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Synchronization of time-varying time delayed neutral-type neural networks for finite-time in complex field

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This paper deals with the problem of finite-time projective synchronization for a class of neutral-type complex-valued neural networks (CVNNs) with time-varying delays. A simple state feedback control protocol is developed such that slave CVNNs can be projective synchronized with the master system in finite time. By employing inequalities technique and designing new Lyapunov–Krasovskii functionals, various novel and easily verifiable conditions are obtained to ensure the finite-time projective synchronization. It is found that the settling time can be explicitly calculated for the neutral-type CVNNs. Finally, two numerical simulation results are demonstrated to validate the theoretical results of this paper.

Keywords: *neutral-type neural network, neutral delay, synchronization, complex field, time-varying time delays.*

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1. Introduction

Recently, various kinds of neural networks (NNs) such as HNNs (Hopfield type NNs), CGNNs (Cohen-Grossberg NNs), CNNs (cellular NNs), BAMNNs (bidirectional associative memory NNs) etc., have been commonly used in solving optimization problems, signal and image processing problems. When modeling a NN for such implementations, it is important to understand the equilibrium and stability of the designed NNs properties, which are dependent on the neural system's network parameters. The deep study of the neural network reveals that practical NNs always have certain complex characteristics, for example unknown parameters, active characteristics, high dimensions, delays and external disturbances (see for example [1–7]). Because of these complexities, solving the neural network problem using conventional research methods is difficult. In recent decades, taken into account of all previous works are deal with real-valued NNs (RVNNs). And since RVNNs can be applied in many fields, they also have limitations. For instance, in signal processing, the data processing, the detection of memory and the XOR problem are in complex valued cannot be deal with real-valued neurons, but they can be addressed by a single complex-valued counterpart.

Complex-valued NNs (CVNNs) are a rapidly developing field and CVNNs are generic extension of classical RVNNs with complex valued states, complex valued activation, complex valued connection weights [8]. However, it's worth noting that both complex-valued neurons and CVNNs have a few primary advantages over their real-valued counterparts. That is they have significantly more features and their increased plasticity and versatility, which helps them to learn and generalize more easily. In addition, the optical computing platforms that encode information both in phase and size can perform complex arithmetical procedures through optical interference with a significantly increased computer performance and power efficiency. However, most optical neural network demonstrations to-date have relied on traditional real-valued architectures built for digital computers, sacrificing many of the benefits of optical computing, such as efficient complex-valued operations. So it is necessary to take complex parameters into consideration of neural network research.

In the study of dynamic NNs, the parameters uncertainty and time delays are two major issues due to the limited feedback information as well as the traffic jam in actual NNs. As a result, in the hardware implementation of dynamic NNs, owing to the finite switching speed of amps and the transmission delays during the communication between neurons, certain time delays in the network likely happen, which affect the dynamic behavior of the neural system. Therefore, recent research has concentrated on the stability and stabilization properties of different NNs in the presence of time delays. In the existing NN models such as HNNs, CGNNs, CNNs, BAMNNs, the time delays are occur in neural system's states. However, because the time derivatives of the states are functions of time, a few other delay parameters should be incorporated into the time derivatives of the system's states in order to effectively determine the stability characteristics of the equilibrium point. The NN model having time delays in the time derivative of state vectors is called neutral-type NNs with time delays [9–12]. For example, in very large scale integration implementation of artificial NNs, delay transmission lines and partial element equivalent circuit are the two major types of components used to generate delays. It should be noted that the circuits formed by partial element equivalent circuit may result in neutral delays. This type of neutral system has been applied to a wide range of applications, including distributed networks with lossless transmission lines [13], propagation, population ecology [13], very large scale integration systems [14] and diffusion models [14]. Many researchers have recently studied the equilibrium and stability characteristics of neutral-type NNs with a single delay and proposed various adequate conditions for the global stability and synchronization of the equilibrium states in the literature.

One of the most important aspects of neural network research is synchronization behaviors of an equilibrium states. Neural network synchronization has become an important problem in neural network research over the last few decades, and it has gotten a lot of attention. Synchronization of NNs is a critical problem in natural science and engineering technology. Synchronization is achieved for the drive-response neural network system when the derived error system states become stable over time. Complete synchronization [15], generalized synchronization [16], quasi-synchronization [17], anti-synchronization, projective synchronization [18], exponential synchronization [19], cluster synchronization [20] and phase synchronization [21] are now just a few examples of synchronization schemes suggested both theoretically and practically. Among all types of chaotic synchronizations, projective synchronization represents that under a suitable controller, the slave neural network is synchronized to the master neural network by a proportional factor, demonstrates that different types of synchronization can be obtained by selecting different projection coefficients. When the projection coefficients are 1, 0 and -1, respectively, complete synchronization, stability, and anti-synchronization can be obtained separately. The scale factor of projective synchronization may be changed flexibly, which increases the uncertainties of the slave system and improves communication security. The authors of the literature [22] said that projective synchronization may be utilized to expand binary digital to M-nary digital communication in order to achieve rapid communication in applications involving secure communications, so the research on projective synchronization has both theoretical and practical relevance. Nevertheless, most synchronization research focuses on infinite-time synchronization control. Infinite-time synchronization cannot calculate the time required to achieve complete synchronization, which may result in discrepancy between the sent and received times. In many practical applications, the inconsistency induced by infinite-time synchronization control is problematic. However, the finite time synchronization, compared to the infinite time synchronization, has a faster convergence speed and a better robustness [23], which is more reasonable in realistic applications and attracts a lot of researchers [24–28]. Therefore, it is essential to understand the synchronization that can be achieved in a finite time. Up to now, the effects of neutral delays on finite time projective synchronization between two CVNNs with time-varying delays have not been investigated, which motivates the current study.

To the best of our knowledge, the projective finite-time synchronization for neutral-type CVNNs with time-varying delays has received very little attention, which means that this interesting topic is still open and challenging. We will do some efforts on it and provide some sufficient conditions to realize master-slave projective synchronization in finite time. The main contributions of this article is given in the following aspects:

(i) The finite time projective synchronization for neutral type CVNNs with time-varying delay is considered and by choosing different projective scaling matrix (βI) we achieve different type of synchronization, i.e. a) complete synchronization is achieved when $\beta = 1$; b) anti-synchronization is achieved when $\beta = -1$; c) projective synchronization is achieved when $\beta = c$, where c is arbitrary constant.

(ii) Sufficient criteria are provide to realize master-slave projective synchronization in finite time based on Lyapunov stability theory and analytic techniques are presented in terms of LMIs which could be easily verified by using LMI tool box in MATLAB. Two numerical examples are provided in order to show the effectiveness of theoretical results.

The remaining part of this article is organized as follows: Section 2 provides the model description of neutral-type CVNNs with time-varying delays, together with the basic preliminary results related to our main results. Then, Section 3 is devoted to provide a sufficient condition based on the adaptive feedback controller, that ensures finite-time projective master-slave synchronization. Two numerical examples are given to demonstrate the viability and efficacy of the obtained theoretical results in Section 4. Finally, the conclusions of the paper is presented in Section 5.

