ are vectors of stator and rotor phase voltages; $\boldsymbol{i}_s = (i_{sa}, i_{sb}, i_{sc})^t$, $\boldsymbol{i}_r = (i_{ra}, i_{rb}, i_{rc})^t$ are vectors of stator phase currents; $\boldsymbol{\lambda}_s = (\lambda_{sa}, \lambda_{sb}, \lambda_{sc})^t$, $\boldsymbol{\lambda}_r = (\lambda_{ra}, \lambda_{rb}, \lambda_{rc})^t$ are vectors of the phase stator and rotor flux linkages, $\boldsymbol{R}_s = \mathrm{diag}\,(R_{sa}, R_{sb}, R_{sc})$, $\boldsymbol{R}_r = \mathrm{diag}\,(R_{ra}, R_{rb}, R_{rc})$ are the diagonal matrices of the phase stator and rotor winding resistances.

It should be noted that in equation (2) the values of all physical quantities are reduced to the stator winding. The components of the flux linkage vectors λ_s, λ_r are complex functions with periodic coefficients of the rotor axis rotation angle relatively to the stator, values of their currents and electromagnetic parameters of the magnetic core and windings. In order to construct an universal mathematical model of IM, it is reasonable to write down the expressions for determining the flux linkages in the following form on the basis of the research results obtained in [1]:

$$\boldsymbol{\lambda}_s = 2/3 \left(\boldsymbol{L}_{\sigma s} \boldsymbol{D} \boldsymbol{i}_s + L_m(i_m)(\boldsymbol{D}\boldsymbol{i}_s + \boldsymbol{D}_{rs}\boldsymbol{i}_r) \right), \tag{3}$$

$$\boldsymbol{\lambda}_r = 2/3 \left(\boldsymbol{L}_{\sigma r} \boldsymbol{D} \boldsymbol{i}_r + L_m(i_m)(\boldsymbol{D}\boldsymbol{i}_r + \boldsymbol{D}_{sr}\boldsymbol{i}_s) \right), \tag{4}$$

where $\boldsymbol{L}_{\sigma s} = \mathrm{diag}\,(L_{\sigma sa}, L_{\sigma sb}, L_{\sigma sc})$, $\boldsymbol{L}_{\sigma r} = \mathrm{diag}\,(L_{\sigma ra}, L_{\sigma rb}, L_{\sigma rc})$ are the diagonal matrices of stator and rotor winding phase leakage inductances, D_{rs}, D_{sr} are matrices of the space vector projection coefficients of rotor currents on the stator winding axes and stator currents on the rotor winding axes, respectively:

$$\boldsymbol{D}_{rs} = \left\| \begin{array}{ccc} \cos{(\theta)} & \cos{(\theta + \rho)} & \cos{(\theta - \rho)} \\ \cos{(\theta - \rho)} & \cos{(\theta)} & \cos{(\theta + \rho)} \\ \cos{(\theta + \rho)} & \cos{(\theta - \rho)} & \cos{(\theta)} \end{array} \right\|, \tag{5}$$

$$\boldsymbol{D}_{sr} = \left\| \begin{array}{ccc} \cos{(\theta)} & \cos{(\theta - \rho)} & \cos{(\theta + \rho)} \\ \cos{(\theta + \rho)} & \cos{(\theta)} & \cos{(\theta - \rho)} \\ \cos{(\theta - \rho)} & \cos{(\theta + \rho)} & \cos{(\theta)} \end{array} \right\|, \tag{6}$$

where θ is a rotor rotation angle, it is the angle between the axes of the rotor and stator phases, $\rho = 2\pi/3$ is the angle between the geometrical axes of the stator and rotor phase windings of IM. $\boldsymbol{D}$ is the matrix of coefficients of projections of the phases of the stator and rotor phase currents on the geometrical a, b, c axes of their windings in the following form:

$$\boldsymbol{D} = \left\| \begin{array}{ccc} \cos{(0)} & \cos{(-\rho)} & \cos{(\rho)} \\ \cos{(\rho)} & \cos{(0)} & \cos{(-\rho)} \\ \cos{(-\rho)} & \cos{(+\rho)} & \cos{(0)} \end{array} \right\| = \left\| \begin{array}{ccc} 1 & -1/2 & -1/2 \\ -1/2 & 1 & -1/2 \\ -1/2 & -1/2 & 1 \end{array} \right\|, \tag{7}$$

where $L_m = L_m\left(|\bar{\lambda}_m|\right) = |\bar{\lambda}_m| / \bar{i}_m\left(|\bar{\lambda}_m|\right)$ is the inductance of the main magnetic circuit of IM, which is determined by its magnetization characteristic $\bar{i}_m\left(|\bar{\lambda}_m|\right)$ as a function of the space vector modulus of the main air gap flux linkages $|\bar{\lambda}_m|$, in the next form

$$\left| \bar{\boldsymbol{\lambda}}_m \right| = \sqrt{2 \left(\lambda_{ma}^2 + \lambda_{mb}^2 + \lambda_{mc}^2 \right)/3}. \tag{8}$$

