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The valuation of knock-out power calls under Black–Scholes framework

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Knock-out power calls are options that incorporate barriers to the valuation of power calls. Introducing barriers to power calls reduces the costs to hold power calls which are known to have higher leverage than the standard vanillas. In this paper, we model the valuation of knock-out power calls using Crank–Nicolson and Monte Carlo simulation under Black–Scholes environment. Results show that Crank–Nicolson is more accurate and more efficient than Monte Carlo simulation for pricing knock-out power calls.

Keywords: *power calls, knock-out power calls, Black–Scholes, Crank–Nicolson, Monte Carlo simulation.*

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1. Introduction

Option is a financial derivative that is based on the value of an underlying security, such as share prices, which can be a European that limits exercise to its expiration date, or an American that allows its holder to exercise at any time before its expiration date (inclusive). Option can be further divided into calls and puts, where the former allows its holder to purchase the underlying at a specific price within a specific time period, and the latter allows its holder to sell the underlying at a specific price within a specific time period.

The basis of an option has been the vanilla option. However, through time, many studies have extended the vanillas to accommodate other investment opportunities, which they referred to as exotic options. One such option is the power option where its payoff at expiry is raised to a power of the underlying price. The work of [1] shows the pricing and hedging of power option and parabola option in particular. The valuation of power option under Black–Scholes model [2] has been obtained using fast Fourier transform, which is an efficient way to price the power option since it is more flexible and reliably fast to price the option since it can produce a range of prices for a range of strikes [3]. Furthermore, [4] studies on general valuation principle for arbitrary payoff and applications for power option under stochastic volatility where they found equivalent martingale measure (EMM) for complete market is equivalent to the physical measure, while in incomplete market, an attainable numeraire for every measure equivalent to the physical measure does not exist. Power options can increase leverage in markets where the underlying trades within narrow limits. In order to make power options cheaper because of its leverage, [5] introduces a barrier to the valuation of power options. Recent studies provided combination of options, such as power exchange options [6], Parisian exchange options [7] and compound exchange options [8].

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In this paper, we aim to estimate the prices of knock-out power call options using Crank-Nicolson and as a benchmark, Monte Carlo simulation, under Black–Scholes framework. Black–Scholes model is a seminal mathematical model for pricing option contracts, which estimates financial derivatives and assumes the derivatives are log-normally distributed. The equation derives the price of a call and puts option by using this assumption and factoring in other important variables. Some studies such as [5] and [9] used Black–Scholes framework to derive pricing formula of exotic options.

The organization of this paper is as follows: Section 2 reviews the closed-form solution for knock-out power call options. Section 3 describes Crank–Nicolson technique for knock-out power calls pricing, while Section 4, Monte Carlo simulation. Section 5 documents some numerical examples, and Section 6 concludes the paper.

2. Knock-out power calls

This section presents the closed-form solution for knock-out power options under Black—Scholes environment. Power option is a non-linear payment option. Let $(\Omega, \mathfrak{F}, \mathbb{Q})$ be a probability space which defines a standard Brownian motion W_t $0 \leqslant t \leqslant T$ that generates a filtration $\mathfrak{F}_t$, $0 \leqslant t \leqslant T$, and $\mathbb{Q}$ is a risk neutral measure, under which the asset price process S_t, $0 \leqslant t \leqslant T$ follows the dynamics below:

$$dS_t = r\, S_t\, dt + \sigma\, S_t\, dW_t.$$

Under Black–Scholes model, the volatility σ and interest rate r are assumed to be constant. Suppose that we raise the underlying asset price to a power of a constant, say β. Itô's lemma implies that S_T^β also follows a geometric Brownian motion [10], such that:

$$dS_T^\beta = \left(\beta\, r + \frac{1}{2}\beta^2\sigma^2 - \frac{1}{2}\beta\,\sigma^2\right) S_T^\beta\, dt + \beta\,\sigma\, S_T^\beta\, dW_t.$$

