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Forced convection of laminar gaseous slip flow near a stagnation point with viscous dissipation and pressure work

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Forced convection problem of laminar quasi-incompressible boundary layer for the stagnation slip flow at a relatively low Mach number, considering the simultaneous effects of viscous dissipation and pressure work, has been investigated. The system of coupled partial differential equations was first transformed into a system of coupled ordinary differential equations through suitable transformations, which was then solved using Runge–Kutta–Fehlberg fourth-fifth order method. The solution obtained here is much better suited to formulating and solving the variable-property of chemically reacting flows that occur in practice, by taking into account the slip boundary conditions at the gas–wall interface. The effects of the Eckert number and the slip parameter on the heat transfer characteristics are presented graphically and discussed. The numerical results show that the pressure work, viscous dissipation play significant role on the heat transfer and could not be neglected under any circumstance for rarefied gas flows.

Keywords: *slip flow, stagnation point flow, viscous dissipation, pressure work, boundary layer, similarity solution.*

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1. Introduction

Stagnation flow in the slip regime is often encountered in many industrial processes of micro-electromechanical systems (MEMS) that have great practical value in multiple engineering fields. As typical example for applications of such flow, one can cite chemical-vapor-deposition reactors that are used in the growth of electronic thin films in the manufacture of semiconductors [1]. In this flow, the flatness of the boundary layer leads to uniform thickness that is independent of the distance along the wall which is characteristic of stagnation boundary layer flow. Therefore, this uniformity is highly desirable, in the deposition of the film along the rigid surface, for the design of the semiconductors. Stagnation flow can also be found in the cooling of electronic devices by fans [2] and used also in micro-scale combustion research, to study the effects of shear stress on flame behavior [3]. In this context, one of the major difficulties in the conception of a micro-scale reactor, is to maintain a stable combustion in a very small device [4, 5].

Lin and Schaaf [6] were the first who studied the effect of the slip on the flow near a stagnation point using perturbation method, they suggested that shear stress or heat transfer would not be affected by the slip condition. Thereafter, the result of Lin and Schaaf [6] was partially contradicted by a more complete thermal analysis that has suggested that heat transfer of a laminar boundary layer decreased in the presence of a slip boundary condition [7]. Wang [8–10] studied in three different works the stagnation slip flow and the heat transfer over an axisymmetric plate with partial slip boundary condition. It was found that slip greatly affects the flow field and the heat transfer. Cao and Baker [11] have used the local non-similar method developed by Sparrow et al. [12, 13] in continuum flow to analyzed mixed convection around an isothermal vertical flat plate with velocity slip and temperature jump at the wall. They have shown that the heat transfer decreases widely in the slip flow regime whether for a liquid or for a gas. Later, Martin and Boyd [14] have numerically studied the heat transfer over an isothermal 2-dimensional vertical flat plate in the presence of a slip boundary condition. A significant investigation of the boundary layer of Falkner–Skan flow, over a wedge in the slip regime, has recently been made by Essaghir et al. [15]. The results of the stagnation flow case included in their study have shown that the local heat transfer exhibits a large decrease as the flow becomes more rarefied due to effect of temperature-jump at the wall, which globally reduces the local heat transfer near the stagnation point. The results of Wang [9], Martin and Boyd [14] and Essaghir et al. [15] show a change in the heat transfer and wall temperature jump on the order of several percent. Thus, the incorporating rarefied-flow effects into stagnation-point heat transfer models will result in a measurable increase in the accuracy of the estimated heat transfer.

All the above mentioned studies of stagnation flow in the rarefied regime do not consider the effects of pressure work and viscous dissipation. However, it can be shown that these factors play significant role on the heat transfer and cannot be neglected under any circumstance in rarefied flows, as shown recently by Essaghir et al. [16]. These authors have studied the effects of viscous dissipation and modified Knudsen number on the heat transfer of laminar incompressible gaseous slip flow over a horizontal flat plate. The non-similar solution of thermal boundary layer equation has revealed that the rarefaction has significant impact on the local heat transfer for large Eckert number, this effect mainly depends if the plate being warmer or colder than the free stream. The present study extends previous analysis to obtain similarity solution for the stagnation slip flow due to combined effects of the velocity slip and thermal jump boundary conditions in the presence of viscous dissipation and pressure work. To our knowledge, the results of such solution have not been reported before. The non-linear partial differential equations of momentum and energy are transformed into ordinary differential equations and solved numerically Runge–Kutta–Fehlberg fourth-fifth order method. The effects of slip parameter and Eckert number on the temperature and local Nusselt number will be presented and discussed. This analysis provides an estimate of change in heat transfer between plate and gas of a rarefied stagnation flow due to viscous dissipation and pressure work effects, which may find great interest in many manufacturing processes in industry and engineering.