To obtain the differential state equations of IM, expressions (3) and (4) should be differentiated in time. Obtained relationships should be substituted into equations (1) and (2). As a result, the nonlinear

system of the differential equations with the periodic coefficients is obtained. So their implementation into the modern software environments is complicated. Simplification of IM equations is carried out by transformation of coordinates for extracting from them periodic coefficients [1, 2, 6]. Usually the IM equation is transformed to rotating orthogonal d-q Park axes, more rarely to the fixed orthogonal α-β Clark axes. Such transformation is possible only under the condition of symmetry of circuit elements parameters of stator and rotor circuits of IM. This fact limits the scope and possibilities of studying the modes and processes of complex electrotechnical objects with non-symmetric parameters and dynamic loads.

Analysis of the possible ways to construct a universal mathematical IM model to study the modes and processes of the complex smart grid or microgrid proves the expediency of the state equations transformation to immovable three-phase a, c, b coordinate axes of stator and rotor windings [8, 9]. During such transformation the element parameters and stator phase axes of AM will remain physically real, and rotating rotor (phase or squirrel) will be replaced by the immovable symmetric three-phase rotor [1].

To obtain the mathematical IM model without the periodic coefficients in equations (1)–(4) it is necessary to transform the real physical coordinates (currents, voltages and flux linkages) of rotor into the stationary (retarded) $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ coordinates [1]. Let's introduce the transformation matrix of the rotor coordinates T_r into the stationary coordinates and vectors of physical quantities in the stationary axes: $\tilde{i}_r = \left(\tilde{i}_{ra}, \tilde{i}_{rb}, \tilde{i}_{rc}\right)^t$ and $\tilde{\lambda}_r = \left(\tilde{\lambda}_{ra}, \tilde{\lambda}_{rb}, \tilde{\lambda}_{rc}\right)^t$.

The relation between the physical values of the rotor in the moving and stationary axes is

$$\tilde{\boldsymbol{i}}_r = \boldsymbol{T}_r \boldsymbol{i}_r, \quad \tilde{\boldsymbol{\lambda}}_r = \boldsymbol{T}_r \boldsymbol{\lambda}_r. \tag{9}$$

The inverse transformations look like

$$\boldsymbol{i}_r = \boldsymbol{T}_r^{-1} \tilde{\boldsymbol{i}}_r, \quad \boldsymbol{\lambda}_r = \boldsymbol{T}_r^{-1} \tilde{\boldsymbol{\lambda}}_r. \tag{10}$$

Transformation by means of matrix $\boldsymbol{T}_r$ from the physical point of view is a replacement of rotating three-phase rotor windings by three-phase stationary windings. Its phase axes $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ are superposed with a, b, c stator phase axes. So, the physical values of stator windings remain unchanged.

Let us convert equation (1) to the stationary coordinates. After substituting (10) into expression (3) one can get

$$\boldsymbol{\lambda}_s = 2/3 \left(\boldsymbol{L}_{\sigma s} \boldsymbol{D} \boldsymbol{i}_s + L_m(i_m)(\boldsymbol{D}\boldsymbol{i}_s + \boldsymbol{D}_{rs}\boldsymbol{T}_r^{-1}\tilde{\boldsymbol{i}}_r)\right). \tag{11}$$

From equation (11) it follows that in order to extract the periodic coefficients, the condition $\boldsymbol{D}_{rs}\boldsymbol{T}_r^{-1} = \boldsymbol{D}$ must be satisfied. Thus, the matrix of transformation of rotor currents in the stationary coordinates is:

$$\boldsymbol{T}_r = \frac{2}{3}\boldsymbol{D}_{rs} = \frac{2}{3} \left\| \begin{matrix} \cos(\theta) & \cos(\theta + \rho) & \cos(\theta - \rho) \\ \cos(\theta - \rho) & \cos(\theta) & \cos(\theta + \rho) \\ \cos(\theta + \rho) & \cos(\theta - \rho) & \cos(\theta) \end{matrix} \right\|. \tag{12}$$

Matrix $\boldsymbol{T}_r$ is an orthogonal one, therefore the inverse matrix inverse coincides with the transposed one, namely

$$\boldsymbol{T}_r^{-1} = \boldsymbol{T}_r^t. \tag{13}$$

Taking into account (13), after substituting (11) into equation (1), it is reasonable to write it in the following form

$$\boldsymbol{v}_s = \frac{d\boldsymbol{\lambda}_m}{dt} + L_{\sigma s}\frac{d\boldsymbol{i}_s}{dt} + \boldsymbol{R}_s \boldsymbol{i}_s, \tag{14}$$

where $\boldsymbol{\lambda}_m = L_m(i_m)\left(\boldsymbol{i}_s + \tilde{\boldsymbol{i}}_r\right)$ is a vector of phase flux linkages of the air gap of IM.

