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Mathematical modeling of centrifugal machines rotors seals for the purpose of assessing their influence on dynamic characteristics

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With an increase of equipment parameters, such as the pressure of the sealing medium and the speed of shaft rotation, the problems ensuring its hermetization efficiency are rising up. In addition to hermetization itself, the sealing system affect the overall operational safety of the equipment, especially vibratory. Groove seals are considered as hydrostatodynamic supports capable of effectively damping rotor oscillations. To determine the dynamic characteristics, models of grooved seals and single-disc rotors with grooved seals are examined. The obtained analytical dependences for computation of dynamic characteristics for the hydromechanical rotor-seals system, describing radial-angular oscillations of the centrifugal machine rotor in groove seals are presented as well as the formulas for computation of amplitude frequency characteristics. An example for the computation dynamic characteristics of one of the centrifugal machine rotor models is drawn.

Keywords: *seals-supports, mathematical model, radial-angular vibrations, frequency characteristics.*

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1. Introduction

The distinguishing characteristics of centrifugal machine is that the tasks of vibration reliability and sealing are interrelated and, in most cases, can be satisfactorily accomplished through the correct choice of the groove seal construction. Therefore, while choosing the construction of groove seals, besides their designated purpose – to reduce volume losses, it is necessary to ensure the required vibration characteristics of rotor, that is also a very important function.

Noncontact seals, on which a huge differential pressure is throttled, can play the role of static, and with the right design approaches, dynamic supports. The last fact must be taken into account as designing critical power equipment [1].

In aviation and space technology, where, in addition to high sealed pressures and rotational speeds of rotors, there are great restrictions on the weight and dimensions of equipment, the use of seals as dynamic supports is especially important. When properly designed, noncontact seals can stiffen a flexible rotor to provide the required vibration reliability.

Current approaches for refinement of mathematical models of oscillatory systems according to experimental data are presented in the work [2]. The monograph [3] evaluates coefficients of the mathematical models for oscillatory systems, including rotary systems for multistage centrifugal machines. The article [4] addresses the phenomena of rotor rotation stability loss at rolling bearing.

Modern approaches in the linear and non-linear rotor dynamics and their practical applications are presented in the paper [5]. The work [6] provides an estimation of segment bearing stiffness with the balancing procedure for the flexible rotors of turbocharge units in the accelerating-balancing stand. Current methods for determination of active magnetic bearings stiffness and damping identification from frequency characteristics of control systems were introduced in [7]. Application of the finite element analysis for stiffness and critical speed calculation of a magnetic bearing-rotor system for electrical machines was described in the paper [8]. Article [9] provides stability and vibration analysis of a complex flexible rotor bearing system. A phenomenon of subharmonic resonance of a symmetric ball bearing-rotor system is investigated in the paper [10]. The paper [11] studies models for the critical frequencies of the centrifugal compressor rotor with taking into account the non-linear stiffness characteristics of bearings and seals.

As indicated in [12], energy of volumetric losses can be converted into net energy, if the groove seals are used simultaneously as hydrostatic bearings, that are able to have not only high radial rigidity but also to effectively damp the rotor fluctuations to the acceptable level even if there is a significant disbalance. This effect is especially considerable if there are existing steep velocity and pressure gradients, which are peculiar to close gaps of the groove seals, on which high pressure differentials are chocked and one of the surfaces belongs to rotor that both rotates and vibrates [13].

The dynamic characteristics of groove seals as intermediate supports have been studied in the paper [14].

However, the problems of rotor dynamics in groove seals are slightly neglected as to solve them it is necessary to account for the hydrodynamic characteristics of groove seals. And this is a separate problem in the hydrodynamics of three-dimensional unsteady viscous fluid flows in annular channels, whereof surfaces rotate and simultaneously perform radial-angular oscillations.

Since the problems of the rotor dynamics without groove seals have been mainly solved, this paper focuses more on the analysis of oscillatory processes caused by the hydrodynamic characteristics of seals.

2. Groove seal model

Fig. 1 shows a model of the groove seal that is an annular throttle formed by inner cylinder (shaft) with a small taper angle ϑ_A and outer cylinder (sleeve) with a taper angle ϑ_B; total taper angle of the channel $\vartheta_0 = \vartheta_B - \vartheta_A$. Taper parameter of the channel

$$\theta_0 = \frac{\vartheta_0 l}{2H}, \quad |\theta_0| \leqslant 1.$$

Shaft and bushing rotate around their own axes with the frequencies of their own rotation ω_1, ω_2. The axes themselves rotate around the fixed center O with precession frequencies Ω_1, Ω_2, and also perform radial and angular oscillations.

Thus, when developing groove seals, it is necessary to consider not only their direct purpose to reduce volumetric losses, but also their equally important function, which is to provide the necessary vibration characteristics of the rotor.

