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Image restoration and contrast enhancement based on a nonlinear reaction-diffusion mathematical model and divide & conquer technique

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In this article, we present a new algorithm for digital image processing noised by mixed Gaussian-impulse noise. Our mathematical model is based on the divide and conquer technique coupled with a reaction-diffusion system. We first decompose our image into low and high-frequency components by convolving each with a predefined convolutional filter. Further, we use a simple scheme of different weights to integrate and collect these processed sub-images into a filtered image. Finally, we apply our Reaction-Diffusion system to increase the contrast in the image. A number of experimental results are described to illustrate the performance of our algorithm and show that it is very effective in eliminating mixed Gaussian-impulse noise, increasing the contrast of the image and preserving the edges.

Keywords: *reaction-diffusion, image processing, mixed Gaussian-impulse noise, divide and conquer technique.*

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1. Introduction

Image processing is a discipline of computer science and applied mathematics that deals with digital images and their transformations in order to improve their quality or to extract information. Image processing involves improving the content of an image, for example, it is used for detection, counting, and measuring in order to help automate the process in manufacturing and machine vision domains. In military and security, image processing and computer vision is used for detecting a target, tracking, recognizing objects, etc.

Moreover, image processing methods are applied to enhance pictures, clean, cut and combine in photography and movie industry. Furthermore, those the methods also find their applications in the analysis of biomedical images. Indeed, the diagnostic study of a disease is either to differentiate normal tissue from abnormal tissue or to identify different human organs.

The most important image processing task is image restoration which is the process of image recovering from a degraded version usually a blurred and noisy image. A different number of methods were proposed for image restoration. The first class of restoration method is focused on spatial domain, bilateral filtering [1] and nonlocal means [2], involving variational models [3–6]. The first pioneer work of the last category is the heat equation. This equation presents a major defect for analysis and image restoration: the contours are suppressed since each pixel is replaced by the average of its neighbourhood.

To preserve image structures (edges and textures), [7] derived a model that incorporates local information from an image within a partial differential equation framework. It has stimulated a great deal of interest by the image processing community. The idea focuses to introduce a nonlinear diffusion equation which is called often anisotropic. Thus, this model behaves like a heat equation in the areas of a low gradient, while diffusion is stopped and guarantee edges preservation in the regions of high gradient. Due to the lack of theoretical proof of this model that justifies the existence and stability of the solution, several improvements have been integrated into this model.

Among them, [8] has offered an interesting nonlinear form of restoration model with solving the Perona-Malik equation with a finite difference method. For the mathematical framework of Perona-Malik equation, [9] introduced a regularization in space and time directly into the continuous equation in order to obtain a well-posed model. In practice, the method do not distinguish the strong gradients that come from noise of those significant objects (high-frequency components). To avoid this confusion, a scale parameter is used for smoothing homogeneous areas. The existence, uniqueness and regularity of this model have been proven in [9]. This model behaves like an inverse heat equation and the presence of a term combining the gradient and its regularized version in the diffusion operator makes difficult the geometric interpretation of the model. For textured images that contain segments or partially hidden parts, this model acts as a linear filter on the one-dimensional objects; and the diffusion disappear from both sides of edges where there is no elimination of the noise.

Moreover, [10] modified Perona-Malik model by basing on Fisher equation. But, the model is sensitive to noise and no proof result of existence and consistency was demonstrated. A modification of this model was proposed in the work of [11] by considering regularization of image gradient and choosing a time variable threshold parameter calculated at each iteration. All the mentioned models used the local or nonlocal similarity of the image in spatial domain in order to reconstruct an ideal one. However, complex degradation problems are not easy to be manipulated without frequency or other transform domains information. Concerning the second category, image restoration method was based on transform domains such as wavelet and curvelet transform. It gives satisfactory results, but it takes a lot of time of calculation as well it uses complex mathematical theories. The last class of restoration methods was based on dictionary learning to represent an image in adaptive and sparse ways. The restored image has a good quality than that of traditional transform domain methods. However, this class is very complex to compute numerically due to a large amount of time calculation.