To reduce notation, we let $Z_t = S_T^\beta$. Then using Itô's lemma, the process followed by the logarithmic asset price $z_t :=\ln S_T^\beta$ is defined by

$$dz_t = \beta\left(r - \frac{1}{2}\sigma^2\right) dt + \beta\,\sigma\, dW_t,$$

with a constant drift $\beta(r - \frac{1}{2}\sigma^2)$ and volatility $\beta\,\sigma$. Therefore, the change in z_t for some time T is normally distributed with mean $\beta(r - \frac{1}{2}\sigma^2)(T-t)$, and variance $\beta^2\sigma^2(T-t)$. Algebraically:

$$\ln S_T^\beta \sim \phi\left[\ln S_T^\beta + \beta\left(r - \frac{1}{2}\sigma^2\right)(T-t), \beta^2\sigma^2(T-t)\right].$$

The payoff function of a power call option is the vanilla call option function that is adjusted by raising the underlying asset to a constant power that is defined by

$$V = (S_T^\beta - K)^+, \tag{1}$$

where Equation (1) is first European power call option. From Equation (1) we get first European call power option as shown below:

$$\mathrm{PC} = e^{-rT} E_Q\left[\left(S_\tau^\beta - K\right)^+\right] = S^\beta e^{\left[(\beta-1)r + \frac{1}{2}\beta(\beta-1)\sigma^2\right]\tau} N\big(d_{1p}\big) - Ke^{-r\tau} N\big(d_{2p}\big), \tag{2}$$

where

$$d_{1p} = \frac{\ln\frac{S^\beta}{K} + \beta(r + \frac{1}{2}\sigma^2)\tau}{\beta\,\sigma\sqrt{\tau}},$$
$$d_{2p} = d_{1p} - \beta\,\sigma\sqrt{\tau}.$$

We consider knock-out power call option where we have S_t as asset price, K as strike price, H is barrier and T is time to expiration. For a down-and-out option, we consider the case where the strike price is higher than the barrier, $K > H$ and for up-and-out option we consider the case where the strike price is lower than the barrier, $K < H$. The payoffs of down-and-out power call DOPC and up-and-out power call UOPC are:

$$\text{DOPC: } \left(S_T^\beta - K\right)^+ \mathbf{1}_{\{m_T > H_d\}}, \tag{3}$$

$$\text{UOPC: } \left(S_T^\beta - K\right)^+ \mathbf{1}_{\{M_T < H_u\}}, \tag{4}$$

where $m_T = \min\{S_t^\beta : t < T\}$ and $M_T = \max\{S_t^\beta : t < T\}$.

The derivation of down-and-out power call option can be obtained by solving its discounted expected payoff given by Equation (5), at a risk-free rate r as follows:

$$\text{DOPC}(K > H) = e^{-r(T-t)} \mathbb{E}^{\mathbb{Q}} \left[\max\left(S_T^\beta - K, 0\right) \mathbf{1}_{\{m_T > H_d\}} \middle| \mathfrak{F}_t \right]. \tag{5}$$

Solving for the expectation requires the restricted density function which are

$$\phi\left(x \middle| m_T > H_d\right) = \begin{cases} f(x) - \left(\frac{H_d}{S_t^\beta}\right)^{2\lambda_\beta} f(x - 2b), & \text{for} \quad x > b; \\ 0, \quad \text{for} \quad x \leqslant b. \end{cases} \tag{6}$$

The price of a down-and-out power call option discounted at risk-free rate r is given as below:

$$\text{DOPC}(K > H) = \text{PC} - \left(\frac{H_d}{S_t^\beta}\right)^{2\lambda_\beta + 2} S_t^\beta e^{\mu_\beta (T-t) + \frac{1}{2}\sigma_\beta^2 (T-t)} N(y_{1p}) - \left(\frac{H_d}{S_t^\beta}\right)^{2\lambda_\beta} K N(y_{2p}), \tag{7}$$

where

$$y_{1P} = y_{2P} + \sigma_\beta \sqrt{T-t}, \qquad y_{2P} = \frac{\ln(H_d^2 / K S_t^\beta) - \mu_\beta (T-t)}{\sigma_\beta \sqrt{T-t}}.$$

Similarly for an up-and-out power call option, this is obtained by solving its discounted expected payoff given by Equation (8), at a risk-free rate r as follows:

$$\text{UOPC}(K < H) = e^{-r(T-t)} \, \mathbb{E}^{\mathbb{Q}} \left[\max\left(S_T^\beta - K, 0\right) \mathbf{1}_{\{M_T < H_u\}} \middle| \mathfrak{F}_t \right]. \tag{8}$$