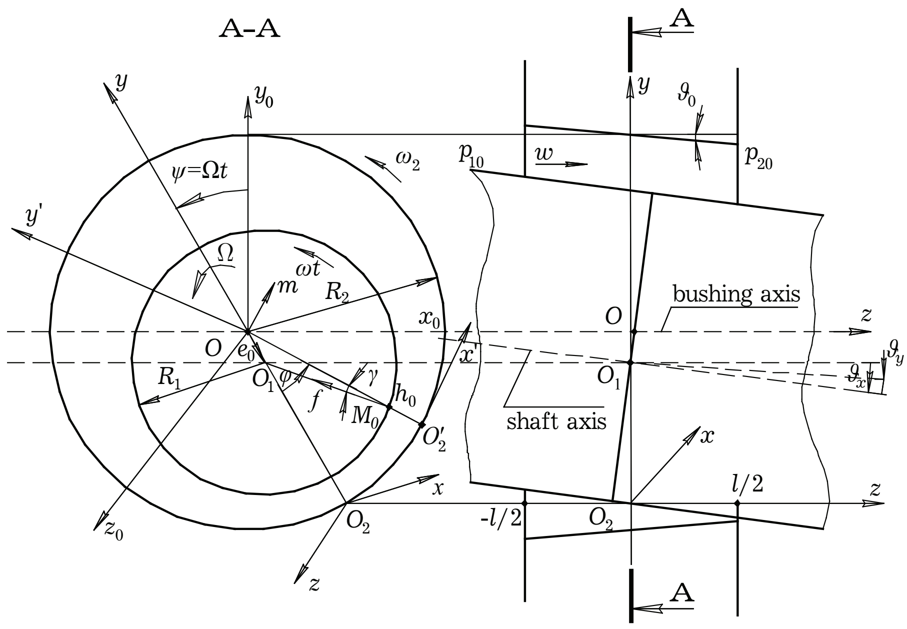

Fig. 1. Model of the groove seal.

3. Radial forces and moments in groove seals

The paper has provided an assessment of the force characteristics for laminar and turbulent flow regimes taking into consideration local resistances and in view of flow swirl at the gap inlet [14].

Projections onto the fixed coordinate axes of individual components of hydrodynamic forces and moments
$$F_{s(x,y)} = F_{g(x,y)} + F_{d(x,y)} + F_{p(x,y)}, M_{s(x,y)} = M_{g(x,y)} + M_{d(x,y)} + M_{p(x,y)},$$
arising in one groove seal and referred to the rotor mass, are as follows:
– forces and moments due to fluid inertia:
$$\begin{aligned} \frac{F_{gx}}{m} &= -k_g \left[\ddot{x} - \frac{12\theta_0 q_0}{Hl\,(2-n)}\dot{x} + \kappa\omega_a \dot{y} - \frac{2}{15}\kappa\omega_a\theta_0\frac{l}{2}\dot{\vartheta}_x \right], \\ \frac{F_{gy}}{m} &= -k_g \left[\ddot{y} - \frac{12\theta_0 q_0}{Hl\,(2-n)}\dot{y} - \kappa\omega_a \dot{x} - \frac{2}{15}\kappa\omega_a\theta_0\frac{l}{2}\dot{\vartheta}_y \right], \end{aligned} \tag{1}$$
$$\begin{aligned} \frac{M_{gx}}{m} &= -k_g\frac{l}{30}\left(\frac{l}{2}\ddot{\vartheta}_x + 2\kappa\omega_a\theta_0\dot{x} + \kappa\omega_a\frac{l}{2}\dot{\vartheta}_y\right), \\ \frac{M_{gy}}{m} &= -k_g\frac{l}{30}\left(\frac{l}{2}\ddot{\vartheta}_y + 2\kappa\omega_a\theta_0\dot{y} - \kappa\omega_a\frac{l}{2}\dot{\vartheta}_x\right); \end{aligned} \tag{2}$$
– forces and moments due to displacement flow:
$$\begin{aligned} \frac{F_{dx}}{m} &= -k_d\left[\dot{x} + \kappa\omega_a y - \frac{l}{5}\theta_0\left(\kappa\omega_a\vartheta_x - \dot{\vartheta}_y\right)\right], \\ \frac{F_{dy}}{m} &= -k_d\left[\dot{y} - \kappa\omega_a x - \frac{l}{5}\theta_0\left(\kappa\omega_a\vartheta_y + \dot{\vartheta}_x\right)\right], \end{aligned} \tag{3}$$
$$\begin{aligned} \frac{M_{dx}}{m} &= -k_d\frac{l}{5}\left[\theta_0\left(\dot{y} - \kappa\omega_a x\right) + \frac{l}{12}\left(\kappa\omega_a\vartheta_y + \dot{\vartheta}_x\right)\right], \\ \frac{M_{dy}}{m} &= k_d\frac{l}{5}\left[\theta_0\left(\dot{x} + \kappa\omega_a y\right) + \frac{l}{12}\left(\kappa\omega_a\vartheta_x - \dot{\vartheta}_y\right)\right]; \end{aligned} \tag{4}$$
– forces and moments due to drop in pressure throttled on the groove seal Δp_0 (pressure flow):
$$\begin{aligned} \frac{F_{px}}{m} &= -k_p\left[\left(\theta_0 + N\chi_m\right)x + \left(1 + 2\Delta\chi\right)\frac{l}{2}\vartheta_y\right], \\ \frac{F_{py}}{m} &= -k_p\left[\left(\theta_0 + N\chi_m\right)y - \left(1 + 2\Delta\chi\right)\frac{l}{2}\vartheta_x\right], \end{aligned} \tag{5}$$
$$\begin{aligned} \frac{M_{px}}{m} &= -k_p\frac{l}{6}\left[N\Delta\chi y - 2\chi_m\frac{l}{2}\vartheta_x\right], \\ \frac{M_{py}}{m} &= k_p\frac{l}{6}\left[N\Delta\chi x + 2\chi_m\frac{l}{2}\vartheta_y\right]. \end{aligned} \tag{6}$$