It should be noted that the restoration process is different from image enhancement. Indeed, the latter makes it possible to highlight some characteristics of the image, which makes it possible to make the image sharper but not necessarily to reproduce or find lost data. Image enhancement techniques provided by imaging packages use no a priori model of the process that created the image. One of the most popular methods in contrast enhancement images is that of histogram equalization (HE). This is a statistical method based on the histogram of pixel intensity levels. The shape of this method gives an idea of the overall perceptual quality of the image. Indeed, a histogram focused on low values corresponds to dark images. Other approaches based on multi-scale signal analysis have been proposed for contrast enhancement. These approaches are based on wavelet decomposition [12].

The principle of these approaches is to apply a linear or non-linear representation function to the different coefficients resulting from wavelet transform decomposition. However, there are different wavelets of many properties that have given rise to several discrete wavelet transforms and a lot of contrast enhancement algorithms for mammography images. Moreover, two approaches have been introduced by [13]: the first is a global enhancement method using the alpha-trimmed medium filter given as a technique to refine medical images. However, the second method is local and involves using a cascade of fuzzy masks to select the high frequencies of the image so that they can be enhanced using the modified adaptive contrast enhancement algorithm.

2. Mixed Gaussian-impulse noised image restoration

In recent years, reaction-diffusion systems have been widely applied in image processing. The first application was pattern formation which is presented by the works of [14] who introduces a model for spatial patterns. This model arise as a result of instability morphogenetic chemicals diffusion in animal skins. Each pattern is considered as the product of certain activator and inhibitor in the form of nonlinear partial differential equations.

A typical example of the functions controlling production rate of two chemical substances are proposed by [15] and [16] models. In addition, [17] applied Fitzhugh-Nagumo system to pattern formation; this model simulates the temporal response of a nerve axon to a stimulus. This response is described by a pair of time-evolving ordinary differential equations with an activator $u(t)$ and an inhibitor $v(t)$ variables, as follows:

$$\begin{cases} \dfrac{du}{dt} = \dfrac{1}{\tau}(u(u-a)(1-u)-v), \\ \dfrac{dv}{dt} = u - bv, \\ u(0) = u_0, \quad v(0) = v_0, \end{cases} \tag{1}$$

where τ is a small positive constant, a represents the threshold value and b is a positive constant.

The set of ordinary differential equations defined Fitzhugh–Nagumo model has two different types of system behaviour: mono-stable and bi-stable one, which depends on the parameter values of a and b. The mono-stable system is obtained when $a = 0.25$ and $b = 1$ and has one steady point $(0,0)$. In this case, the solution (u,v) starts from an initial data (u_0,v_0) on a phase plot and returns to the stable point $(0,0)$. In the other side, the system becomes bi-stable when $a = 0.25$ and $b = 10$ and it possess two stable points. When the initial point (u_0,v_0) is over the threshold $(u > a, v = 0)$, the solution (u,v) converges to the steady point in this set while if the initial point is in the threshold $(u < a, v = 0)$, (u,v) goes to the other stable point.

The idea to use the FitzHugh–Nagumo reaction-diffusion system for gray scale image edge detection was suggested by [18] wherein a new system with a simple diffusion equation of a is proposed:

$$\begin{cases} \dfrac{\partial u}{\partial t} - d_u \Delta u = \dfrac{1}{\tau}(u(u-a)(1-u)-v), \\ \dfrac{\partial v}{\partial t} - d_v \Delta v = u - bv, \\ \dfrac{\partial a}{\partial t} - d_a \Delta a = 0, \\ u(0) = a(0) = a_0 I, \quad v(0) = 0. \end{cases} \tag{2}$$

The two diffusion coefficients d_u and d_v need to satisfy $d_u \ll d_v$ for static edge patterns; the rest one d_a needs to be enough large for the computation of local average level. The image brightness distribution function I is normalized.