2. Analysis

We consider the steady two dimensional stagnation point flow of a quasi-incompressible viscous gas for relatively low Mach number ($M < 0.3$). The problem is restricted to the case of a flat plate perpendicular to the stream. The surface of the wall maintained at constant temperature T_w. It is well known that, far from the stagnation point the motion of fluid corresponds to a region where the effect of viscosity is small with zero vorticity. Thus, the flow can be assumed as irrotational and the external flow may be described by the inviscid flow solution [17]

$$\psi_\infty = a\,x\,y, \tag{1}$$

$$U_\infty = a\,x, \tag{2}$$

$$V_\infty = -a\,y, \tag{3}$$

$$\frac{\partial P}{\partial x} = -\rho\,U_\infty \frac{dU_\infty}{dx}, \tag{4}$$

$$\frac{\partial T_\infty}{\partial x} = \frac{1}{\rho\,C\,p}\frac{\partial P}{\partial x}. \tag{5}$$

By using the usual boundary layer approximations, the governing equations of the hydrodynamic and thermal boundary layers, taking into account the effects of the Pressure work and viscous dissipation, may be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}, \tag{7}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{C\,p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{1}{\rho\,C\,p}u\frac{\partial P}{\partial x}. \tag{8}$$

Where a in Eqs. (1)–(3) is the velocity coefficient. For slip flow, the appropriate physical boundary conditions of Eqs. (8)–(10) are:

$$u(x,\infty)=U_\infty, \tag{9}$$

$$u(x,0)=\lambda\frac{2-\sigma_M}{\sigma_M}\left.\frac{\partial u}{\partial y}\right|_{y=0}, \tag{10}$$

$$v(x,0)=0, \tag{11}$$

$$T(x,\infty)=T_\infty, \tag{12}$$

$$T(x,0)=T_w+\frac{\lambda}{\mathrm{Pr}}\frac{2-\sigma_T}{\sigma_T}\frac{2\gamma}{\gamma+1}\left.\frac{\partial T}{\partial y}\right|_{y=0}. \tag{13}$$

The boundary conditions (12) and (15) represent respectively the velocity-slip and the temperature-jump at the gas-wall interface characterizing the slip flow [18].

In order to transform the above equations (8)–(10) into nonlinear ordinary differential equations we introduce the following similarity transformation:

$$\eta=y\sqrt{\frac{a}{\nu}},\quad \mathrm{K}=\lambda\frac{2-\sigma_M}{\sigma_M}\sqrt{\frac{a}{\nu}}, \tag{14}$$

$$f(\eta)=\frac{1}{x\sqrt{a\nu}}\psi\left(x,y\right),\quad \Theta(\eta)=\frac{T(x,y)-T_\infty}{T_w-T_\infty} \tag{15}$$

$$u*(\eta)=\frac{u(x,y)}{U_\infty}=f'(\eta), \tag{16}$$

$$v*(\eta)=\frac{v(x,y)}{\sqrt{\frac{\nu\,U_\infty}{x}}}=-f(\eta) \tag{17}$$

The variable K defined in (16) is the modified boundary layer Knudsen number or the slip parameter. With the substitutions of the above variables in equations (8)–(10) and using (11)–(15), we obtain the following system of ordinary differential equations with the associated boundary conditions:

$$f'''+f\,f''+1-(f')^2=0, \tag{18}$$

$$\Theta''+\mathrm{Pr}\,f\,\Theta'+\mathrm{Ec}\,\mathrm{Pr}\left[\left(f''\right)^2-f'\,\Theta\right]=0, \tag{19}$$

$$f'(\infty)=1, \tag{20}$$

$$f'(0)=\mathrm{K}\,f''(0), \tag{21}$$

$$f(0)=0, \tag{22}$$

$$\Theta(\infty)=0, \tag{23}$$

$$\Theta(0)=1+\frac{\mathrm{K}}{\mathrm{Pr}}\frac{2\gamma}{\gamma+1}\,\Theta'(0). \tag{24}$$

Where $\mathrm{Ec} = U_\infty{}^2/Cp(T_w - T_\infty)$ is the Eckert number.