Let us move on in formulas (1) – (6) to dimensionless coordinates and reduced forces and moments also introducing the additional representations
$$K_i = \frac{12q_0}{Hl\,(2-n)}, \quad j = \frac{ml^2}{60I},$$
where K_i is the parameter considering local component of fluid inertia force; j is dimensionless parameter characterizing the hydrodynamic moments in the groove seal: it converts the radial rigidity coefficients k_g, k_d, k_p into the corresponding angular rigidity coefficients $k_g j$, $k_d j$, $k_p j$.

In our notations

$$k_p = \frac{k_p'}{mH}, \quad M_{px}^* = \frac{M_{px} l}{2HI} = 10 k_p j \chi_m \theta_x.$$

The initial swirl of the flow is estimated by the coefficient

$$\kappa = \frac{\omega_c}{\omega_a},$$

where $\omega_a = 0.5(\omega_1 + \omega_2)$, ω_c is the average angular velocity of the fluid in the channel.

Further the rotor rotating in two symmetrically located groove seals with fixed outer races ($\omega_2 = 0$), is considered, therefore, $\omega_a = 0.5\omega_1 = 0.5\omega$, where $\omega_1 = \omega$ is rotor speed.

The values of forces and moments will be doubled by the number of seals. For convenience of further transformations, the components will be grouped according to their dependence on the generalized coordinates (F_3^*, M_3^*), generalized velocities (F_2^*, M_2^*) and generalized accelerations (F_1^*, M_1^*):

$$-F_{1x}^* = 2k_g \ddot{u}_x, \quad -F_{1y}^* = 2k_g \ddot{u}_y, \quad -M_{1x}^* = -2k_g j \ddot{\theta}_x, \quad -M_{1y}^* = 2k_g j \ddot{\theta}_y,$$

$$-F_{2x}^* = 2\left(k_d + k_g K_i \theta_0\right) \dot{u}_x + k_g \kappa \omega \dot{u}_y - \frac{2}{15} k_g \kappa \omega \theta_0 \dot{\theta}_x + \frac{4}{5} k_d \theta_0 \dot{\theta}_y,$$
$$-F_{2y}^* = -k_g \kappa \omega \dot{u}_x + 2\left(k_d + k_g K_i \theta_0\right) \dot{u}_y - \frac{4}{5} k_d \theta_0 \dot{\theta}_x - \frac{2}{15} k_g \kappa \omega \theta_0 \dot{\theta}_y,$$
$$-M_{2x}^* = j\left(2k_g \kappa \omega \theta_0 \dot{u}_x + 12 k_d \theta_0 \dot{u}_y + 2k_d \dot{\theta}_x + k_g \kappa \omega \dot{\theta}_y\right),$$
$$-M_{2y}^* = j\left(-12 k_d \theta_0 \dot{u}_x + 2k_g \kappa \omega \theta_0 \dot{u}_y - k_g \kappa \omega \dot{\theta}_x + 2k_d \dot{\theta}_y\right);$$

$$-F_{3x}^* = 2k_p\left(\theta_0 + N\chi_m\right) u_x + k_d \kappa \omega u_y - \frac{2}{5} k_d \kappa \omega \theta_0 \theta_x + 2k_p\left(1 + 2\Delta\chi\right) \theta_y,$$
$$-F_{3y}^* = -k_d \kappa \omega u_x + 2k_p\left(\theta_0 + N\chi_m\right) u_y - 2k_p\left(1 + 2\Delta\chi\right) \theta_x - \frac{2}{5} k_d \kappa \omega \theta_0 \theta_y,$$

$$-M_{3x}^* = j\left(-6k_d \kappa \omega \theta_0 u_x + 10 k_p N \Delta\chi u_y - 20 k_p \chi_m \theta_x + k_d \kappa \omega \theta_y\right),$$
$$-M_{3y}^* = j\left(-10 k_p N \Delta\chi u_x - 6k_d \kappa \omega \theta_0 u_y - k_d \kappa \omega \theta_x - 20 k_p \chi_m \theta_y\right).$$