This algorithm is effective for edge detection of clear images, but it produces false edges for noisy images. To solve this problem, [6] introduced a nonlinear anisotropic diffusion system for removing noise, edge preservation and contrast enhancement.

$$\begin{cases} \dfrac{\partial u}{\partial t} - \operatorname{div}(g(|\nabla(G_\sigma * u)|, \lambda(t))\nabla u) = \dfrac{1}{\tau}(u(u-a)(1-u)-v), \\ \dfrac{\partial v}{\partial t} - d_v \Delta v = u - bv, \\ u(0) = u_0, \quad v(0) = v_0, \\ \dfrac{\partial u}{\partial n} = 0, \quad \dfrac{\partial v}{\partial n} = 0, \end{cases}$$

where G_σ is the Gaussian kernel of variance $\sigma > 0$ defined as:

$$G_\sigma(x) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{|x|^2}{2\sigma^2}\right), \quad x \in \mathbb{R}^2$$

and $\nabla(G_\sigma * u)$ is the regularized version of image gradient. The diffusivity g is a decreasing and nonnegative function that satisfies

$$g(0, \lambda(t)) = 1 \text{ and } \lim_{s \to \infty} g(s, \lambda(t)) = 0 \tag{3}$$

with $|\nabla u| = \sqrt{\sum_{i=1}^{2}\left(\frac{\partial u}{\partial x_i}\right)^2}$ is the Euclidean norm of gradient. [19] suggested a regularized version of g given as:

$$g(s, \lambda(t)) = \frac{d}{\sqrt{1 + \left(\frac{s}{\lambda(t)}\right)^2}}, \tag{4}$$

where d is positive constant and the parameter $\lambda(t)$ is chosen as:

$$\lambda(t) = 1.4826\,\mathrm{MAD}(|\nabla u(t)|)/\sqrt{2} \tag{5}$$

MAD denotes the median absolute deviation that can be computed by:

$$\mathrm{MAD}(|\nabla u|) = \mathrm{median}(|\nabla u - \mathrm{median}(|\nabla u|)|),$$

where median$(|\nabla u|)$ represents median value over the image u of gradient norm.

Precisely, the diffusivity g depends on two parameters; the first one is the regularized gradient magnitude of the image u; which plays an important role in noise removal due to the Gaussian smoothing. Indeed, in the areas where $|\nabla(G_\sigma * u)| \ll \lambda(t)$, the value of g is close to d_u and consequently these regions will be well smoothed and therefore the noise is removed. Inversely, the diffusion function g is close to zero in the image parts that are characterized by $|\nabla(G_\sigma * u)| \gg \lambda(t)$; which means that the diffusion process will be stopped and therefore these parts are kept.

In addition, the diffusivity g depends also on the parameter $\lambda(t)$ which is calculated on each iteration of the diffusion process because ∇u norm becomes small. When the norm of $\nabla(G_\sigma * u)$ decreases, the parameter $\lambda(t)$ decrease since edges are preserved in the regions where $|\nabla(G_\sigma * u)|$ is superior than $\lambda(t)$.

Several experiments results shows the efficiency and performance of the algorithm in noise elimination, edge preservation and contrast enhancement for Gaussian noisy image. Unfortunately, this model presents a problem when we consider Gaussian-salt and pepper noise to image; the restored image gives an unsatisfactory results since the noise is still kept in the image. This later introduces very large oscillations of image gradient and therefore the model will stop the diffusion as in Perona–Malik approach.

In most of the previous works, noise is assumed to be Gaussian distribution. Nevertheless, the noise model is more complicated and difficult to practice in many applications due to various imaging environments. Therefore, noise distribution in images differs frequently from a single Gaussian type and that may affects on the result performance of image denoising. Besides this type of noise, there are many methods for image restoration that aims to remove salt and pepper noise. Median filter is considered as the most popular and classical method developed in impulse noise removal. This filter replaces each pixel by the median value of its neighbourhood and the L^1- based fidelity term is appropriated in many denoising variational methods. However, the median filter differs from the mean filters for Gaussian noise denoising and destroys image details, such as edges and textures.