2. Model description and preliminaries

Consider the following neutral-type CVNNs with time-varying delays:

$$\frac{dw(t)}{dt} = -Dw(t) + Mf(w(t)) + Ng(w(t-\tau(t))) + \tilde{M}(\dot{w}(t-\sigma(t))) + U, \tag{1}$$

where $w(t) = (w_1(t), w_2(t), \ldots, w_n(t))^T \in \mathbb{C}^n$ is state vector with $w_k(t) = a_k^R(t) + ib_k^I(t)$; $\dot{w}(t-\tau(t)) = (\dot{w}_1(t-\tau(t)), \dot{w}_2(t-\tau(t)), \ldots, \dot{w}_n(t-\tau(t)))^T \in \mathbb{C}^n$; $D = \operatorname{diag}\{d_1, d_2, \ldots, d_n\}$ is the self-feedback connection weight matrix with $d_k > 0$, $k = 1, 2, \ldots, n$; $M = (m_{kq}) \in \mathbb{C}^{n\times n}$, $N = (h_{kq}) \in \mathbb{C}^{n\times n}$, $\tilde{M} = (\tilde{m}_{kq}) \in \mathbb{C}^{n\times n}$ are connection weights matrices; $f(w(t)) = (f_1(w_1(t)), f_2(w_2(t)), \ldots, f_n(w_n(t)))^T \in \mathbb{C}^n$, $g(w(t-\tau(t))) = (g_1(w_1(t-\tau(t))), g_2(w_2(t-\tau(t))), \ldots, g_n(w_n(t-\tau(t))))^T \in \mathbb{C}^n$ are activation functions without and with time-delay respectively. $\tau(t)$ is the time-varying delay satisfying $0 \leqslant \tau(t) \leqslant \bar{\tau}$ and $\dot{\tau}(t) \leqslant \mu$; $\sigma(t)$ is the neutral delay with $0 \leqslant \sigma(t) \leqslant \sigma$ and let $\tau = \max\{\bar{\tau}, \sigma\}$; $U = (u_1, u_2, \ldots, u_n)^T \in \mathbb{C}^n$ is the external input vector. The initial condition associated with the neutral-type CVNNs (1) is $w(s) = \phi(s), \dot{w}(s) = \Phi(s)$, $s \in [-\tau, 0]$, where $\phi(s) = \phi^R(s) + i\phi^I(s)$, $\Phi(s) = \Phi^R(s) + i\Phi^I(s)$, $\phi(s), \Phi(s) \in \mathcal{C}([-\tau, 0]; \mathbb{C}^n)$.

Next, we give some definitions, lemmas and assumptions to find the finite-time projective synchronization of neutral-type CVNNs (1) with time-varying delays.

Assumption 1. For $w(t) = a^R(t) + ib^I(t) \in \mathbb{C}$, $a^R(t), b^I(t) \in \mathbb{R}$, the activation functions $f_k(w_k(t))$ can be separated into its real and imaginary part as: $f_k(w_k(t)) = f_k^R(a_k^R(t)) + if_k^I(b_k^I(t))$, $k = 1, 2, \ldots, n$ and there exist positive constants o_k and $\tilde{o}_k$ such that $|f_k^R(a_k(t))| \leqslant o_k$, $|f_k^I(b_k(t))| \leqslant \tilde{o}_k$. Therefore the activation functions $f_k^R(a_k^R(t))$, $f_k^I(b_k^I(t))$, $g_k^R(a_k^R(t-\tau(t))), g_k^I(b_k^I(t-\tau(t)))$ satisfies $|f_k^R(a_k^R(t))| \leqslant o_k^f$, $|f_k^I(b_k^I(t))| \leqslant \tilde{o}_k^f$, $|g_k^R(a_k^R(t-\tau(t)))| \leqslant o_k^g$, $|g_k^I(b_k^I(t-\tau(t)))| \leqslant \tilde{o}_k^g$, where o_k^f, $\tilde{o}_k^f$, o_k^g, $\tilde{o}_k^g$ are Lipchitz constants. $k = 1, 2, \ldots, n$.

Assumption 2. Suppose that there exists some constants $\hat{\xi}_k^-, \check{\xi}_k^-, \hat{\xi}_k^+, \check{\xi}_k^+, \hat{\zeta}_k^-, \check{\zeta}_k^-, \hat{\zeta}_k^+, \check{\zeta}_k^+$, $k = 1, 2, \ldots, n$ such that for any $a, b \in \mathbb{R}$ and $a \neq a$, $f_k^R(\cdot), f_k^I(\cdot), g_k^R(\cdot), g_k^I(\cdot)$ satisfies that

$$\hat{\xi}_k^- \leqslant \frac{f_k^R(a) - f_k^R(b)}{a-b} \leqslant \hat{\xi}_k^+, \quad \check{\xi}_k^- \leqslant \frac{f_k^I(a) - f_k^I(b)}{a-b} \leqslant \check{\xi}_k^+,$$
$$\hat{\zeta}_k^- \leqslant \frac{g_k^R(a) - g_k^R(b)}{a-b} \leqslant \hat{\zeta}_k^+, \quad \check{\zeta}_k^- \leqslant \frac{g_k^I(a) - g_k^I(b)}{a-b} \leqslant \check{\zeta}_k^+.$$

Denote

$$G_1 = \operatorname{diag}\{\hat{\xi}_1^+\hat{\xi}_1^-,\dots,\hat{\xi}_n^+\hat{\xi}_n^-\},\quad G_2 = \operatorname{diag}\left\{\frac{\hat{\xi}_1^+ + \hat{\xi}_1^-}{2},\dots,\frac{\hat{\xi}_n^+ + \hat{\xi}_n^-}{2}\right\},$$
$$G_3 = \operatorname{diag}\left\{\check{\xi}_1^+\check{\xi}_1^-,\dots,\check{\xi}_n^+\check{\xi}_n^-\right\},\quad G_4 = \operatorname{diag}\left\{\frac{\check{\xi}_1^+ + \check{\xi}_1^-}{2},\dots,\frac{\check{\xi}_n^+ + \check{\xi}_n^-}{2}\right\},$$
$$G_5 = \operatorname{diag}\{\hat{\zeta}_1^+\hat{\zeta}_1^-,\dots,\hat{\zeta}_n^+\hat{\zeta}_n^-\},\quad G_6 = \operatorname{diag}\left\{\frac{\hat{\zeta}_1^+ + \hat{\zeta}_1^-}{2},\dots,\frac{\hat{\zeta}_n^+ + \hat{\zeta}_n^-}{2}\right\},$$
$$G_7 = \operatorname{diag}\{\check{\zeta}_1^+\check{\zeta}_1^-,\dots,\check{\zeta}_n^+\check{\zeta}_n^-\},\quad G_8 = \operatorname{diag}\left\{\frac{\check{\zeta}_1^+ + \check{\zeta}_1^-}{2},\dots,\frac{\check{\zeta}_n^+ + \check{\zeta}_n^-}{2}\right\}.$$