To transform equation (2) into the stationary coordinates, The flux linkages and currents of rotor phases are to be transformed. After substituting (10) into formula (2)

$$\boldsymbol{v}_r = \frac{d(\boldsymbol{T}_r^{-1}\tilde{\boldsymbol{\lambda}}_r)}{dt} + \boldsymbol{R}_r\boldsymbol{T}_r^{-1}\tilde{\boldsymbol{i}}_r. \tag{15}$$

Multiplying equation (15) by the matrix $\boldsymbol{T}_r$ at the left-hand side and taking into account that the time derivative of the fluxlinkage vector in the fixed coordinate axes is

$$\frac{d(\boldsymbol{T}_r^{-1}\tilde{\boldsymbol{\lambda}}_r)}{dt} = \frac{d\boldsymbol{T}_r^{-1}}{dt}\tilde{\boldsymbol{\lambda}}_r + \boldsymbol{T}_r^{-1}\frac{d\tilde{\boldsymbol{\lambda}}_r}{dt}, \tag{16}$$

one can obtain

$$\boldsymbol{T}_r\boldsymbol{v}_r = \boldsymbol{T}_r\frac{d\boldsymbol{T}_r^{-1}}{dt}\tilde{\boldsymbol{\lambda}}_r + \boldsymbol{T}_r^{-1}\frac{d\tilde{\boldsymbol{\lambda}}_r}{dt} + \boldsymbol{T}_r\boldsymbol{R}_r\boldsymbol{T}_r^{-1}\tilde{\boldsymbol{i}}_r. \tag{17}$$

Time derivative of matrix $\boldsymbol{T}_r^{-1}$ taking into account the equation (13) looks as

$$\frac{d\boldsymbol{T}_r^{-1}}{dt} = -\frac{2}{3}\omega_r \left\| \begin{array}{ccc} \sin(\theta) & \sin(\theta - \rho) & \sin(\theta + \rho) \\ \sin(\theta + \rho) & \sin(\theta) & \sin(\theta - \rho) \\ \sin(\theta - \rho) & \sin(\theta + \rho) & \sin(\theta) \end{array} \right\|, \tag{18}$$

where $\omega_r = d\theta/dt$ is reduced to the pole section the angular rotation speed of the rotor. Taking into account (13) and (18) we find

$$\boldsymbol{T}_r\frac{d\boldsymbol{T}_r^{-1}}{dt} = \Omega_r = \frac{\omega_r}{\sqrt{3}} \left\| \begin{array}{ccc} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{array} \right\|. \tag{19}$$

The expression for the rotor flux linkage required to convert equation (17) to the stationary coordinates will be found from the expression (4) by multiplying its terms by the matrix $\boldsymbol{T}_r$ on the left-hand side. Taking into account (7) and (9)

$$\tilde{\boldsymbol{\lambda}}_r = 2/3\,C\left(\boldsymbol{T}_r\boldsymbol{L}_{\sigma r}\boldsymbol{T}_r^{-1}\boldsymbol{i}_r + L_m(i_m)(\boldsymbol{i}_s + \tilde{\boldsymbol{i}}_r)\right). \tag{20}$$

Substituting expressions (19) and (20) into equation (17) provides

$$\boldsymbol{T}_r\boldsymbol{v}_r = d\boldsymbol{\lambda}_m/dt + \boldsymbol{T}_r\boldsymbol{L}_{\sigma r}\boldsymbol{T}_r^{-1}d\tilde{\boldsymbol{i}}_r/dt + \boldsymbol{T}_r\boldsymbol{R}_r\boldsymbol{T}_r^{-1}\tilde{\boldsymbol{i}}_r + \Omega_r\tilde{\boldsymbol{\lambda}}_r. \tag{21}$$

In the equation (21) expressions $\boldsymbol{T}_r\boldsymbol{L}_{\sigma r}\boldsymbol{T}_r^{-1}$ and $\boldsymbol{T}_r\boldsymbol{R}_r\boldsymbol{T}_r^{-1}\tilde{\boldsymbol{i}}_r$ are leakage inductance and rotor resistance matrices in the stationary coordinate system. The elements of these matrices are values with the periodic coefficients, which makes the modeling of equations with such matrices more difficult.