Let us introduce notations of the doubled force coefficients

$$a_{11} = 2k_g, \quad a_{21} = 2\left(k_d + k_g K_i \theta_0\right), \quad a_{41} = k_g \kappa \omega, \quad \alpha_2 = \frac{2}{15} k_g \kappa \omega \theta_0, \quad \alpha_4 = \frac{4}{5} k_d \theta_0,$$
$$a_{31} = 2k_p\left(\theta_0 + N\chi_m\right), \quad a_{51} = k_d \kappa \omega, \quad \alpha_3 = \frac{2}{5} k_d \kappa \omega \theta_0, \quad \alpha_5 = 2k_p\left(1 + 2\Delta\chi\right).$$

Expressions of relative forces and moments for two groove seals now become:

$$\begin{gathered} -F_{1x}^* = a_{11} \ddot{u}_x, \quad -F_{1y}^* = a_{11} \ddot{u}_y, \quad -M_{1x}^* = a_{11} j \ddot{\theta}_x, \quad -M_{1y}^* = a_{11} j \ddot{\theta}_y, \\ -F_{2x}^* = a_{21} \dot{u}_x + a_{41} \dot{u}_y - \alpha_2 \dot{\theta}_x + \alpha_4 \dot{\theta}_y, \quad -F_{2y}^* = a_{41} \dot{u}_x + a_{21} \dot{u}_y - \alpha_4 \dot{\theta}_x - \alpha_2 \dot{\theta}_y, \end{gathered} \tag{7}$$

$$\begin{gathered} -M_{2x}^* = j\left[15\alpha_2 \dot{u}_x + 15\alpha_4 \dot{u}_y + 2k_d \dot{\theta}_x + a_{41} \dot{\theta}_y\right], \\ -M_{2y}^* = j\left[-15\alpha_4 \dot{u}_x + 15\alpha_2 \dot{u}_y - a_{41} \dot{\theta}_x + 2k_d \dot{\theta}_y\right]. \end{gathered} \tag{8}$$

$$\begin{gathered} -F_{3x}^* = a_{31} u_x + a_{51} u_y - \alpha_3 \theta_x + \alpha_5 \theta_y, \quad -F_{3y}^* = -a_{51} u_x + a_{31} u_y - \alpha_5 \theta_x - \alpha_3 \theta_y, \\ -M_{3x}^* = j\left(-15\alpha_3 u_x + 5\alpha_5 \frac{N\Delta\chi}{1 + 2\Delta\chi} u_y - 10 a_{31} \frac{\chi_m}{\theta_0 + N\chi_m} \theta_x + a_{51} \theta_y\right), \\ -M_{3y}^* = j\left(-5\alpha_5 \frac{N\Delta\chi}{1 + 2\Delta\chi} u_x - 15\alpha_3 u_y - a_{51} \theta_x - 10 a_{31} \frac{\chi_m}{\theta_0 + N\chi_m} \theta_y\right). \end{gathered} \tag{9}$$

4. The models of rotors

In the models of rotors of single-stage pumps, impellers are situated between two identical seals.

Two models of single-disk rotor: with the disk between fixed bearings (Fig. 2**a**) of asymmetrical (P-1 model), symmetrical $l_1 = l_2$ (P-1c model) and overhung (Fig. 2**b**, P-2 model) are considered. Identical groove seals are situated from the both sides of the disk (impeller).

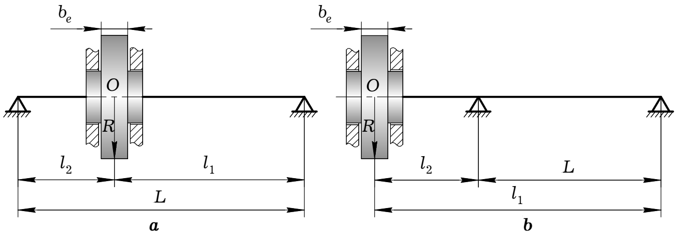

Fig. 2. Models of single-disk rotors in groove seals: **a** — with a disk between the bearings (P-1, P-1c models); **b** — overhung (P-2 model) with mass m, radius R and effective thickness b_e.

The first model imitates the rotor of single-stage pump with double-entry impeller and the second one – the rotor of overhang pump.

In the both models' radial displacements of disk are accompanied with its rotation in the plane of defected shaft axis. Inertial resistance to rotation is characterized with the corresponding gyroscopic disk moment. Rotor mass is concentrated in the center of disk masses and a weightless elastic shaft rotates in fixed bearings.

The rotor is statically and dynamically unbalanced: mass center is displaced from the mechanical center for the amount of eccentricity $\boldsymbol{a}(a_x, a_y)$ that stands for static unbalance. The main central axes of disk inertia due to fit tilt or other instrument accuracy are deviated from the principal shaft axes of section (main flexural shaft axes) to angles γ_x, γ_y, characterizing dynamic unbalance of the rotor. Unbalance parameters are considered to be preset small values.

For the symmetrical statistically unbalanced rotors, for example, rotors of double inlet impeller pump, radial oscillations are predominant. Small angular oscillations are caused by unavoidable dynamic unbalance and probable disturbance of the rotor symmetry in regard to the transverse vertical plane passing through the center of masses. In this case, useful preliminary results can be received, when considering only radial oscillations. Herewith, the coefficients of hydrodynamic forces should be doubled (according to the number of seals).