If $f_k(w_k(t))$, $g_k(w_k(t-\tau(t)))$, $\tilde{M}_k(w_k(t-\sigma(t)))$ can be separated into its real and imaginary part by using Assumption 1, then neutral-type CVNNs (1) can be separated into its real and imaginary part as follows:

$$\begin{cases} \dot{a}^R(t) = -Da^R(t) + M^R f^R(a^R(t)) - M^I f^I(b^I(t)) + N^R g^R(a^R(t-\tau(t))) - N^I g^I(a^I(t-\tau(t))) \\ \qquad\quad +\tilde{M}^R\dot{a}^R(t-\sigma(t)) - \tilde{M}^I\dot{b}^I(t-\sigma(t)) + U^R, \\ \dot{b}^I(t) = -Db^I(t) + M^R f^I(b^I(t)) + M^I f^R(a^R(t)) + N^R g^I(b^I(t-\tau(t))) + N^I g^R(a^R(t-\tau(t))) \\ \qquad\quad +\tilde{M}^R\dot{b}^I(t-\sigma(t)) + \tilde{M}^I\dot{a}^R(t-\sigma(t)) + U^I. \end{cases} \tag{2}$$

Considering (2) as master system and the corresponding slave system of (2) is given bellow:

$$\begin{cases} \dot{\hat{a}}^R(t) = -D\hat{a}^R(t) + M^R f^R(\hat{a}^R(t)) - M^I f^I(\hat{b}^I(t)) + N^R g^R(\hat{a}^R(t-\tau(t))) - N^I g^I(\hat{a}^I(t-\tau(t))) \\ \qquad\quad +\tilde{M}^R\dot{\hat{a}}^R(t-\sigma(t)) - \tilde{M}^I\dot{\hat{b}}^I(t-\sigma(t)) + U^R + \mu^R(t), \\ \dot{\hat{b}}^I(t) = -D\hat{b}^I(t) + M^R f^I(\hat{b}^I(t)) + M^I f^R(\hat{a}^R(t)) + N^R g^I(\hat{b}^I(t-\tau(t))) + N^I g^R(\hat{a}^R(t-\tau(t))) \\ \qquad\quad +\tilde{M}^R\dot{\hat{b}}^I(t-\sigma(t)) + \tilde{M}^I\dot{\hat{a}}^R(t-\sigma(t)) + U^I + \mu^I(t), \end{cases} \tag{3}$$

where $\hat{a}^R(t)$, $\hat{b}^I(t) \in \mathbb{R}^n$ and $\mu^R(t)$, $\mu^I(t)$ are the controller to be designed and all the other parameters are same as those defined in the master system (2).

Let the error of projective synchronization between master system (2) and slave system (3) is $e^R(t) = \hat{a}^R(t) - \beta a^R(t)$, $e^I(t) = \hat{b}^I(t) - \beta b^I(t)$, i.e. $\dot{e}^R(t) = \dot{\hat{a}}^R(t) - \beta\dot{a}^R(t)$, $\dot{e}^I(t) = \dot{\hat{b}}^I(t) - \beta\dot{b}^I(t)$, where β is the scaling factor. Then the state feedback controller scheme is designed for finite-time projective synchronization between the master (2) and slave (3) system, for which the controllers $\mu^R(t)$, $\mu^I(t)$ are given by

$$\begin{aligned} \mu^R(t) &= K^R e^R(t) + (D\beta - \beta D)a^R(t) - M^R f^R(\beta(a^R(t))) + \beta M^R f^R(a^R(t)) + M^I f^I(\beta b^I(t)) \\ &- \beta M^I f^I(b^I(t)) - N^R f^R(\beta a^R(t-\tau(t))) + \beta N^R g^R(a^R(t-\tau(t))) + N^I g^I(\beta b^I(t-\tau(t))) \\ &- \beta N^I g^I(b^I(t-\tau(t))) - (\tilde{M}^R\beta - \beta\tilde{M}^R)\dot{a}(t-\sigma(t)) + (\tilde{M}^I\beta - \beta\tilde{M}^I)\dot{a}^I(t-\sigma(t)) + \beta U^R - U^R, \end{aligned} \tag{4}$$

$$\begin{aligned} \mu^I(t) &= K^I e^I(t) + (D\beta - \beta D)b^I(t) - M^R f^I(\beta(b^I(t))) + \beta M^R f^I(b^I(t)) - M^I f^R(\beta a^R(t)) \\ &+ \beta M^I f^R(\beta a^R(t)) - N^R g^I(\beta a^R(t-\tau(t))) + \beta N^R g^I\beta(a^R(t-\tau(t)) - N^I g^R(\beta a^R(t-\tau(t)) \\ &+ \beta N^I g^R(a^R(t-\tau(t))) - \dot{b}^I(t-\sigma(t))(\tilde{M}^R\beta - \beta\tilde{M}^R) - \dot{a}^R(t-\sigma(t))(\tilde{M}^I\beta - \beta\tilde{M}^I) + \beta U^I - U^I, \end{aligned} \tag{5}$$

where K^R, K^I are controller gain matrices.

Definition 1. *For a designed controller $\mu^R(t)$, $\mu^I(t)$, if there exists a constant $t_1 > 0$, such that $||b(t_1) - \beta a(t_1)||_1 = 0$ and $||b(t) - \beta a(t)||_1 \equiv 0$ for all $t > t_1$, where $||b(t) - \beta a(t)||_1 = \sum_{k=1}^n |b_k(t) - \beta a_k(t)| = \sum_{k=1}^n \sqrt{(b_k^R(t) - \beta a_k^R(t))^2 + (b_k^I(t) - \beta a_k^I(t))^2}$, then the neutral-type CVNNs (3) is said to be finite-time projective synchronization with neutral-type CVNNs system (2).*

Lemma 1. *Let $r, s \in \mathbb{C}^n$, P be a positive definite Hermitian matrix, then $r^T s + s^T r \leqslant r^T P^{-1} r + s^T P s$.*

Remark 1. Recently, many scholars have concentrated their research on synchronization of drive-response NNs, for example complete synchronization, Lag synchronization, impulsive synchronization, generalized synchronization and projective synchronization. Briefly speaking, complete synchronization is characterized by the equality of state variables while evolving in time, that is $b(t) \to a(t)$, $t \to \infty$. Lag synchronization is defined as the concurrence of shifted-in-time states of two systems, that is $b(t) \to a(t-\tau)$, $t \to \infty$, with a propagation delay $\tau > 0$. The sudden changes in the system dynamics at some instants describe impulsive synchronization. The existence of certain functional relationship between the states of the drive and response systems is termed as generalized synchronization, that is $b(t) \to 6(a(t))$, $t \to \infty$. Projective synchronization is distinguished by the fact that the drive and response systems can be synchronized up to a scaling factor β, that is $b(t) \to \beta a(t)$, $t \to \infty$. When the projection coefficients (β) are -1, 0, and 1, respectively, complete synchronization, stability, and anti-synchronization can be obtained separately.

3. Main results

First, we provide a sufficient condition based on the controller (4) and (5) that ensures finite-time projective synchronization of (2) and (3).