In general, images corrupted by different types of noise (with several means and variances), namely mixed noise, are not easy to be reconstructed because the noise levels of each pixel are dissimilar and there is no standard fidelity term which can be used to handle a mixed noise restoration models. Thus, all the above single type noise removal model are not adapted to restore mixed noise images.

To deal with Gaussian-impulse noise image restoration models, [20] proposed a kernel estimation method focused on Bayesian classification of the input pixel in order to eliminate Gaussian-impulse noise. [21] introduced an adaptive technique for recovering image from mixed noisy data. Moreover, a simple energy functional of total variation regularization term and L^2 and L^1 data fidelity terms, was suggested by [22] for Gaussian and impulse noise removing. In addition, [23] derived a model from the regularized maximum likelihood estimation of the noise and sparse representations over a trained dictionary.

In this paper, we propose a modification of the usual discretization of laplacian operator in classical FitzHugh–Nagumo system by using a divide and conquer technique. Precisely, we divide the mixed Gaussian-salt and pepper noisy image into four sub-images by using some linear filters in order to extract both of smooth areas and texture information from the image. Then, we collect these sub-images by multiplying each one by a different balancing weight. Finally, we apply Fitzhugh–Nagumo system so as to obtain the restored and enhanced image.

3. Proposed framework

Divide-and-conquer is an important algorithm technique which has been used in numerous problems, and has stimulated a great deal of interest by image processing community. In particular, divide and conquer algorithm was applied to achieve parallel computers [24], cellular automata [25], image enhancement [26].

Previous literatures on the decomposition models [4,5] shown the difference between low (cartoon) and high-frequency (texture) components of an image; the smoothing areas represent low-frequency portions, while edges and details are contained in high-frequency parts. Afterwards, each component will be implemented separately. To this end, we decompose an observed image U into different low and high-frequency by convolving u with a predefined convolutional filter h_i:

$$u_i = u \otimes h_i \quad \text{for} \quad i = 1, 2, 3, 4, \tag{6}$$

where $h_3 = [1, -1]$ and $h_4 = [1; -1]$ are two high frequency filters and the symbol $\otimes$ is the two-dimensional convolution operator. The sub-images u_3 and u_4 contain the high-frequency component of the entire image u whereas u_1 and u_2 forms regions with low frequency. Then, we define frequency responses H_1 and H_2 of low-frequency filters h_1 and h_2 as follow:

$$H_1 = I - H_3, \quad H_2 = I - H_4, \tag{7}$$

where H_3 and H_4 are respectively the Fourier transform of h_3 and h_4. I is a matrix where each element of it equals to one. Once the decomposition process of the image U is obtained, we use an integration scheme of different weights to collect these sub-images, given as:

$$\Delta u = \sum_{i=1}^{4} w_i u_i, \tag{8}$$

where $\{w_i\}_{i=1}^4$ are the weights of balancing different sub-images. The nonlinearity of the first equation in the proposed model allows to enhance image contrast.

The overview of proposed framework for image restoration and contrast enhancement is mentioned in the following Algorithm.

Algorithm 1 Outline of our proposed Reaction diffusion and divide and conquer framework for image restoration and enhancement

Require: Observed image U, filters h_i, balancing weights w_1, w_2, w_3 and w_4
Ensure: restored and contrasted image ;
1: Generate observed sub-images $\{u_i\}_{i=1}^4$ via (6);
2: Collect these sub-images via Eq. (8) ;
3: Compute the system (2) by replacing the Laplacian operator by the formula (8) ;

4. Numerical results

In this section, different experimental results are presented to show the effectiveness and robustness of the proposed divide and conquer approach on mixed Gaussian and salt and pepper noise removal and contrast enhancement. As for this part, we assume that the noise model is followed by mixed distribution with Gaussian noise and salt and pepper (impulse) noise. To compare the proposed algorithm with that of classical Fitzhugh-Nagumo for noise removal and contrast enhancement, we set the parameters of the nonlinearity a to 0.5 to keep the symmetry and b to 1, dt to 0.001, τ to 0.001, $d_u = 150$ and $d_v = 250$.