Once the solutions of Eqs. (20) and (21) subject to the boundary conditions (22)–(26) determined, we can compute the local Nusselt number which is given by:

$$Nu_x = \frac{q_w}{k(T_w - T_\infty)/x} = -\mathrm{Re}_x^{1/2}\Theta'(0). \tag{25}$$

Where $q_w = -k\frac{\partial T}{\partial y}\Big|_{y=0}$ is the total heat flux from the wall which is given by Fourier's law.

3. Results and Discussion

The system governing boundary layers equations (20)–(21) can be solved numerically by using the Runge–Kutta–Fehlberg fourth-fifth order (RFK45) method. The asymptotic boundary conditions (22) and (25) at $\eta = \infty$ were replaced by those at $\eta = 10.5$ in accordance with standard practice in the boundary layer analysis. The RKF45 algorithm has been well tested for its accuracy and robustness.

Figures 1, 2 and 3 show the variation of the dimensionless temperature profiles inside the boundary layer as a function of Eckert number $\mathrm{Ec} = U_\infty{}^2/Cp(T_w - T_\infty)$, for Pr = 0.7, $\gamma = 10.4$ (air) and for three different values of slip parameter, K = 0, 0.1 and 0.3. Note that for Ec < 0 the flat plate is colder than the free stream, while for Ec > 0 the plate is hotter. It is observed that for fixed values of slip parameter K, increasing the number of Eckert decreases the temperature of the gas inside the boundary layer. This is due to the computational effects of viscous dissipation and pressure work. For positive values of Eckert number Ec > 0 the pressure work term in eq. (21) plays the role of a sink while the viscous dissipation plays the role of a heat source, thus the pressure work dominance leads to a decrease in temperature. However, in the opposite case (for negative values of Ec) the pressure term plays the role of a source while the viscous dissipation plays the role of a sink, and consequently the thermal energy stored in the gas becomes more important. One notes also that when the flow becomes more rarefied the behavior of the temperature remains the same but the thermal boundary layer thickness and the temperature at each point decrease considerably with slip parameter K.

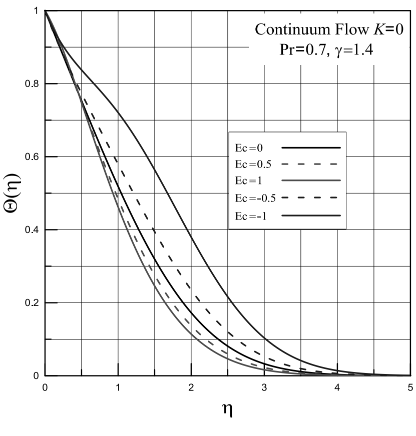


Fig. 1. Effect of Eckert number on the temperature distribution in the thermal boundary layer for $K = 0$ (Continuum flow).

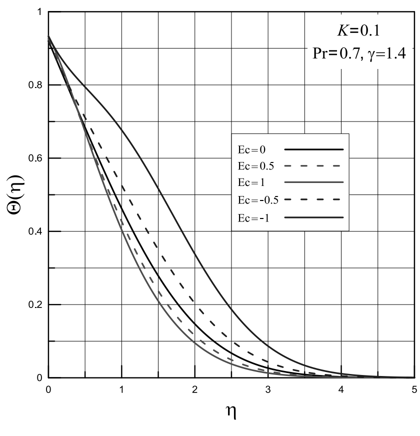


Fig. 2. Effect of Eckert number on the temperature distribution in the thermal boundary layer for $K = 0.1$.

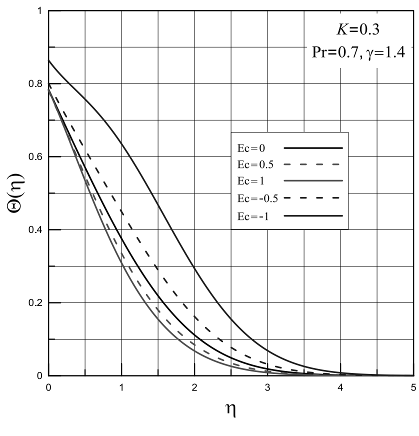
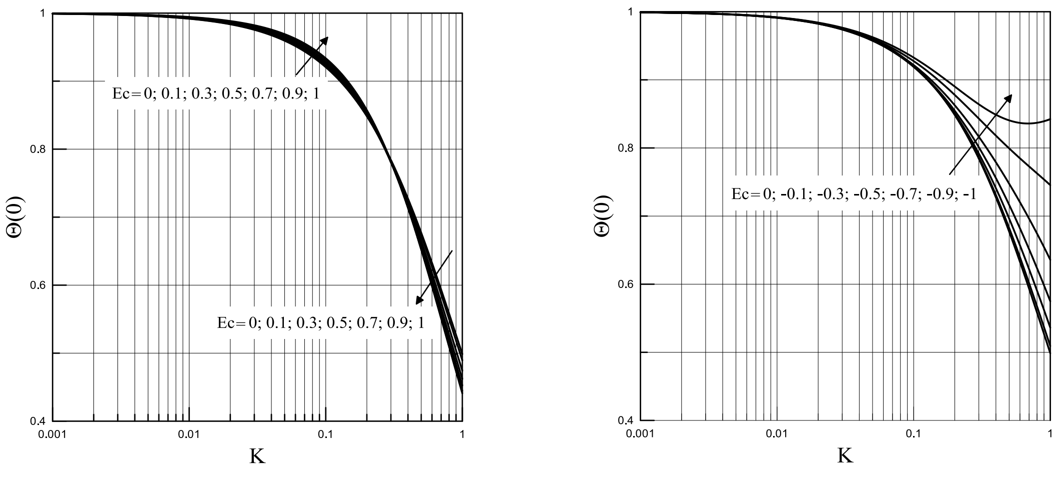