Theorem 1. *Suppose that Assumptions 1 and 2 are satisfied and there exist positive definite matrices P_s, Q_s, S_s, R_s, $s = 1, 2$ and diagonal matrices λ_p, $p = 1, \dots, 7$ such that the following LMI condition is hold with given positive constants ε_q, $q = 1, \dots, 4$,*

$$\Omega = \begin{pmatrix} \Pi_{1,1} & 0 & 0 & 0 & P_1M^R + \lambda_1 G_2 & 0 & \lambda_3 G_6 & 0 & P_1 N^R & -P_1 N^I & 0 & 0 \\ \star & \Pi_{2,2} & 0 & 0 & 0 & \Pi_{2,6} & 0 & \lambda_4 G_8 & P_2 N^I & P_2 N^R & 0 & 0 \\ \star & \star & \Pi_{3,3} & 0 & 0 & 0 & 0 & 0 & \lambda_5 G_6 & 0 & 0 & 0 \\ \star & \star & \star & \Pi_{4,4} & 0 & 0 & 0 & 0 & 0 & \lambda_6 G_8 & 0 & 0 \\ \star & \star & \star & \star & -\lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -\lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -\lambda_3 + Q_1 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & -\lambda_4 + Q_2 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \Pi_{9,9} & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \Pi_{10,10} & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \Pi_{11,11} & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \Pi_{12,12} \end{pmatrix} < 0 \tag{6}$$

where,

$$\Pi_{1,1} = P_1(K^R - D) + (K^R - D)^T P_1 + S_1 - \lambda_1 G_1 - \lambda_3 G_5 + \varepsilon_1^{-1} P_1 P_1^T - \varepsilon_3^{-1} P_1 P_1^T,$$
$$\Pi_{2,2} = P_2(K^I - D) + (K^I - D)^T P_2 + S_2 - \lambda_2 G_3 - \lambda_4 G_7 + \varepsilon_2^{-1} P_2 P_2^T - \varepsilon_4^{-1} P_2 P_2^T$$
$$\Pi_{3,3} = -(1-\dot{\tau}(t))S_1 - \lambda_5 G_5, \quad \Pi_{4,4} = -(1-\dot{\tau}(t))S_2 - \lambda_6 G_7, \quad \Pi_{9,9} = -(1-\dot{\tau}(t))Q_1 - \lambda_5,$$
$$\Pi_{10,10} = -(1-\dot{\tau}(t))Q_2 - \lambda_6, \quad \Pi_{2,6} = P_2 M^R + \lambda_2 G_4, \quad \Pi_{11,11} = \varepsilon_1 \tilde{M}^R \tilde{M}^{R^T} - \varepsilon_4 \tilde{M}^R \tilde{M}^{R^T},$$
$$\Pi_{12,12} = \varepsilon_2 \tilde{M}^I \tilde{M}^{I^T} - \varepsilon_3 \tilde{M}^I \tilde{M}^{I^T}, \quad e(t) = [e^R(t), e^I(t)]^T, \quad \mu(t) = [\mu^R(t), \mu^I(t)]^T.$$

Then for a scaling matrix βI_n, the neutral-type CVNNs (2) and (3) are projective synchronized in finite-time under the control inputs $\mu^R(t)$ and $\mu^I(t)$.

Proof. Construct the following Lyapunov function for the synchronization errors $e^R(t)$ and $e^I(t)$:

$$\begin{aligned} V(t) = {} & e^{R^T}(t) P_1 e^R(t) + e^{I^T}(t) P_2 e^I(t) + \int_{t-\tau(t)}^{t} e^{R^T(t)} S_1 e^R(s)\, ds + \int_{t-\tau(t)}^{t} e^{I^T(t)} S_2 e^I(s)\, ds \\ & + \int_{t-\tau(t)}^{t} (\tilde{g}^R(e^R(s)))^T Q_1 \tilde{g}^R(e^R(s))\, ds + \int_{t-\tau(t)}^{t} (\tilde{g}^I(e^I(s)))^T Q_2 \tilde{g}^I(e^I(s))\, ds \\ & + \int_{t-\sigma(t)}^{t} (\dot{e}^R(s))^T R_1 (\dot{e}^R(t))\, ds + \int_{t-\sigma(t)}^{t} (\dot{e}^I(s))^T R_2 (\dot{e}^I(t))\, ds. \end{aligned} \tag{7}$$

Calculating the derivative of $V(t)$ along the trajectories $e^R(t)$ and $e^I(t)$, we can obtain

$$\begin{aligned}\dot{V}(t) = {} & 2e^{R^T}(t)P_1\dot{e}^R(t) + 2e^{I^T}(t)P_2\dot{e}^I(t) + e^{R^T}(t)S_1e^R(t) - (1-\dot{\tau}(t))e^{R^T}(t-\tau(t))S_1e^R(t-\tau(t)) \\ & + e^{I^T}(t)S_2e^R(t) - (1-\dot{\tau}(t))e^{I^T}(t-\tau(t))S_2e^I(t-\tau(t)) + (\tilde{g}^R(e^R(t))^TQ_1\tilde{g}^R(e^R(t)) \\ & - (1-\dot{\tau}(t))(\tilde{g}^R(e^R(t-\tau(t)))^TQ_1\tilde{g}^R(e^R(t-\tau(t))) + (\tilde{g}^I(e^I(t))^TQ_2\tilde{g}^I(e^I(t)) \\ & - (1-\dot{\tau(t)})(\tilde{g}^I(e^I(t-\tau(t)))^TQ_1\tilde{g}^I(e^I(t-\tau(t))) + (\dot{e}^R(t))^TR_1(\dot{e}^R(t)) \\ & - (1-\dot{\sigma}(t))(\dot{e}^R(t-\sigma(t)))^TR_1(\dot{e}^R(t-\sigma(t))) + (\dot{e}^I(t))^TR_2(\dot{e}^I(t)) \\ & - (1-\dot{\sigma}(t))(\dot{e}^I(t-\sigma(t)))^TR_2(\dot{e}^I(t-\sigma(t))). \end{aligned} \tag{8}$$

From Assumption 2, there exist positive diagonal matrices λ_p $(p = 1, 2, \ldots, 6)$ such that

$$\begin{aligned} 0 &\leqslant \begin{pmatrix} e^R(t) \\ \tilde{f}^R(e^R(t)) \end{pmatrix}^T \begin{pmatrix} -\lambda_1G_1 & \lambda_1G_2 \\ * & -\lambda_1 \end{pmatrix} \begin{pmatrix} e^R(t) \\ \tilde{f}^R(e^R(t)) \end{pmatrix}, \\ 0 &\leqslant \begin{pmatrix} e^I(t) \\ \tilde{f}^I(e^I(t)) \end{pmatrix}^T \begin{pmatrix} -\lambda_2G_3 & \lambda_2G_4 \\ * & -\lambda_2 \end{pmatrix} \begin{pmatrix} e^I(t) \\ \tilde{f}^I(e^I(t)) \end{pmatrix}, \\ 0 &\leqslant \begin{pmatrix} e^R(t) \\ g^R(e^R(t)) \end{pmatrix}^T \begin{pmatrix} -\lambda_3G_5 & \lambda_3G_6 \\ * & -\lambda_3 \end{pmatrix} \begin{pmatrix} e^R(t) \\ g^R(e^R(t)) \end{pmatrix}, \\ 0 &\leqslant \begin{pmatrix} e^I(t) \\ g^I(e^I(t)) \end{pmatrix}^T \begin{pmatrix} -\lambda_4G_7 & \lambda_4G_8 \\ * & -\lambda_4 \end{pmatrix} \begin{pmatrix} e^I(t) \\ g^I(e^I(t)) \end{pmatrix}, \\ 0 &\leqslant \begin{pmatrix} e^R(t-\tau(t)) \\ \tilde{g}^R(e^R(t-\tau(t))) \end{pmatrix}^T \begin{pmatrix} -\lambda_5G_5 & \lambda_5G_6 \\ * & -\lambda_5 \end{pmatrix} \begin{pmatrix} e^R(t-\tau(t)) \\ \tilde{g}^R(e^R(t-\tau(t))) \end{pmatrix}, \\ 0 &\leqslant \begin{pmatrix} e^I(t-\tau(t)) \\ \tilde{g}^I(e^I(t-\tau(t))) \end{pmatrix}^T \begin{pmatrix} -\lambda_6G_7 & \lambda_6G_8 \\ * & -\lambda_6 \end{pmatrix} \begin{pmatrix} e^I(t-\tau(t)) \\ \tilde{g}^I(e^I(t-\tau(t))) \end{pmatrix}. \end{aligned} \tag{9}$$