The another extreme case of predominantly angular vibrations is possible for a symmetrical statically balanced rotor under the influence of dynamic unbalance. In this case, it is necessary to double the hydrodynamic moments. Besides, the radial hydrodynamic forces F^*_{su} arising when the rotor axis is skewed towards to the seal axis are different in value due to the difference in eccentricities, radial speeds and accelerations. Therefore, they create an additional moment relative to the impeller center. The azimuth angles in the both seals with equally spaced bushings remain the same, therefore, the components of forces due to the angular vibrations (with coefficients α_i), do not create additional moments.

More details on the additional moments from elastic forces one can find in the paper [14]. For the right and left seals, dimensionless generalized coordinates, velocities and accelerations of the shaft center, which will be needed to calculate the moments of inertial, dissipative and gyroscopic forces:

$$\begin{aligned}
&u'_x = u_x + \Delta u_x, \quad u''_x = u_x - \Delta u_x, \quad \Delta u_x = \frac{1}{H} l_c \vartheta_y = 2\frac{l_c}{l}\theta_y;\\
&u'_y = u_y - \Delta u_y, \quad u''_y = u_y + \Delta u_y, \quad \Delta u_y = \frac{1}{H} l_c \vartheta_x = 2\frac{l_c}{l}\theta_x;\\
&\ddot{u}'_x = \ddot{u}_x + \Delta\ddot{u}_x, \quad \ddot{u}''_x = \ddot{u}_x - \Delta\ddot{u}_x, \quad \Delta\ddot{u}_x = 2\frac{l_c}{l}\ddot{\theta}_y;\\
&\ddot{u}'_y = \ddot{u}_y - \Delta\ddot{u}_y, \quad \ddot{u}''_y = \ddot{u}_y + \Delta\ddot{u}_y, \quad \Delta\ddot{u}_y = 2\frac{l_c}{l}\ddot{\theta}_x.
\end{aligned} \tag{10}$$

Correlation $\theta_{x,y} = \vartheta_{x,y} l/(2H)$ is used here. The centripetal components are not indicated in the formulas for accelerations, since they are directed along the rotor axis and do not affect its radial-angular vibrations.

Based on (5), the dimensional forces in the right F_3' and left F_3'' seals are as follows:

$$F'_{3x} = -mH\frac{a_{31}}{2}(u_x + \Delta u_x) = -F_{3x} - \Delta F_{3x},$$

$$F''_{3x} = -\frac{a_{31}}{2}mH(u_x - \Delta u_x) = -F_{3x} + \Delta F_{3x},$$

$$F'_{3y} = -mH\frac{a_{31}}{2}(u_y - \Delta u_y) = -F_{3y} + \Delta F_{3y},$$

$$F''_{3y} = -\frac{a_{31}}{2}mH(u_y + \Delta u_y) = -F_{3y} - \Delta F_{3y},$$

$$\Delta F_{3x} = a_{31}Hm\frac{l_c}{l}\theta_y, \quad \Delta F_{3y} = a_{31}Hm\frac{l_c}{l}\theta_x.$$

Additions to the forces are equal in value and oppositely directed, so their total projections on the corresponding axes are equal to zero and do not affect the radial vibrations.

Additional moments from the elastic forces are as follows:

$$\Delta M_{3x} = -2l_c\Delta F_{3y} = -2a_{31}Hm\frac{l_c^2}{l}\theta_x, \quad \Delta M_{3y} = -2l_c\Delta F_{3x} = -2a_{31}Hm\frac{l_c^2}{l}\theta_y,$$

and after dividing by the equatorial moment of inertia and multiplying by $l/(2H)$,

$$\Delta M^*_{3x} = \frac{\Delta M_{3x}l}{2HI} = -a_{31}j_c\theta_x, \quad \Delta M^*_{3y} = \frac{\Delta M_{3y}l}{2HI} = -a_{31}j_c\theta_y, \quad j_c = \frac{ml_c^2}{I}.$$

Similarly, additional moments of other components of the radial force are calculated, due to the difference in the generalized coordinates, velocities and accelerations (7) of the shaft centers in the right and left seals:

$$\Delta M^*_{1x} = -a_{11}j_c\ddot{\theta}_x, \quad \Delta M^*_{1y} = -a_{11}j_c\ddot{\theta}_y, \quad \Delta M^*_{2x} = -a_{21}j_c\dot{\theta}_x, \quad \Delta M^*_{2y} = -a_{21}j_c\dot{\theta}_y,$$

$$\Delta M^*_{4x} = -a_{41}j_c\dot{\theta}_y, \quad \Delta M^*_{4y} = a_{41}j_c\dot{\theta}_x, \quad \Delta M^*_{5x} = -a_{51}j_c\theta_y, \quad \Delta M^*_{5y} = a_{51}j_c\theta_x.$$