The finite difference scheme of the classical Fitzhugh–Nagumo is given as follows:

$$\frac{u_{i,j}^{n+1} - u_{i,j}^n}{dt} - d_u \Delta u_{i,j}^n = \frac{1}{\tau}\left(u_{i,j}^n(u_{i,j}^n - a)(1 - u_{i,j}^n) - v_{i,j}^n\right),$$

$$\frac{v_{i,j}^{n+1} - v_{i,j}^n}{dt} - d_v \Delta v_{i,j}^n = u_{i,j}^n - b v_{i,j}^n, \tag{9}$$

where

$$\Delta u_{i,j}^n = \frac{\partial^2 u_{i,j}^n}{\partial x^2} + \frac{\partial^2 u_{i,j}^n}{\partial y^2} = (u^n \otimes D)(i,j)$$

and the kernel D is defined as:

$$D = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \text{or} \qquad D = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

For the sake of brevity, we will denote it by the classical FN model. To measure enhancement level in the image, we use a technique called measure of enhancement EME or measure of improvement to compare the image enhancement. Let an image $u(N, M)$ be split into $k_1 k_2$ blocks $w_{k,l}(i,j)$ of sizes $l_1 l_2$ then we define

$$\mathrm{EME} = \frac{1}{k_1 k_2} \sum_{l=1}^{k_1} \sum_{k=1}^{k_2} 20 \log \left(\frac{u^w_{\max;k,l}}{u^w_{\min;k,l}} \right),$$

where $u^w_{\max;k,l}$ and $u^w_{\min;k,l}$ are respectively maximum and minimum values of the image $u(N, M)$ inside the block $w_{k,l}$. A higher value of EME indicates that the image is enhanced very well.

5. Results and discussion

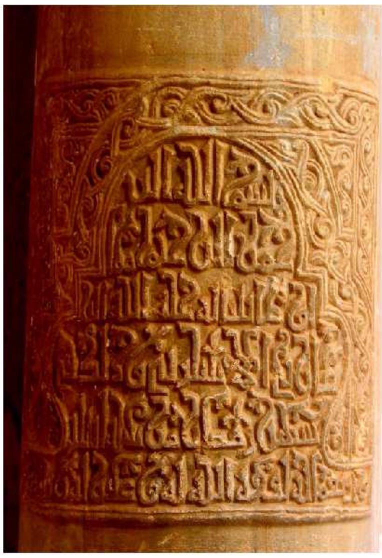

Fig. 1. Mixed Gaussian and salt-and-pepper noisy image.

In this section, we consider different test images that are corrupted by mixed Gaussian and salt-and-pepper noise (see Figures 1, 4 and 7). Meanwhile, the denoising results of different methods on these images are illustrated in Figures 2, 5 and 8. In order to evaluate the result of restoration, we take firstly a zoom on both restored images obtained by the two models. Then, we represent line profile of each recovered data so as to show which model enhances very well image contrast and preserves its details. Later, a texture component is used to distinguish the efficient model in noise removal without elimination and blurring of edges and structures.

Fig. 2. Restored image, left: classical Fitzhugh–Nagumo model, right: Proposed model.

Fig. 3. Zoom on the restored image, left: classical Fitzhugh-Nagumo model, right: Proposed model.

According to this result, we show that details of the restored image obtained by classical Fitzhugh–Nagumo model are blurred due to the regularization effect of the laplacian operator.