In view of Lemma 1, the following inequalities hold,

$$\begin{aligned} 2e^{R^T}(t)P_1\tilde{M}^R\dot{e}(t-\sigma) &\leqslant \varepsilon_1^{-1}e^{R^T}P_1P_1^Te^R(t) + \varepsilon_1\dot{e}^{R^T}(t-\sigma(t))\tilde{M}^R\tilde{M}^{R^T}\dot{e}^R(t-\sigma), \\ 2e^{I^T}(t)P_2\tilde{M}^I\dot{e}^I(t-\sigma) &\leqslant \varepsilon_2^{-1}e^{I^T}P_2P_2^Te^I(t) + \varepsilon_2\dot{e}^{I^T}(t-\sigma(t))\tilde{M}^I\tilde{M}^{I^T}\dot{e}^I(t-\sigma), \\ -2e^{R^T}(t)P_1\tilde{M}^I\dot{e}^I(t-\sigma) &\leqslant -\varepsilon_3^{-1}e^{R^T}P_1P_1^Te^R(t) - \varepsilon_3\dot{e}^{I^T}(t-\sigma(t))\tilde{M}^I\tilde{M}^{I^T}\dot{e}^I(t-\sigma), \\ -2e^{I^T}(t)P_2\tilde{M}^R\dot{e}^R(t-\sigma) &\leqslant -\varepsilon_4^{-1}e^{I^T}(t)P_2P_2^Te^I(t) - \varepsilon_4\dot{e}^{R^T}(t-\sigma(t))\tilde{M}^R\tilde{M}^{R^T}\dot{e}^R(t-\sigma). \end{aligned} \tag{10}$$

From (8)–(10), we get

$$\dot{V}(t) \leqslant \Xi^T\Omega\,\Xi, \tag{11}$$

where Ω is given in (6) and $\Xi^T = [e^{R^T}(t)e^{I^T}(t)e^{R^T}(t-\tau(t))e^{I^T}(t-\tau(t))f^{R^T}(e^R(t))f^{I^T}(e^I(t))g^{R^T}(e^R(t))$ $g^{I^T}(e^I(t))\,g^{R^T}(e^R(t-\tau(t)))\,g^{I^T}(e^I(t-\tau(t)))\,\dot{e}^{R^T}(t-\sigma(t))\,\dot{e}^{I^T}(t-\sigma(t))]^T$. Therefore substituting the condition (6) into (11), one can obtain $\dot{V}(t) \leqslant 0$, combine with $V(t)$ that is the positive definite, therefore it has constant $V^* > 0$ and $t_1 \in (0, +\infty)$ such that $\lim_{t\to t_1} V(t) = V^*$ and $V(t) \equiv V^*$, for all $t \geqslant t_1$. Then we illustrate that $\|e(t_1)\|_1 = 0$ and $\|e(t)\|_1 \equiv 0$ for $t \geqslant t_1$. First, we prove $\|e(t_1)\|_1 = 0$. Otherwise $\|e(t_1)\|_1 > 0$, there exists a small positive number θ such that $\|e(t_1)\|_1 > 0$, $\forall t \in [t_1, t_1+\theta]$, that is $\left|e^R(t)\right| > 0$ or $\left|e^I(t)\right| > 0$ for $t \in [t_1, t_1+\theta]$, which gives that $\dot{V}(t) < 0$ holding $\forall t \in [t_1, t_1+\varepsilon]$, which contradicts to $\lim_{t\to t_1} V(t) = V^*$ and $V(t) \equiv V^*$, $\forall t \geqslant t_1$. Therefore, $\|e(t_1)\|_1 = 0$. Next, we prove $\|e(t)\|_1 \equiv 0$ for $t \geqslant t_1$. In the contradiction, without loss of generality, if there exists $t_2 > t_1$ such that $\left|e^R(t_2)\right|_1 > 0$ or $\left|e^I(t_2)\right|_1 > 0$.

Let $t_s = \sup\{t \in [t_1, t_2] \colon \|e(t)\|_1 = 0\}$, from this we obtain that $t_s < t_2$, $\|e(t_s)\|_1 = 0$ and $\left|e^R(t)\right|_1 > 0$ or $\left|e^I(t)\right|_1 > 0$ for all $t \in (t_s, t_2]$. In addition, there exists $t_3 \in (t_s, t_2]$ such that $\left|e^R(t)\right|$ or $\left|e^I(t)\right|$ is monotonically non decreasing to the interval $[t_s, t_3]$; therefore, $V(t)$ is also monotonically non decreasing to $[t_s, t_3]$, i.e., $\dot{V}(t) > 0$ for $t \in (t_s, t_3]$. According to the discussion in previous part, we have $\dot{V}(t) \leqslant -\delta_{k_0} < 0$ holds for all $t \in [t_s, t_3]$, which gives a contradiction. This shows, $\|e(t)\|_1 \equiv 0$ for $t \geqslant t_1$.

Therefore, $\|e(t_1)\|_1 = 0$ and $\|e(t)\|_1 \equiv 0$ for $t \geqslant t_1$ hold. From Definition 1, the neutral-type CVNNs (3) is projective synchronized with CVNNs (2) in finite time under the control inputs (4) and (5). The proof is completed. ■

Remark 2. Under the Assumptions 1 and 2 the activation function can be separated into real and imaginary part. For neutral type CVNNs with time-varying delay, Theorem 1 provides some requirements to ensure finite time projective synchronization. When the real and imaginary parts of an activation function can not be distinguished, it is considered invalid. Following that, we will discuss the results for such cases in future.

Remark 3. From the proof of Theorem 1, by introducing the state derivative with neutral delay $\sigma(t)$ named as neutral term in Lyapunov functional produces an effective solution and plays a significant role to directly analyzing the finite time projective synchronization of the neutral type CVNNs.

Remark 4. To the best of authors knowledge, the projective synchronization behaviors of master-slave system does not affected by the neutral delay term $\dot{w}(t-\sigma(t))$.