In real IM, the leakage inductances and resistances of the rotor phases are the same. Taking into account the equality of rotor inductances $L_{\sigma ra} = L_{\sigma rb} = L_{\sigma rc} = L_{\sigma r}$ and resistances $R_{ra} = R_{rb} = R_{rc} = R_{\sigma r}$ we have

$$\tilde{\boldsymbol{L}}_{\sigma r} = \boldsymbol{T}_r\boldsymbol{L}_{\sigma r}\boldsymbol{T}_r^{-1} = \boldsymbol{L}_{\sigma r}, \tag{22}$$

$$\tilde{\boldsymbol{R}}_r = \boldsymbol{T}_r\boldsymbol{R}_r\boldsymbol{T}_r^{-1} = \boldsymbol{R}_r. \tag{23}$$

After substituting expressions (22) and (23) into equation (21), taking into account (20), it will look like

$$\tilde{\boldsymbol{v}}_r = d\boldsymbol{\lambda}_m/dt + \boldsymbol{L}_{\sigma r}d\tilde{\boldsymbol{i}}_r/dt + \boldsymbol{R}_r\tilde{\boldsymbol{i}}_r + \Omega_r(\boldsymbol{L}_{\sigma r}\tilde{\boldsymbol{i}} + \boldsymbol{\lambda}_m), \tag{24}$$

where $\boldsymbol{\lambda}_m = L_m(i_m)\left(\boldsymbol{i}_s + \tilde{\boldsymbol{i}}_r\right) = (\lambda_{ma}, \lambda_{mb}, \lambda_{mc})^t$ is a vector of phase flux linkages of the IM air gap.

Accounting of active power losses in the magnetic core of IM caused by eddy currents and hysteresis is carried out by introducing three additional resistors into the circuit with the additional winding, which wraps the main magnetic flux λ_m, similarly as for the transformer described in [10]. The differential equations for the main magnetic fluxes will be obtained by Ampere's law for the main circuit of the magnetic core and Kirchhoff's law. Let us write these equations in the following form:

$$\boldsymbol{D}\left(\boldsymbol{i}_s + \tilde{\boldsymbol{i}}_r + \boldsymbol{i}_{si} - R_m(\lambda_m)\boldsymbol{\lambda}_m\right) = 0, \tag{25}$$

$$\frac{d\boldsymbol{\lambda}_m}{dt} + \boldsymbol{R}_{si}\boldsymbol{i}_{si} = 0, \tag{26}$$

where $R_m(\lambda_\delta) = i_m(\lambda_\delta)/\lambda_\delta = 1/L_m(\lambda_\delta)$ is magnetic resistance of IM main circuit, $\boldsymbol{R}_{si} = \mathrm{diag}\,(R_{sia}, R_{sib}, R_{sic})$ is the diagonal matrix of resistors considering also the active power losses in the magnetic core, $\boldsymbol{i}_{si} = (i_{sia}, i_{sib}, i_{sic})^t$ is a vector of currents in resistors considering also active power losses in the IM magnetic core.

Taking into account that in equation (25) $\boldsymbol{D} \neq 0$, it can be rewritten

$$\boldsymbol{i}_s + \tilde{\boldsymbol{i}}_r + \boldsymbol{i}_{si} - R_m(\lambda_m)\boldsymbol{\lambda}_m = 0. \tag{27}$$

Vector of currents i_{si} via equation (27), after substitution into (26) is

$$\frac{d\boldsymbol{\lambda}_m}{dt} = R_{si}\left(\boldsymbol{i}_s + \tilde{\boldsymbol{i}}_r - R_m\left(\lambda_m\right)\boldsymbol{\lambda}_m\right), \tag{28}$$

The equation of mechanical motion of the induction motor rotor looks as follows:

$$\frac{J}{p_0}\frac{d\omega_r}{dt} = \pm T_{em}\left(\boldsymbol{i}_r, \boldsymbol{\lambda}_m\right) \mp T_m\left(\omega_r\right), \tag{29}$$

where p_o, J are the number of pole pairs and the moment of the AM rotor inertia respectively, $T_m(\omega_r)$ is mechanical torque applied to the AM rotor, $T_{em}\left(\boldsymbol{i}_r, \boldsymbol{\lambda}_m\right)$ is electromagnetic moment of IM, which should be found out by the formula [1]

$$T_{em}(\boldsymbol{\lambda}_m, \tilde{\boldsymbol{i}}_r) = 3p_o|\bar{\boldsymbol{\lambda}}_m \times \bar{\tilde{\boldsymbol{i}}}_r|/2 = \sqrt{3}p_o\big(\lambda_{ma}(\tilde{i}_{rb} - \tilde{i}_{rc}) + \lambda_{mb}(\tilde{i}_{rc} - \tilde{i}_{ra}) + \lambda_{mc}(\tilde{i}_{ra} - \tilde{i}_{rb})\big)/2. \tag{30}$$

where $\bar{\lambda}_m$, $\bar{\tilde{i}}_r$ are the spatial vectors of the main flux linkage and rotor current in the stationary coordinates.