The given additions depend on the angular coordinates, so they should be introduced into the moment components determined by the angles of the disk rotation, angular velocities and accelerations. Having denoted total components of the hydrodynamic moment in two seals

$$M_{i(x,y)} = M^*_{i(x,y)} + \Delta M^*_{i(x,y)}, \quad i = 1, 2, 3,$$

taking into consideration (2) – (9) one can obtain:

$$-M_{1x} = a_{11}(j + j_c)\ddot{\theta}_x, \quad -M_{1y} = a_{11}(j + j_c)\ddot{\theta}_y;$$

$$-M_{2x} = 15j(\alpha_2\dot{u}_x + \alpha_4\dot{u}_y) + (2k_dj + a_{21}j_c)\dot{\theta}_x + a_{41}(j + j_c)\dot{\theta}_y,$$
$$-M_{2y} = 15j(-\alpha_4\dot{u}_x + \alpha_2\dot{u}_y) - a_{41}(j + j_c)\dot{\theta}_x + (2k_dj + a_{21}j_c)\dot{\theta}_y;$$

$$-M_{3x} = j\left(-15\alpha_3u_x + 5\alpha_5\frac{N\Delta\chi}{1 + 2\Delta\chi}u_y\right) - a_{31}\left(\frac{10\chi_m}{\theta_0 + N\chi_m}j - j_c\right)\theta_x + a_{51}(j + j_c)\theta_y,$$
$$-M_{3y} = j\left(-5\alpha_5\frac{N\Delta\chi}{1 + 2\Delta\chi}u_x - 15\alpha_3u_y\right) - a_{51}(j + j_c)\theta_x - a_{31}\left(\frac{10\chi_m}{\theta_0 + N\chi_m}j - j_c\right)\theta_y.$$

It should be noted that additional moments were found for seals symmetrically located regarding the mass center of the impeller. If this symmetry is violated, values $\Delta u_{x,y}$ (10) will slightly change, causing a change in the numerical coefficient in expression of the parameter j_c.

5. Joint radial-angular oscillations of the rotor in groove seals

The considered rotor in groove seals is an eighth-order oscillatory system with four generalized coordinates: u_x, u_y, θ_x, θ_y. The system oscillates about a stable equilibrium position, so the roots of the characteristic equation are four pairs of complex adjoined numbers.

The pressure developed by the centrifugal stage is throttled on the front groove seal of the stage. This pressure is proportional to the square of the impeller rotary velocity. These conditions are peculiar to centrifugal machines. This affects the form of frequency characteristics, which are dependencies of natural frequencies on the rotary speed. In this case, the pressure differential ceases to be an independent external influence, it is associated with the additional ratio $\Delta p_0 = B\omega^2$. As a result, only the rotary speed is external influence, and self-toughening effect of the rotor is enhanced.

Forced joint radial-angular oscillations of the rotor at a constant pressure drop across the seals are described by equations [14],

$$\begin{aligned} &a_1\ddot{u} + a_2\dot{u} + a_3u \mp i\,(a_4\dot{u} + a_5u)\,\omega - \big(\alpha_2\dot{\theta} + \alpha_3\theta\big)\omega \\ &\qquad\qquad \mp i\big(\alpha_4\dot{\theta} + \alpha_5\theta - \alpha_0\theta\big) = \omega^2 a^* = \omega^2\,|a^*|\,e^{\pm i\omega t}, \\ &b_1\ddot{\theta} + b_2\dot{\theta} + b_3\theta \mp i\big(b_4\dot{\theta} + b_5\theta\big)\omega + (\beta_2\dot{u} - \beta_3u)\,\omega \\ &\qquad\qquad \mp i\,(\beta_4\dot{u} + \beta_5u + \beta_0u) = (1-j_0)\,\omega^2\gamma^* = (1-j_0)\,\omega^2\,|\gamma^*|\,e^{\pm i\omega t}. \end{aligned} \tag{11}$$

Using standard programs, one can immediately find numerical solution of these equations. However, the traditional described approach makes it possible to estimate the influence of the various forces and moments on amplitudes and phases via their analytical expressions (by evaluating the coefficients of the proper operator and operators of external influences).

6. Determination of the amplitude and phase frequency characteristics

Substituting the solution of equations (11) in the form

$$u = u_a e^{i(\omega t + \varphi_u)} = \tilde{u}e^{i\omega t},$$

$$\theta = \theta_a e^{i(\omega t + \varphi_\vartheta)} = \tilde{\theta}e^{i\omega t},$$

we shall obtain a system of algebraic equations for complex amplitudes A and Γ:

$$\begin{aligned} &\big[-a_1\omega^2 + a_3 + a_4\omega^2 + i\,(a_2 - a_5)\,\omega\big]\,\tilde{u} - \big[(\alpha_3 - \alpha_4)\,\omega + i\,\big(\alpha_2\omega^2 + \alpha_5 - \alpha_a\big)\big]\,\tilde{\theta} = \mathrm{A}\omega^2, \\ &\big[-(\beta_3 - \beta_4)\,\omega + i\,\big(\beta_2\omega^2 - \beta_5 - \beta_a\big)\big]\,\tilde{u} + \big[-b_1\omega^2 + b_3 + b_4\omega^2 + i\,(b_2 - b_5)\,\omega\big]\,\tilde{\theta} = \Gamma\omega^2. \end{aligned} \tag{12}$$