Remark 5. The synchronization problem of delayed NNs has been a popular issue in recent years, with rapid growth due to its potential uses in biological systems (eg. Central Pattern Generation (CPG) models), chemical processes, secure communications and information processing. Up to date, there are many important synchronization results focusing mainly on some well-known models such as dynamical models, biological systems, etc. In biological systems the CPG is a biological notion that describes brain networks that create rhythmic motions for both invertebrate and vertebrate animals, such as chewing, digesting, walking, swallowing, and breathing [29]. Because the periodic oscillation of CPGs is dependent on internal information inside neurons as well as connectivity information between neurons, cyclic motions can be created with just extremely basic and non-rhythmic input signals, or even without sensory inputs [30]. On the other hand, the cluster synchronization, a more feasible collective behavior than complete synchronization, is thought to be important in biological science [31] and communication engineering [32]. Typically, if all of the neurons in NNs are separated into numerous clusters, and nodes within the same cluster may achieve complete synchronization, there is no consistent behavior between various clusters. In addition, the Hindmarsh and Rose's (HR) [33, 34] model is a common low-dimensional neuron model capable of recreating numerous qualitative spiking neuron behaviors. It may be divided into a slow fast system that exhibits regular or chaotic bursting depending on the system characteristics. In particular, the projective synchronization, through which the drive and response system can be synced with a factor of scaling. The projective synchronization may be utilized to expand binary digital to M-nary digital communication in order to achieve rapid communication in applications involving secure communications, so the research on projective synchronization has both theoretical and practical importance.

4. Numerical Example

In this simulation section, we will propose two numerical examples to illustrate the effectiveness of the theoretical results derived in the previous section.

Example 1. Consider the following neutral-type CVNNs with time-varying delays model as the master system:

$$\frac{dw(t)}{dt} = -Dw(t) + Mf(w(t)) + Ng(w(t-\tau(t))) + \tilde{M}(\dot{w}(t-\sigma(t))) + U \tag{12}$$

and the corresponding slave system is

$$\frac{d\hat{w}(t)}{dt} = -D\hat{w}(t) + Mf(\hat{w}(t)) + Ng(\hat{w}(t-\tau(t))) + \tilde{M}(\dot{\hat{w}}(t-\sigma(t))) + U + \mu, \tag{13}$$

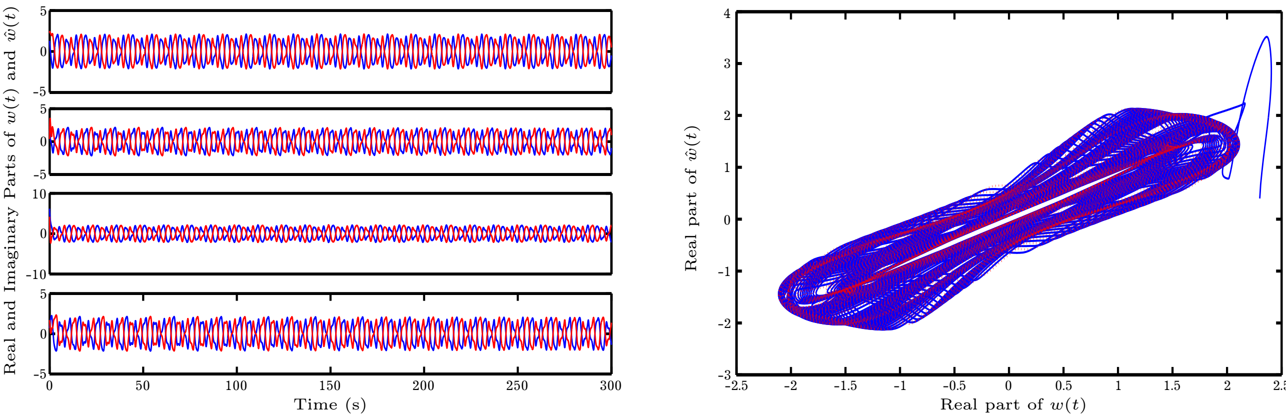

Fig. 1. The state trajectories and phase portrait position of master and slave systems (2) and (3) for scaling factor $\beta = -1$.

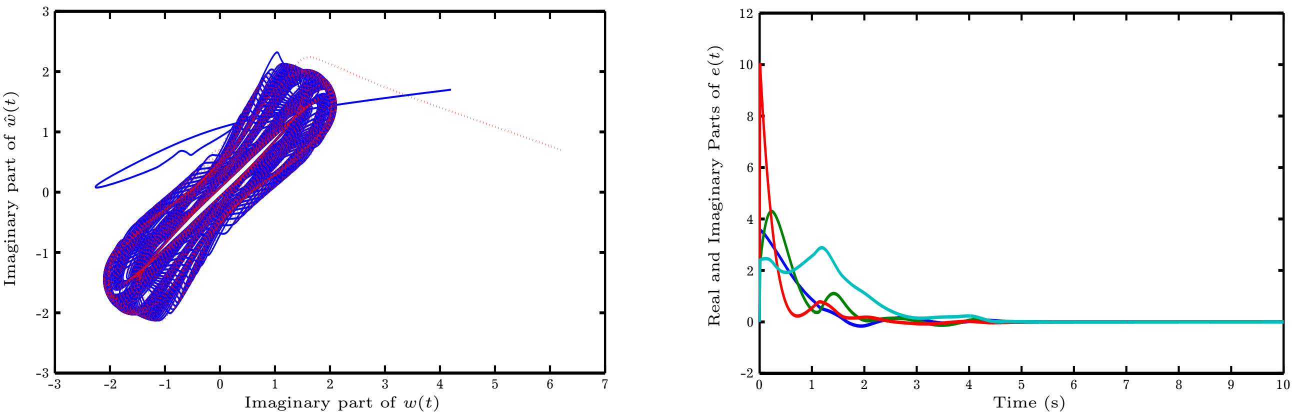

Fig. 2. Phase portrait position and error state trajectories of master and slave systems (2) and (3) for scaling factor $\beta = -1$.

where,

$$M = \begin{pmatrix} 2.4 - 0.7i & 0.6 + 1.5i \\ 3.2 - 2.3i & -1.3 + 0.5i \end{pmatrix}, \qquad N = \begin{pmatrix} -1.3 - 0.4i & -0.5 + 1.8i \\ 0.2 + 1.5i & 3.2 + 1.7i \end{pmatrix},$$

$$\tilde{M} = \begin{pmatrix} 0.3 + 0.3i & 0.6 + 0.8i \\ 2.4 + 1.4i & 3.4 + 0.3i \end{pmatrix}, \qquad D = \operatorname{diag}(1, 1), \quad U = (u_1, u_2)^T$$

$\mu(t) = \mu^R(t) + i\mu^I(t)$ is control input and $\mu^R(t)$, $\mu^I(t)$ are defined in (4) and (5), $f(w_k(t)) = \tanh(w_k(t))$, $g(w_k(t - \tau(t))) = \tanh(w_k(t - \tau(t)))$, $u_k = 0 + 0i$, the time-varying delay $\tau(t) = 0.3 + 0.2\cos^2 t$.