3. Implementation of the induction machine model in MATLAB/Simulink

System of the nonlinear differential and algebraic equations of electromechanical IM state (14), (24), (28) and (29) obtained above, taking into account expressions (8) and (30), can be implemented in the software environments. The implementation of such a system in the available software tools used for the modeling states and processes is a complicated problem.

To implement the mathematical IM model in MATLAB/Simulink differential equations (14), (24), (28) and (29) must be expressed in the integral form

$$\boldsymbol{i}_s = \boldsymbol{L}_{\sigma s}^{-1}\left(\int_0^t \left(\boldsymbol{v}_s - \boldsymbol{R}_s\boldsymbol{i}_s\right)dt - \boldsymbol{\lambda}_m\right) + \boldsymbol{i}_s(0), \tag{31}$$

$$\tilde{\boldsymbol{i}}_r = \boldsymbol{L}_{\sigma r}^{-1}\left(\int_0^t \left(\tilde{\boldsymbol{v}}_r - \boldsymbol{R}_r\tilde{\boldsymbol{i}}_r - \boldsymbol{\Omega}_r(\boldsymbol{L}_{\sigma r}\tilde{\boldsymbol{i}}_r + \boldsymbol{\lambda}_m)\right)dt - \boldsymbol{\lambda}_m\right) + \tilde{\boldsymbol{i}}_r(0), \tag{32}$$

$$\boldsymbol{\lambda}_m = \boldsymbol{R}_{si} \int_0^t \left(\boldsymbol{i}_s + \tilde{\boldsymbol{i}}_r - R_m(|\bar{\lambda}_m|)\boldsymbol{\lambda}_m\right) dt + \boldsymbol{\lambda}_m(0), \tag{33}$$

$$\omega_r = \frac{J}{p_0} \int_0^t (\pm\sqrt{3}p_0(\lambda_{ma}(\tilde{i}_{rb} - \tilde{i}_{rc}) + \lambda_{mb}(\tilde{i}_{rc} - \tilde{i}_{ra}) + \lambda_{mc}(\tilde{i}_{ra} - \tilde{i}_{rb}))/2 \mp T_m(\omega_r))dt + \omega_r(0). \tag{34}$$

The stator phase voltages $\boldsymbol{v}_s$ are determined by the phase voltages of the busbars supplying the IM, according to the following formulas

$$\boldsymbol{v}_s = \left| \begin{array}{c} (2v_a - v_b - v_c)/3 \\ (2v_b - v_a - v_c)/3 \\ (2v_c - v_a - v_b)/3 \end{array} \right|, \tag{35}$$

where v_a, v_b, v_c are the phase voltages of microgrid buses connected to the induction machine (Fig. 1).
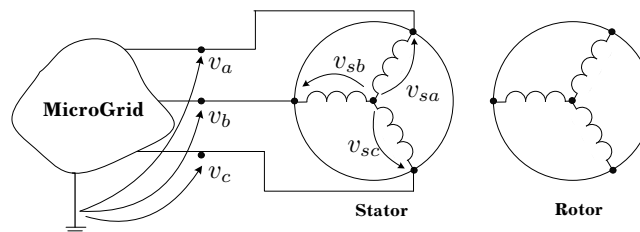
Fig. 1. Scheme of IM connection with a microgrid.

For a squirrel-cage induction motor in equation (32) it is necessary to take that $\tilde{\boldsymbol{v}}_r = 0$. Structural diagram of the mathematical model of squirrel-cage induction motor for one phase b in MATLAB / Simulink environment is shown in Fig. 2. Simulation results of the induction motor direct switching from a powerful source are shown in Fig. 3.

To verify the adequacy and the capability of the proposed mathematical model, a model of induction motor [11–13] of 4A180M4 type neglecting the saturation of the magnetic core and losses of active power in the magnetic core using built-in SimScape/Electrical model was developed. Parameters of the induction motor were as follows: $U_{phnom} = 220\,\text{V}$, $2p = 4$, $P_2 = 30\,\text{kW}$, $J = 0.225\,\text{kg}\,\text{m}^2$, $R_s = 0.16\,\text{Ohm}$, $L_s = 0.169\,\text{H}$, $L_{s\sigma} = 0.005\,\text{H}$, $L_{r\sigma} = 0.0075\,\text{H}$, $R_r = 0.078\,\text{Ohm}$, $L_m = 0.049\,\text{H}$.