After transformation to dimensionless frequencies $\bar{\omega} = \omega/\Omega_{u0}$ and introduction of some notations, the equations (12) take the form:

$$\begin{aligned} &(U_{11} + iV_{11})\,\tilde{u} + (U_{12} + iV_{12})\,\tilde{\theta} = \mathrm{A}\bar{\omega}^2, \\ &(U_{21} + iV_{21})\,\tilde{u} + (U_{22} + iV_{22})\,\tilde{\theta} = \Gamma\bar{\omega}^2. \end{aligned} \tag{13}$$

Here $U_{11} + iV_{11}$, $U_{22} + iV_{22}$ are proper operators of the independent radial and angular oscillations correspondingly. Cross sectional operators $U_{12} + iV_{12}$, $U_{21} + iV_{21}$ characterize the influence of angular oscillations on radial and the effect of radial on angular, i.e., interconnection of these oscillations.

From the system of non-homogenous algebraic equations (13) after a series of transformations can be obtained the amplitudes and phases expressed in terms of external disturbances:

$$
\begin{aligned}
u_0 &= \bar{\omega}^2 \sqrt{\frac{(\mathrm{A}U_{22} - \Gamma U_{12})^2 + (\mathrm{A}V_{22} - \Gamma V_{12})^2}{U_0^2 + V_0^2}}, \\
\theta_a &= \bar{\omega}^2 \sqrt{\frac{(\Gamma U_{11} - \mathrm{A}U_{21})^2 + (\Gamma V_{11} - \mathrm{A}V_{21})^2}{U_0^2 + V_0^2}}, \\
\varphi_u &= -\arctan \frac{(\mathrm{A}U_{22} - \Gamma U_{12})\, V_0 - (\mathrm{A}V_{22} - \Gamma V_{12})\, U_0}{(\mathrm{A}U_{22} - \Gamma U_{12})\, U_0 + (\mathrm{A}V_{22} - \Gamma V_{12})\, V_0}, \\
\varphi_\vartheta &= -\arctan \frac{(\Gamma U_{11} - \mathrm{A}U_{21})\, V_0 - (\Gamma V_{11} - \mathrm{A}V_{21})\, U_0}{(\Gamma U_{11} - \mathrm{A}U_{21})\, U_0 + (\Gamma V_{11} - \mathrm{A}V_{21})\, V_0}.
\end{aligned}
\tag{14}
$$

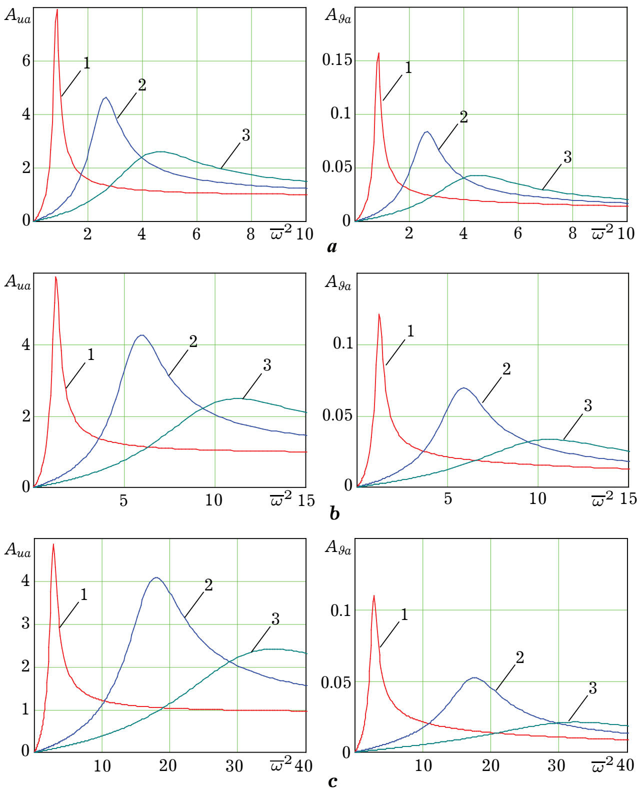

Fig. 3. Amplitude frequency characteristics as a response to statistic unbalance, model P-1: ***a*** – $\Delta p_0 = 1.5\,\text{MPa} = \text{const}$, ***b*** – $\Delta p_0 = 4\,\text{MPa} = \text{const}$, ***c*** – $\Delta p_0 = 13.3\,\text{MPa} = \text{const}$.