Fig. 3. Effect of Eckert number on the temperature distribution in the thermal boundary layer for $K = 0.3$.

Figure 4 represents the variation of dimensionless gas temperature at the wall $\Theta(0)$ with slip parameter for different values of Eckert number Ec and for Pr = 0.7, $\gamma = 1.4$ and $\sigma_m = \sigma_T$. It can be seen that, for hot plate Ec > 0 the gas temperature at the wall decreases monotonically with K for any fixed value of Ec. By increasing the value of Ec, the combined effect of viscous dissipation and

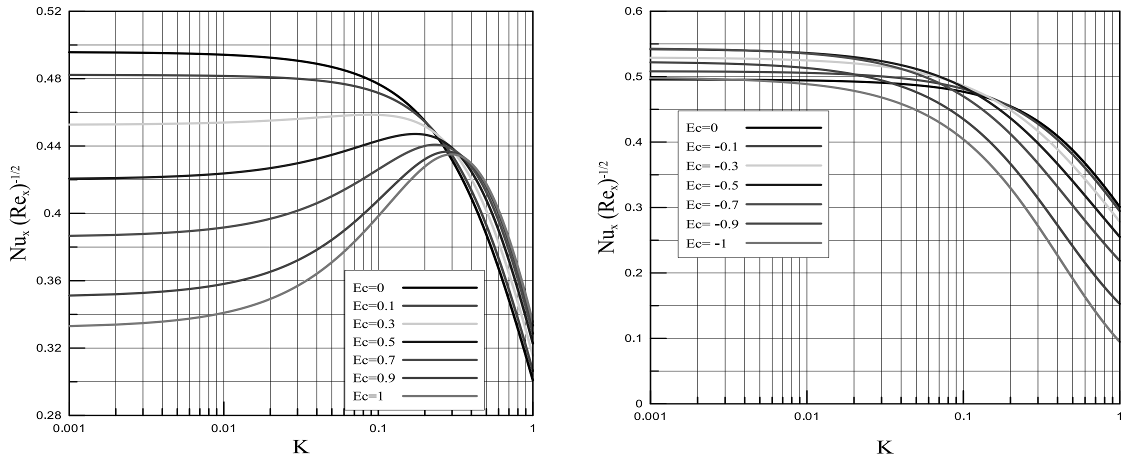


a ***b***

Fig. 4. Effect of Eckert number and slip parameter on the gas temperature at the wall.

pressure work becomes apparent. These effects contributes to slightly decrease the values of the gas temperature at the wall within the interval of continuum ($K < 0.01$) and slip flow ($0.01 < \text{K} < 0.1$) regimes, while according to global energy conservation in the thermal boundary layer, the opposite effect occurs after some value of slip parameter ($K = 0.29$) as the gas becomes more rarefied. The decrease of $\Theta(0)$ for large K ($\text{K} > 0.29$), especially at higher values of Ec, is due to the decrease of temperature gradient at the wall ($\Theta'(0)$) according to Eq. (26). However, in the case of a cold plate hot plate $\text{Ec} < 0$ the temperature at the wall decreases with the variation of the Eckert number Ec for any fixed value of the slip parameter K. This decrease is more important for highly rarefied flow due to the effect of temperature jump that becomes important at wall surface. In addition, a remarkable inflection will appear on the gas temperature at the wall when the number of Eckert is equal to -1. This is particularly due to the effect of pressure work which becomes important, leading to an increase in the level of the heat exchange between the wall and the gas. These results are fully highlighted in Figure 5, in which we have plotted the evolution of the local heat transfer given by Eq. (27) in the same condition of Fig. 4.