Mechanical torque $T_m(t)$ for the mechanism with fan characteristics is given in the form of the following function presented in Fig. 3. Block diagram of this model is presented in Fig. 4.

$$T_m(\omega_r) = 176\left(0.2 + 0.08\,(\omega_r/308)^2\right). \tag{36}$$

The simulation results of direct motor starting fed by high-power voltage source with the linear voltage 380 V are shown in Fig. 4. The figure shows the angular rotor speed, the stator phase current and the electromagnetic torque of the motor.

The simulation results obtained with the proposed model (Fig. 2) are shown in the Fig. 4 and Fig. 5. In Fig. 4 the electromagnetic coordinates of the transient process at induction motor start-up are shown. Parameters of the power supply and mechanical torque were the same as used for the model given in Fig. 3. Here the following oscillograms are presented: stator phase current, modulus of spatial vector of flux linkage $|\bar{\lambda}_m|$, phase flux linkage from the air gap magnetic flux λ_{ma} and magnetic resistance $R_m(|\bar{\lambda}_m|)$ of the main magnetic circuit of induction motor.

The offered structural diagram (Fig. 2) takes into account the magnetization characteristic of the main magnetic circuit, approximated by the dependence of its magnetic resistance on the modulus of the spatial vector $|\bar{\lambda}_m|$ by a polynom of the following type

$$R_m(|\bar{\lambda}_m|) = 11.7 + 1.21\left|\bar{\lambda}_m\right|^4 + 0.497\left|\bar{\lambda}_m\right|^8 \tag{37}$$

and active power losses in the magnetic core by introducing of resistors $R_{si} = 500\,\text{Ohm}$.
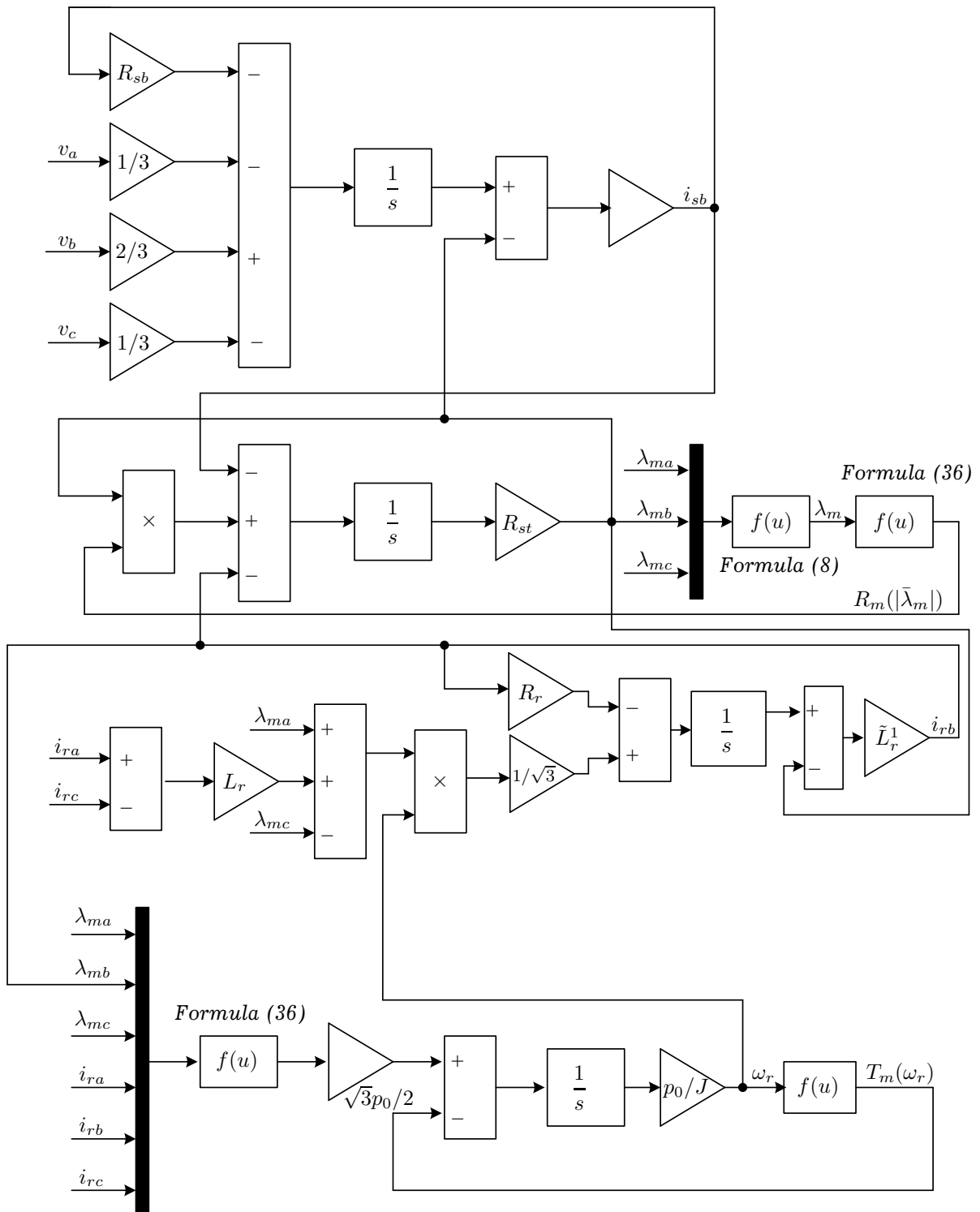
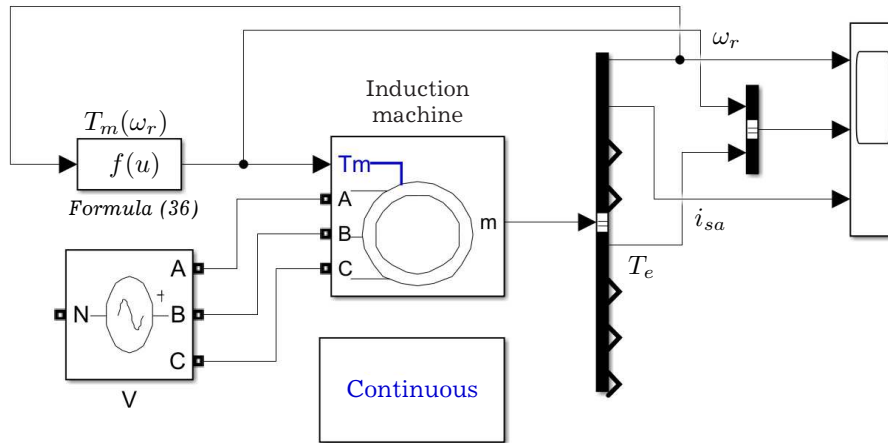**Fig. 2.** Block diagram of the mathematical IM model in Simulink.