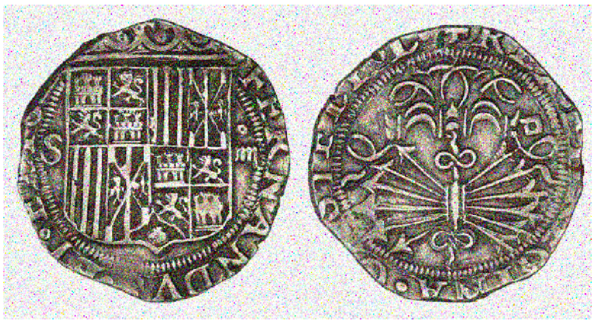

Fig. 4. Mixed noisy image.

Fig. 5. Restored image by classical Fitzhugh–Nagumo model.

Fig. 6. Restored image by classical Fitzhugh–Nagumo model.

In this Figure, classical Fitzhugh–Nagumo model eliminates some of image details and that justify the fact that this model does not enhances image contrast very well. However, the proposed algorithm follows these details at the same time with those of the original image and therefore the contrast is very enhanced.

The results obtained by the proposed model are more satisfactory than those obtained by classical Fitzhugh–Nagumo model since structures of image stocked in texture component; are preserved and not blurred unlike to classical one.

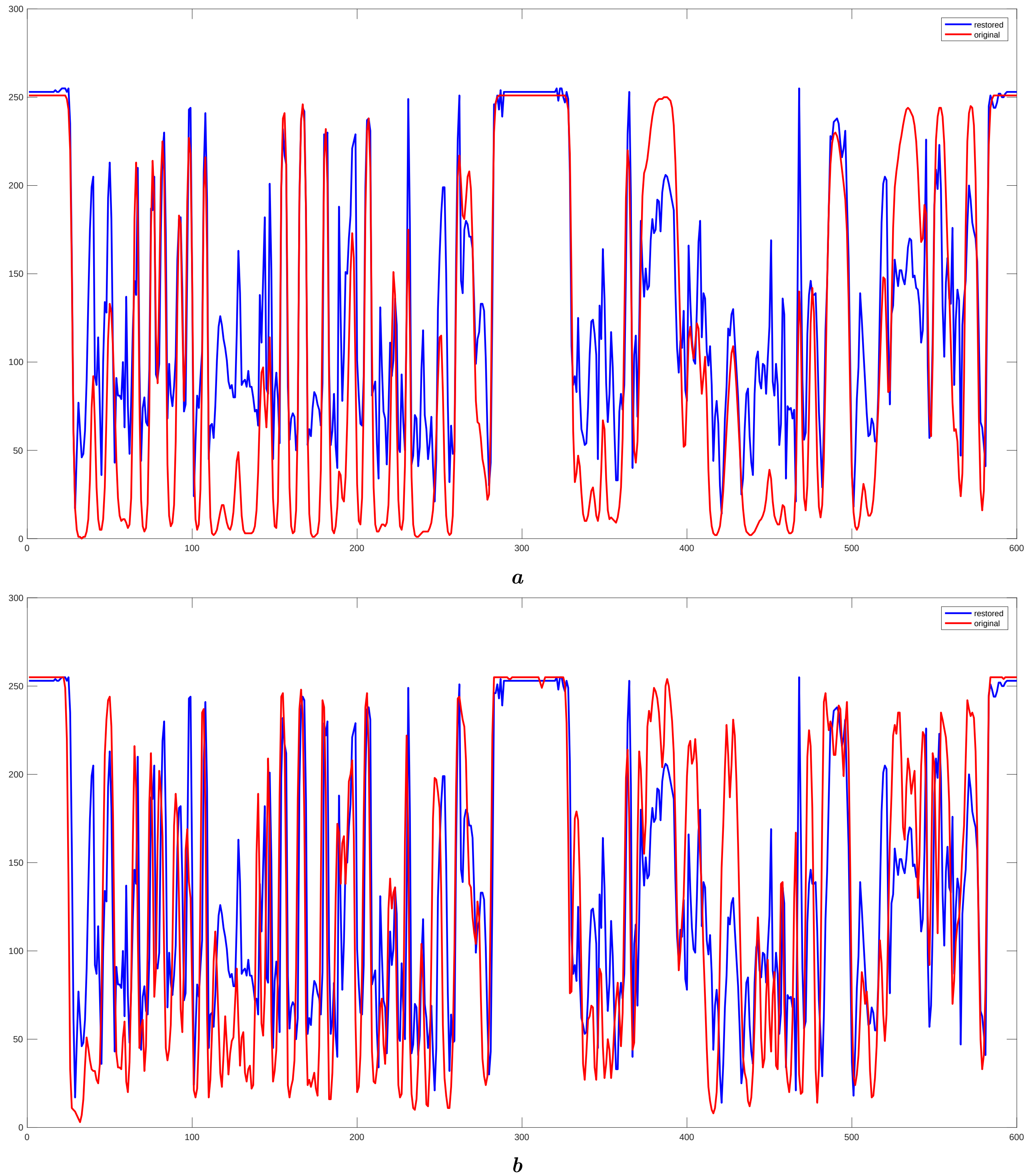

Fig. 7. Profile of line number 100 for (***a***) classical Fitzhugh–Nagumo model, (***b***) proposed model.

Fig. 8. Mixed noisy image.

Fig. 9. Restored image by classical Fitzhugh–Nagumo.

Fig. 10. Restored image by the proposed model.

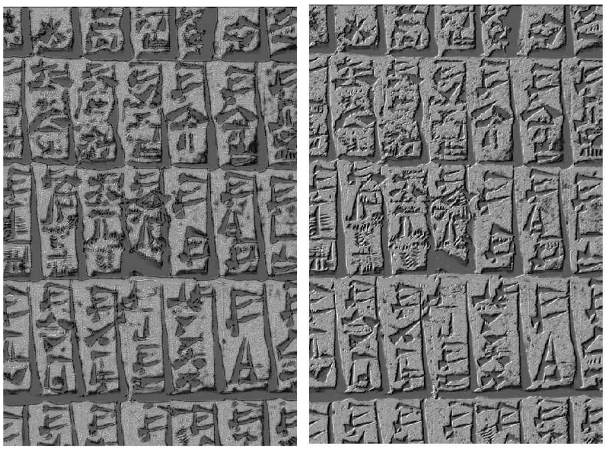

Fig. 11. Texture component: (left) classical Fitzhugh–Nagumo model, (right) proposed model.

The following Table 1 shows the value of measures enhancement of the two models: the proposed method and the classical Fitzhugh–Nagumo model.

Table 1. EME values of each image by the two models.

EME	Classiacal Fitzhugh–Nagumo	Proposed
Image 1	9.76	11.57
Image 2	9.36	10.30
Image 3	12.34	14.28