Figure 5 shows particularly that for all $\text{Ec} = 0$, the heat transfer exhibits a slight decrease in the interval of slip flow regime and a large decrease as the flow becomes more rarefied. This is due to the temperature jump which reduces the heat transfer and dominates the effect of enhancing by the wall slip velocity. This result is similar to that obtained in Ref. [16] for slip low over a flat plate. Under heating condition $\text{Ec} > 0$, the Nusselt number increases with K and presents a maximum at some value of K depending of Ec, this increase is more pronounced when the Eckert number increases, followed by a large decrease for highly rarefied flow. This maximum is due to the balance between the combined effect of thermal jump condition and pressure work which reduces the Nusselt number and the combined effect of viscous dissipation and velocity slip at the wall which enhance the local heat transfer. We also notice that for any fixed value of slip parameter in the interval of continuum and slip flow regimes, the heat transfer decreases with the increase of Eckert number; while the opposite effect is observed after some value of K when the flow becomes more rarefied. This result permits to understand the behavior of the wall temperature with Eckert number given in Fig. 1. Under cooling condition $\text{Ec} < 0$, the Nusselt number undergoes a slight decrease in the region of the slip flow followed by a large decrease as the flow becomes highly rarefied for any fixed value of Ec. Furthermore, one notes that for values of $\text{Ec} > -0.7$ the heat transfer increases as a function of the decrease of Ec, whereas an inverse behavior is observed at the level of the local transfer for values of $\text{Ec} < -0.7$. This behavior is mainly due to the competition between the combined effect of thermal jump condition and viscous dissipation which reduces the heat exchange and those of pressure work and velocity slip at the wall which favors the heat exchange between the gas and the wall.



a Under heating condition ***b*** Under cooling condition

Fig. 5. Effects of the Eckert number and slip parameter on local heat transfer

4. Conclusion

This study is devoted on extension previous analysis of the fluid flow and heat transfer of the impinging stagnation flow towards a rigid surface or a flat plate, known as the Hiemenz flow, extended to micro-scale in the slip flow regime in the presence of viscous dissipation and pressure work. The numerical results of the governing equations of the hydrodynamic and thermal boundary layers in the slip flow regime are validated with the existing works in the limiting continuum case of no slip and very good agreement are found.

The main results show that the heat transfer and the temperature field in the boundary layer are both governed by the behavior of four different physical aspects: viscous dissipation, pressure work, velocity slip and temperature jump at the wall. The slip boundary effects on the thermal characteristics become more important when the flow is moderately or severely rarefied, while the effect of viscous dissipation and pressure work is related to the direction of heat exchange between the wall and the fluid. As the plate is hotter than the flow, the pressure work plays the role of a sink while the viscous dissipation plays the role of a heat source. On the other hand, when the plate is cooler than the flow the pressure work plays the role of a heat source while the viscous dissipation plays the role of a sink. Thus, we conclude that the coupled action of these different quantities has an important effect on the thermal characteristics of the flow, especially in the presence of rarefaction. And therefore it is necessary to take into account the expressions of the pressure work and the viscous dissipation in the governing equations of the flow.

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Вимушена конвекція ламінарного газоподібного потоку ковзання поблизу точки застою з в'язкою дисипацією та роботою тиску

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Досліджено проблему вимушеної конвекції ламінарного квазінестисливого граничного шару для потоку зі застійним ковзанням за відносно низького числа Маха, враховуючи одночасно ефекти в'язкої дисипації та роботи тиску. Система зв'язаних диференціальних рівнянь у частинних похідних спочатку була перетворена в систему зв'язаних звичайних диференціальних рівнянь за допомогою відповідних перетворень, яка потім була розв'язана за допомогою методу Рунге–Кутта–Фельберга четвертого-п'ятого порядку. Отриманий тут розв'язок набагато краще підходить для формулювання та опису непостійних властивостей хімічно реагуючих потоків, що виникають на практиці, з урахуванням граничних умов ковзання на межі поділу "газ–стінка". Вплив числа Екерта та параметра ковзання на характеристики теплопередачі представлені графічно та обговорені. Чисельні результати показують, що робота тиску та в'язке розсіювання відіграють значну роль у теплопередачі і за будь-яких обставин не можна ними нехтувати для потоків розрідженого газу.

Ключові слова: *ковзання, потік у точці застою, в'язка дисипація, робота тиску, граничний шар, розв'язок подібності.*