The activation functions $\tanh(w_k(t))$, $\tanh(w_k(t - \tau(t)))$ satisfies the Assumptions 1 and 2 with $\hat{\xi}_k^- = \check{\xi}_k^- = \hat{\zeta}_k^- = \check{\zeta}_k^- = 0$, $\hat{\xi}_k^+ = \check{\xi}_k^+ = \hat{\zeta}_k^+ = \check{\zeta}_k^+ = 1$, $k = 1, 2$. By numerical computations, the feasible solutions of the LMI (6) can be given as follows:

$$P = \begin{pmatrix} 0.0657 + 0.0000i & -0.0335 + 0.0344i \\ -0.0335 + 0.0344i & 0.0474 + 0.0000i \end{pmatrix}, \quad S = \begin{pmatrix} 1.2330 - 0.0000i & -0.0001 + 0.0001i \\ -0.0001 - 0.0001i & 1.2324 + 0.0000i \end{pmatrix},$$

$$R = \begin{pmatrix} -1.0282 + 0.0000i & 0.0044 - 0.0055i \\ 0.0044 + 0.0055i & -0.9997 + 0.0000i \end{pmatrix}, \quad Q = \begin{pmatrix} 0.3829 + 0.0000i & 0.0015 - 0.0011i \\ 0.0015 + 0.0011 & 0.3763 + 0.0000i \end{pmatrix}$$

$$W_1 = \begin{pmatrix} -0.0164 + 0.0000i & 0.0031 + 0.0039i \\ 0.0031 - 0.0039i & -0.0059 + 0.0000i \end{pmatrix}, \qquad \lambda_1 = \lambda_2 = \begin{pmatrix} 0.6507 & 0 \\ 0 & 0.6616 \end{pmatrix},$$

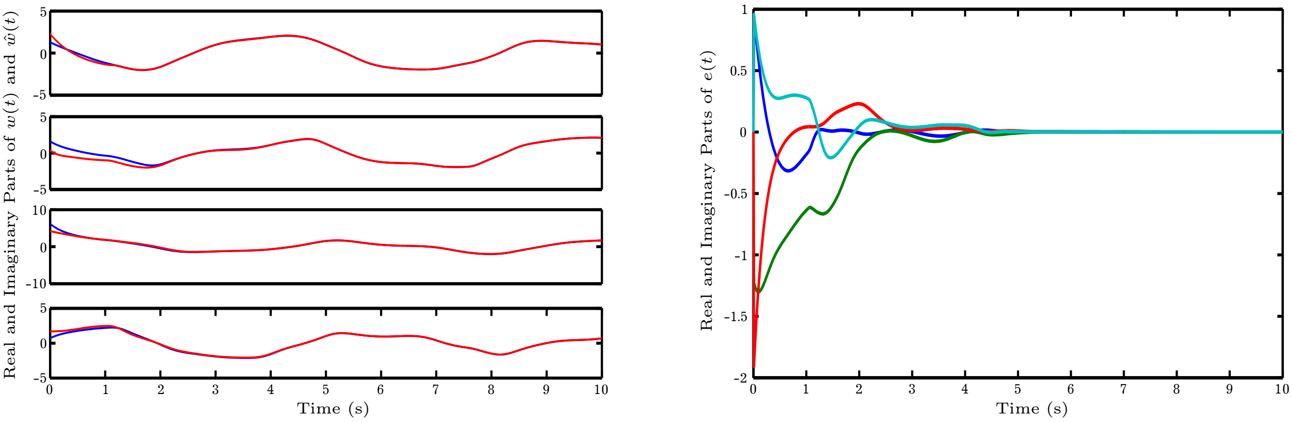

Fig. 3. The state trajectories and error state trajectories of master and slave systems (2) and (3) for scaling factor $\beta = 1$.

$$\lambda_3 = \lambda_4 = \begin{pmatrix} 0.8830 & 0 \\ 0 & 0.8828 \end{pmatrix}, \qquad \lambda_5 = \lambda_6 = \begin{pmatrix} 0.8830 & 0 \\ 0 & 0.8828 \end{pmatrix},$$

where $P = P_1 + P_2 i$, $Q = Q_1 + Q_2 i$, $R = R_1 + R_2 i$, $S = S_1 + S_2 i$. Therefore, in view of Theorem 1, the neutral-type CVNNs (13) is projective synchronized in finite-time with CVNNs (12) under the control input $\mu(t)$.

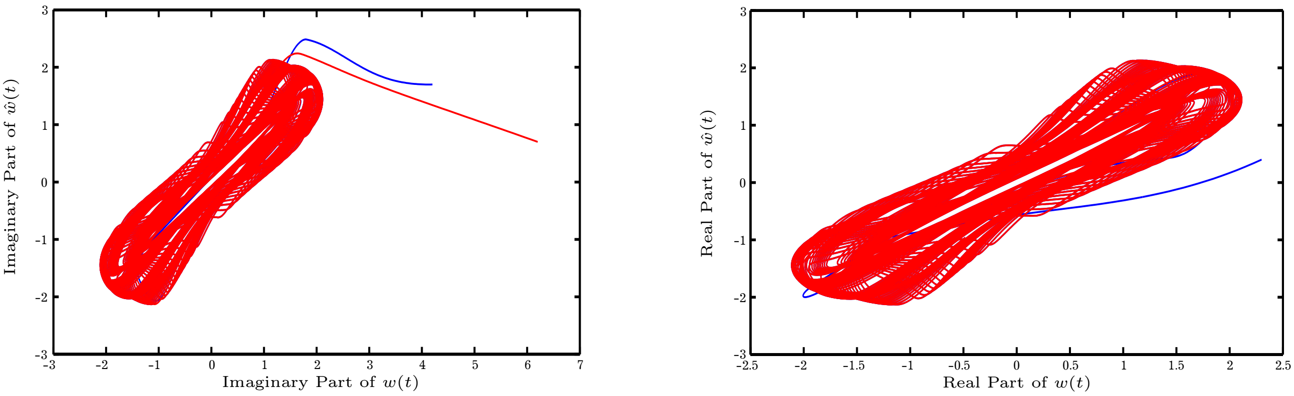

Fig. 4. Phase portrait position of master and slave systems (2) and (3) for scaling factor $\beta = 1$.

Example 2. In this example, we consider the neutral-type CVNNs (12) as master system and (13) as slave system with time-varying delays and without neutral delay model has the following complex parameters:

$$M = \begin{pmatrix} 1.2 + 0.7i & 2.6 + 1.5i \\ 2.2 - 1.3i & -1.3 + i0.5 \end{pmatrix}, \quad N = \begin{pmatrix} 1.3 + 0.4i & -1.5 + i0.8 \\ 0.3 + 2.5i & 2.2 + 1.7i \end{pmatrix}, \quad D = \operatorname{diag}(3, 3)$$

$\tau(t) = 2.3 + 1.3\sin^2(t)$, $f(w_k(t)) = \tanh(w_k(t))$, $g(w_k(t - \tau(t))) = \tanh(w_k(t - \tau(t)))$. The activation functions defined in the network is satisfy the Assumptions 1 and 2 with $\hat{\xi}_k^- = \check{\xi}_k^- = \hat{\zeta}_k^- = \check{\zeta}_k^- = 0$, $\hat{\xi}_k^+ = \check{\xi}_k^+ = \hat{\zeta}_k^+ = \check{\zeta}_k^+ = 1$, $k = 1, 2$. The solutions for LMI (6) without neutral term can be derived using the Matlab software as follows:

$$P = \begin{pmatrix} 0.0465 + i0.0000 & -0.0265 - i0.0123 \\ -0.0265 + i0.0123 & 0.0227 + i0.0000 \end{pmatrix}, \quad Q = \begin{pmatrix} 1.0587 + i0.0000 & -0.0001 - i0.0001 \\ -0.0001 + i0.0001 & 1.0591 + i0.0000 \end{pmatrix},$$

$$\lambda_1 = \begin{pmatrix} 0.5779 & 0 \\ 0 & 0.5750 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0.4984 & 0 \\ 0 & 0.4990 \end{pmatrix}.$$