Image 4	19.14	23.26

Through this result, the proposed model has always a high value of enhancement measure EME than classical Fitzhugh–Nagumo system for different images and that explains the good quality of the image enhanced by our method.

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Відновлення зображення та покращення контрастності на основі нелінійної реакційно-дифузійної математичної моделі та техніки "розділяй і володарюй"

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У статті представлено новий алгоритм цифрової обробки зображень, які зашумлені змішаним гаусово-імпульсним шумом. Математична модель базується на техніці "розділяй і володарюй" у поєднанні з реакційно-дифузійною системою. Спочатку зображення розкладається на низькочастотні та високочастотні компоненти, згортаючи кожну із заздалегідь визначеним згортковим фільтром. Далі використовується проста схема з різними вагами, щоб інтегрувати та зібрати ці оброблені фрагменти зображення у відфільтроване зображення. Нарешті, застосовується наша реакційно-дифузійна система, щоб збільшити контрастність зображення. Описано ряд експериментальних результатів, щоб проілюструвати роботу запропонованого алгоритму та показати, що він дуже ефективний при усуненні змішаного гаусово-імпульсного шуму, для збільшення контрастності зображення та збереження країв.

Ключові слова: *реакція-дифузія, обробка зображення, змішаний гаусово-імпульсний шум, техніка "розділяй і володарюй".*