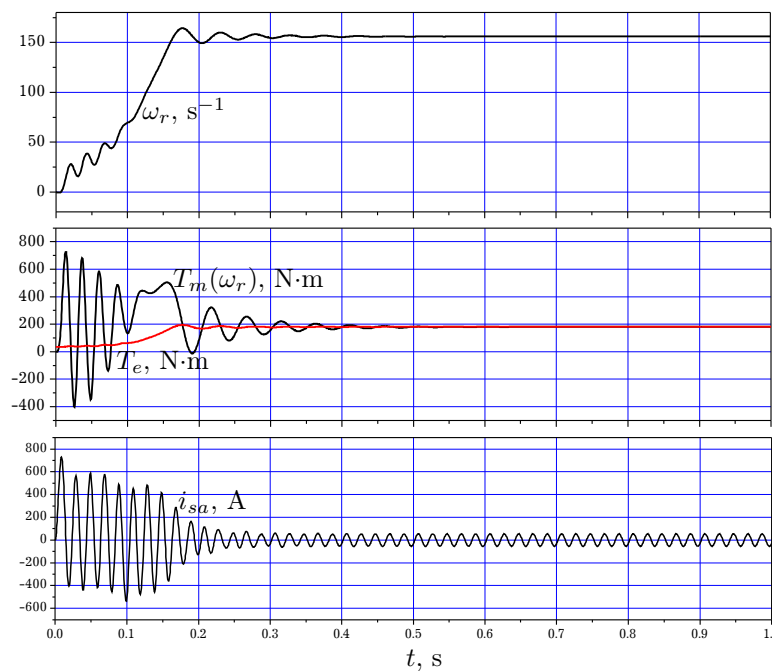**Fig. 3.** Block diagram of the mathematical IM model in Simscape.

Fig. 4. Waveforms of coordinates of the IM transient in Simscape.