In the following various kinds of projective scaling factors are chosen to illustrate the various types of synchronization. If the projection scaling factor is chosen as $\beta = -1$, then the projective synchronization of (12) and (13) is equivalent to the anti-synchronization of (12) and (13). With the help of Theorem 1, we can obtain that the neutral-type CVNNs (13) can be anti synchronized in finite time to the neutral-type CVNNs (12) with the controller $\mu(t)$. Figures 1 and 2 illustrate the dynamic behaviors of synchronization errors and state trajectories of (12) and (13) (where blue and red lines are referred as master and slave state trajectories respectively). As is shown, given random initial conditions in the neutral-type CVNNs, the projective synchronization errors approaches zero eventually, showing that CVNNs (12) and (13) achieve anti synchronization in finite time. If the projection scaling factor is chosen as $\beta = 1$, then the projective synchronization of (12) and (13) is equivalent to the complete synchronization of (12) and (13). With the help of Theorem 1, we can obtain that the neutral-type CVNNs (13) can be anti synchronized in finite time to the neutral-type CVNNs (12) with the controller $\mu(t)$. Figures 3 and 4 illustrate the dynamic behaviors of synchronization errors and state trajectories of (12) and (13). As is shown, given random initial conditions in the neutral-type CVNNs, the projective synchronization errors approaches zero eventually, showing that CVNNs (12) and (13) achieve complete synchronization in finite time. If the projection scaling factor is chosen as $\beta = 1.8$, then the projective synchronization of (12) and (13) is achieved. With the help of Theorem 1, we can obtain that the neutral-type CVNNs (13) can be projective synchronized in finite time to the neutral-type CVNNs (12) with the controller $\mu(t)$. Figures 5 and 6 illustrate the dynamic behaviors of synchronization errors and state trajectories of (12) and (13). As is shown, given random initial conditions in the neutral-type CVNNs, the projective synchronization errors approaches zero eventually, showing that CVNNs (12) and (13) is projective synchronized in finite time.

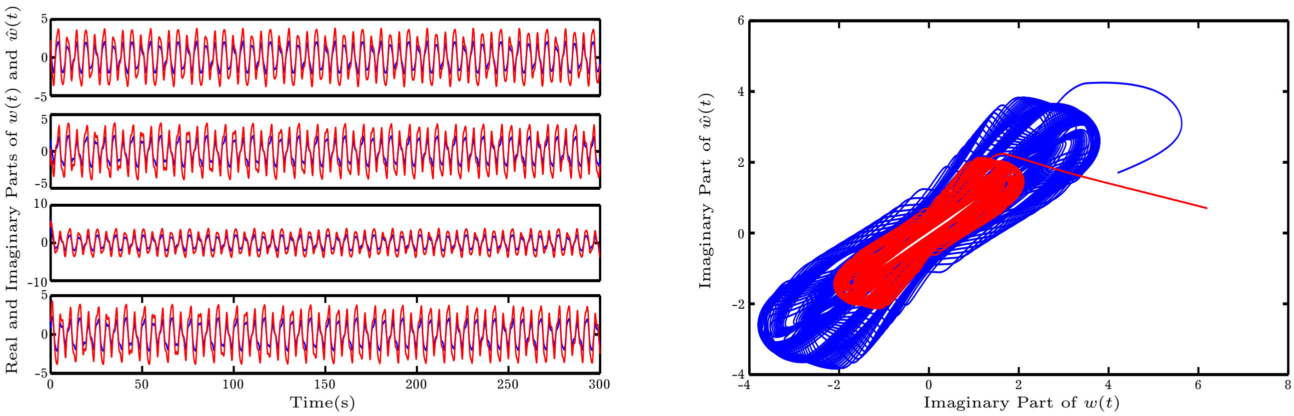

Fig. 5. The state trajectories and phase portrait position of master and slave systems (2) and (3) for scaling factor $\beta = 1.8$.

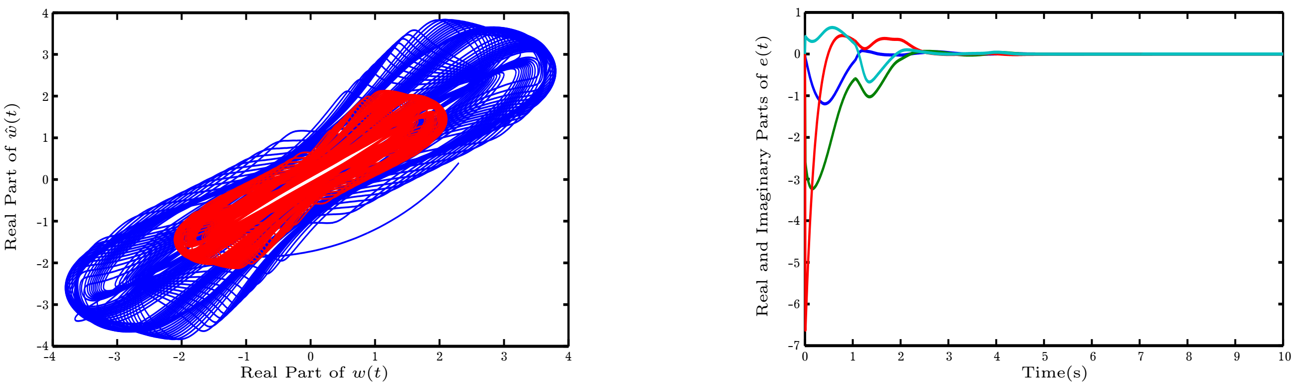

Fig. 6. Phase portrait position and error state trajectories of master and slave systems (2) and (3) for scaling factor $\beta = 1.8$.

5. Conclusion

In this article, the finite-time projective synchronization of neutral type CVNNs with time delays is investigated. By employing the real-imaginary complex separation method the addressed complex valued neural network is separated into its real and imaginary parts. A new sufficient condition is developed in terms of linear matrix inequality (LMI) for the considered neutral type CVNNs based on the Lyapunov stability theory and adaptive control strategy. Moreover, the MATLAB LMI toolbox is to verify the feasibility of the developed LMIs are less conservative than the previous literature's. Finally, two simulation results are given to illustrate the efficiency of our theoretical findings.

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Синхронізація нестаціонарних нейронних мереж нейтрального типу з часовою затримкою для обмеженого часу в складному полі

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У роботі розглядається проблема проективної синхронізації за скінченний час для класу комплексних нейронних мереж нейтрального типу (КНМН) зі змінними у часі затримками. Розроблено простий протокол керування зі зворотним зв'язком за станом так, що підпорядковані КНМН можуть бути проективно синхронізованими з головною системою за скінченний час. Застосовуючи техніку нерівностей та розробляючи нові функціонали Ляпунова–Красовського, отримано різні нові умови, які легко перевіряються, для забезпечення проективної синхронізації за скінченний час. Встановлено, що час усталення можна явно розрахувати для КНМН. Під кінець, продемонстровано два результати чисельного моделювання для підтвердження теоретичних результатів цієї статті.

Ключові слова: *нейронна мережа нейтрального типу, нейтральна затримка, синхронізація, комплексне поле, змінні з часом часові затримки.*