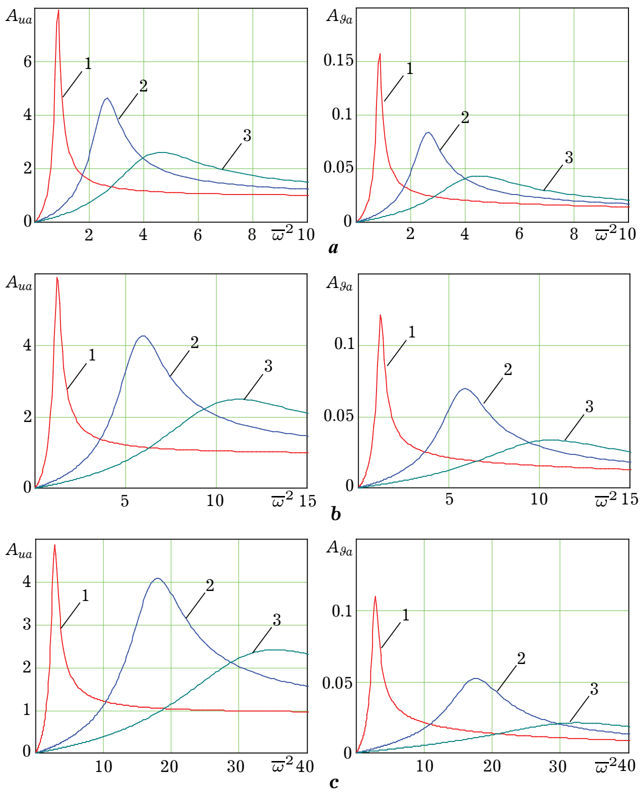

Fig. 4. Amplitude frequency characteristics as a response to dynamic unbalance, model P-1: ***a*** – $\Delta p_0 = 1.5\,\text{MPa} = \text{const}$, ***b*** – $\Delta p_0 = 4\,\text{MPa} = \text{const}$, ***c*** – $\Delta p_0 = 13.3\,\text{MPa} = \text{const}$.

Using formulas (14), one can build amplitude frequency characteristics as amplitude ratio of the corresponding oscillations to the amplitudes of external excitements:

$$
A_{ua} = \frac{u_{aa}}{\mathrm{A}}, \quad A_{\vartheta a} = \frac{\theta_{aa}}{\mathrm{A}}, \quad A_{u\gamma} = \frac{u_{a\gamma}}{\Gamma}, \quad A_{\vartheta\gamma} = \frac{\theta_{a\gamma}}{\Gamma}.
$$

Numerical calculations were carried out for the rotor model with a disc between the bearings. The groove seals with three taper parameters were considered: $\theta_0 = -0.3; 0; 0.3$. Diagrams for these parameters in Figs. 3, 4 are designated respectively by numbers 1, 2, 3. Calculations were carried out for the constant pressure differences $\Delta p_0 = (1.5; 4.0; 13.3)$ MPa. The following values of unbalance were considered in the calculations: $A = a^* = a/H = 0.05$, $a = 12.5\,\mu\text{m}$, $a\omega_n = 3.75\,\text{mm/s}$.

The obtained dependences are confirmed by the results of the experimental studies, published works [1, 5, 12, 13].

7. Conclusions

Based on the study of hydromechanical model of groove seal and models of rotors in groove seals, analytical dependences that describe the radial-angular vibrations of the rotor are obtained.

Force coefficients of groove seals are determined by geometric (clearance, radius, length, taper, shape of the input edges) and operational (pressure drop, operating speed range, physical properties of the pumped medium) parameters. A purposeful choice of these parameters can influence the vibration state of the rotor and the machine itself. An important feature of centrifugal machines is that the pressure drops that throttled on the groove seals are proportional to the rotor speed. This leads to the effect of the rotor self-toughening, and to the fact that in most cases there are no critical frequencies. Self-toughening is enhanced by the gyroscopic moments of groove seals, and, for rotors of a disk design, by the gyroscopic moment of the disk.

This is especially important for machines with high parameters. For example, in space technology, where high shaft speeds and sealing pressure must be provided in combination with the requirement to minimize the weight and dimensions of the unit. That is, the initially "flexible" in the dynamic sense rotor, in combination with properly designed seals, becomes "tough".

The problems of rotor dynamics are of great practical importance, and the range of these problems is unlimited, as is the unlimited number of constructive types of rotary machines, features of rotor designs, and the conditions for their operation. Use of numerical methods is very promising for solving problems of joint radial-angular oscillations of the rotor, that takes into account all hydrodynamic forces and moments arising in groove seals.

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Математичне моделювання ущільнень роторів відцентрових машин з метою оцінки їх впливу на динамічні характеристики

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З ростом параметрів обладнання, таких як тиск ущільнюваного середовища і швидкість обертання ротора, зростають і проблеми, пов'язані із забезпеченням ефективності його герметизації. Крім власне герметизації системи ущільнення впливають на загальну експлуатаційну безпеку обладнання, особливо вібраційну. Щілинні ущільнення розглядаються як гідростатодинамічні опори, здатні ефективно демпфувати коливання ротора. Для визначення динамічних характеристик розглянуто моделі щілинного ущільнення та однодискових роторів з щілинними ущільненнями. Наведено отримані аналітичні залежності для розрахунку динамічних характеристик гідромеханічної системи "ротор – ущільненняєє, що описують радіально-кутові коливання ротора відцентрової машини в щілинних ущільненнях, а також формули для розрахунку амплітудних частотних характеристик. Наведено приклад розрахунку динамічних характеристик однієї з моделей ротора відцентрової машини.

Ключові слова: *ущільнення-опори, математична модель, радіально-кутові коливання, частотні характеристики.*