Based on a comparative analysis of the calculation results of the process coordinates by two models, we can conclude that the nonlinearity of the magnetization characteristic of the main magnetic circuit significantly influences on the values of the transient process coordinates: the surge current and the damping duration of the aperiodic components of its coordinates. In the steady-state mode, the stator current parameters and mechanical coordinates, namely, the rotor rotation speed and electromagnetic moment practically coincide.

The proposed model is the universal one and open to the further improvement due to the additional nonlinear phenomena — the influence of power losses because of hysteresis and eddy currents, the effect of current displacement in the winding conductors and the nonlinearity of stator and rotor leakage inductance.
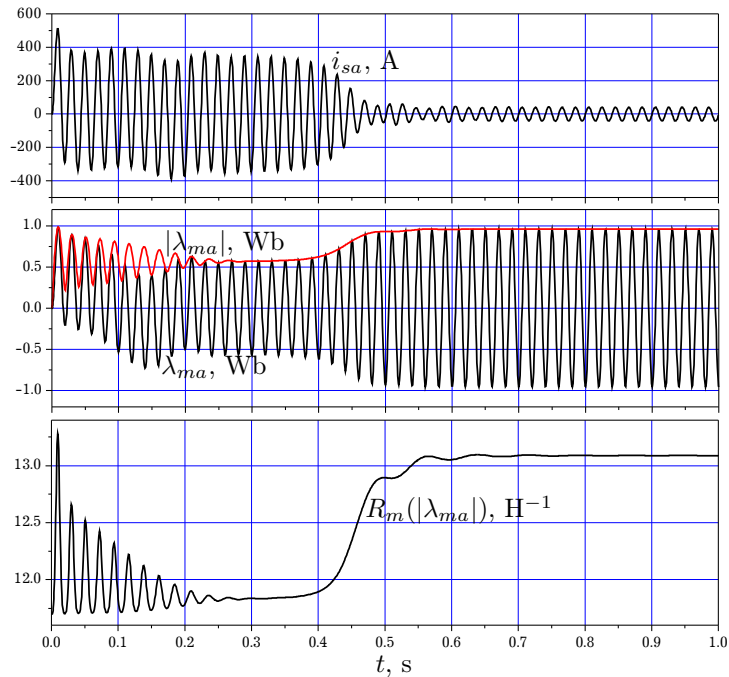
Fig. 5. Waveforms of electromagnetic coordinates of the IM transient in Simulink.
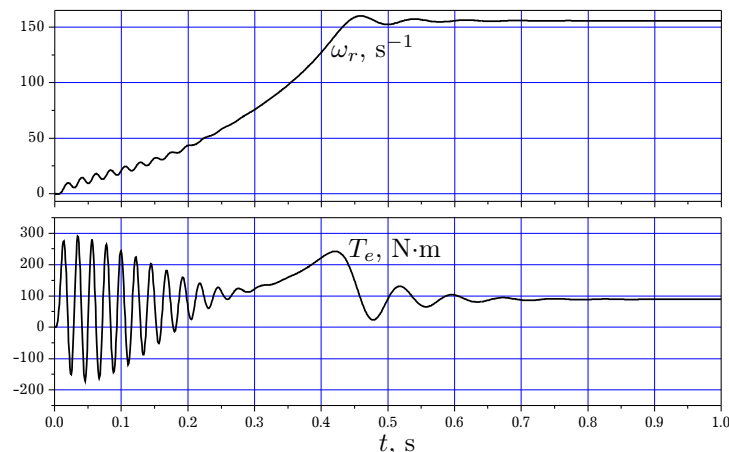

Fig. 6. Waveforms of mechanical coordinates of the IM transient in Simulink.

4. Conclusion

The universal mathematical model for transient processes analysis in the induction machine has been constructed. Presented model considered the influence of schemes of connection and parameters of electric and magnetic circuits of smart grid and microgrid elements in MATLAB / Simulink visual modeling environment. The model provides wide opportunities for the study of transient electromagnetic and electromechanical processes in the complex electrical systems, especially with the nonlinear elements and dynamic loads. The model can be used to solve specific engineering problems of analysis and synthesis of modes and processes of microgrid in smart grid.

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Універсальна математична модель асинхронної машини як елемента інтелектуальної електропостачальної системи

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Створено універсальну математичну модель асинхронної машини (АМ) з адекватним урахуванням насичення основного магнетного кола її магнетопроводу та втрат активної потужності в ньому. Запропонований підхід до моделювання АМ розширює сферу використання середовища MATLAB/Simulink для аналізу режимів і процесів інтелектуальних електропостачальних систем (microgrid in smart grid) з нелінійними елементами та динамічним навантаженням у анормальних, несиметричних і аварійних ситуаціях.

Ключові слова: *індукційна машина, математична модель, інтелектуальна електропостачальна система, моделювання.*