Solving for the expectation requires the restricted density function which are

$$\phi\left(x \middle| M_T > H_u\right) = \begin{cases} f(x) - \left(\frac{H_u}{S_t^\beta}\right)^{2\lambda_\beta} f(x - 2b), & \text{for} \quad x < a; \\ 0, \quad \text{for} \quad x \geqslant b. \end{cases} \tag{9}$$

The price of an up-and-out power call barrier option discounted at a risk-free rate r:

$$\begin{aligned} \text{UOPC}(K < H) = \text{PC} &+ \left(\frac{H_u}{S_t^\beta}\right)^{2\lambda_\beta + 2} S_t^\beta e^{\mu_\beta (T-t) + \frac{1}{2}\sigma\beta^2 (T-t)} N(y_{1p}) - \left(\frac{H_u}{S_t^\beta}\right)^{2\lambda_\beta} K N(y_{2p}) \\ &- \left[S_t^\beta e^{\mu_\beta (T-t) + \frac{1}{2}\sigma\beta^2 (T-t)} N(Z_{1p}) - K e^{-r(T-t)} (Z_{2p}) \right] \\ &- \left(\frac{H_u}{S_t^\beta}\right)^{2\lambda_\beta} \left[\left(\frac{H_u}{S_t^\beta}\right)^2 S_t^\beta e^{\mu_\beta (T-t) + \frac{1}{2}\sigma\beta^2 (T-t)} N(w_{1p}) - K e^{-r(T-t)} (w_{2p}) \right], \end{aligned} \tag{10}$$

where

$$z_{1P} = z_{2P} + \sigma_\beta \sqrt{T-t}, \qquad z_{2P} = \frac{\ln(S_t^\beta / H_u) + \mu_\beta (T-t)}{\sigma_\beta \sqrt{T-t}},$$

$$w_{1P} = w_{2P} + \sigma_\beta \sqrt{T-t}, \qquad w_{2P} = \frac{\ln(H_u / S_t^\beta) + \mu_\beta (T-t)}{\sigma_\beta \sqrt{T-t}}.$$

3. Crank–Nicolson method

Consider Black–Scholes differential equation:

$$\frac{\partial f}{\partial t} + r\,S\frac{\partial f}{\partial S} + \frac{1}{2}\delta^2 S^2\frac{\partial^2 f}{\partial S^2} = rf, \tag{11}$$

where f is the option value for a given underlying asset price S at a given risk-free interest rate r and constant volatility σ.

Given the explicit and implicit methods, respectively, as follows: implicit scheme is given by:

$$\frac{f_{i+1,j} - f_{i,j}}{\delta t} + rj\delta S\frac{f_{i+1,j+1} - f_{i+1,j-1}}{2\delta S} + \frac{1}{2}\sigma^2 j^2\delta S^2\frac{f_{i+1,j+1} - 2f_{i+1,j} + f_{i+1,j-1}}{\delta S^2} = rf_{i,j}, \tag{12}$$

$$\frac{f_{i+1,j} - f_{i,j}}{\delta t} + rj\delta S\frac{f_{i,j+1} - f_{i,j-1}}{2\delta S} + \frac{1}{2}\sigma^2 j^2\delta S^2\frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{\delta S^2} = rf_{i+1,j}. \tag{13}$$

Taking the average of these two equations, and rearranging terms, yields:

$$-\bar{\alpha}_j f_{i,j-1} + (1-\bar{\beta}_j)f_{i,j+1} - \bar{\gamma}_j f_{i,j+1} = \bar{\alpha}_j f_{i+1,j-1} + (1+\bar{\beta}_j)f_{i+1,j} + \bar{\gamma}_j f_{i+1,j+1}, \tag{14}$$

where

$$\bar{\alpha}_j = \frac{\delta t}{4}\big(\sigma^2 j^2 - rj\big), \tag{15}$$

$$\bar{\beta}_j = -\frac{\delta t}{2}\big(\sigma^2 j^2 + r\big), \tag{16}$$

$$\bar{\gamma}_j = \frac{\delta t}{4}\big(\sigma^2 j^2 + rj\big). \tag{17}$$

Expressing Equation (14) as $Cf_i = Df_{i+1}$ yields the following tridiagonal system:

$$\begin{pmatrix} 1+\bar{\beta}_1 & \bar{\gamma}_1 & & & & \\ \bar{\alpha}_2 & 1+\bar{\beta}_2 & \bar{\gamma}_2 & & & \\ & \bar{\alpha}_3 & 1+\bar{\beta}_3 & \bar{\gamma}_3 & & \\ & & \cdots & \cdots & & \\ & & \bar{\alpha}_{M-2} & 1+\bar{\beta}_{M-2} & \bar{\gamma}_{M-2} & \\ & & & \bar{\alpha}_{M-1} & 1+\bar{\beta}_{M-1} & \end{pmatrix} \begin{pmatrix} f_{i+1,1} \\ f_{i+1,2} \\ f_{i+1,3} \\ \vdots \\ f_{i+1,M-2} \\ f_{i+1,M-1} \end{pmatrix}$$

which can be solved using Thomas algorithm [11].

In order to implement the Crank–Nicolson method, we first set up the grid that contains (t, S) points such that $S = 0, \delta S, \ldots, M\delta S = S_{\max}$ on the horizontal line, and $S = 0, \delta t, \ldots, N\delta S = T$ on the vertical line, where $\delta t = \frac{T}{N}$, $\delta S = \frac{S_{max}}{M}$.

Let $f_{i,j}$ be the grid notation for the option price at point (i, j) on the grid that corresponds to time $i\delta t$ and stock price $j\delta S$. Then we set up the boundary conditions such that the domain for power calls is $f(t, S_{\max}) = 0$ and $f(t, 0) = 0$ with payoff $\big(S_T^\beta - K\big)^+$. For knock-out power calls, the domain for down-and-out power call is $H_d \leqslant S_T^\beta \leqslant S_{\max}$ with boundary conditions $f(t, S_{\max}) = 0$ and $f(t, H_d) = 0$ with payoff $\big(S_T^\beta - K\big)^+ \mathbf{1}_{\{m_T > H_d\}}$, while the domain for up-and-out power call is $H_u \geqslant S_T^\beta \geqslant S_{\max}$ with boundary conditions $f(t, S_{\max}) = 0$ and $f(t, H_u) = 0$ with payoff $\big(S_T^\beta - K\big)^+ \mathbf{1}_{\{M_T < H_u\}}$.

Next, we set up the coefficients as given by Equations (15), (16) and (17), and solve the sequence of linear systems using LU-decomposition. This will return the option price using linear interpolation for asset price that is not equal to a value on the grid.

4. Monte Carlo simulation

In this section, Monte Carlo simulation [12] is utilized to price the knock-out power calls. Given the pricing functions of the knock-out power calls as in Equations (3) and (4), the options are priced at strike price K with maturity T. Suppose $S_{T,j}$ is the asset price at time T on the jth path, the prices of the knock-out power calls are as follows:

$$\mathrm{DOPC}(t, x_T) = \frac{e^{-r(T-t)}}{n} \sum_{j=1}^{n} \left(S_T^\beta - K\right)^+ \mathbf{1}_{\{m_T > H_d\}},$$

$$\mathrm{UOPC}(t, x_T) = \frac{e^{-r(T-t)}}{n} \sum_{j=1}^{n} \left(S_T^\beta - K\right)^+ \mathbf{1}_{\{M_T < H_u\}},$$

where $m_T = \min\{S_t^\beta : t < T\}$, $M_T = \max\{S_t^\beta : t < T\}$, and n is the number of simulations. Let $x_t = \ln S_t$ with the following process:

$$x_t = \left(r - \frac{\sigma^2}{2}\right) dt + \sigma\, dW_t. \tag{18}$$

The asset path is discretized using the Euler scheme as:

$$x_{j+1} = x_j + \left(r - \frac{\sigma^2}{2}\right) \Delta t + \sigma\, \Delta W_j, \tag{19}$$
$$\Delta W_j = W_{t_j} - W_{t_j} = Z\sqrt{\Delta t},$$

with $Z(0,1)$, over time interval $[t,T]$. A random sample is withdrawn from a normal distribution for each $j = 0, 1, \ldots, m$ to simulate (19), thus generates a sample path for x_T by simulating x_j for $j = 1$ to $j - m$. This is repeated to generate many paths to estimate the price of the knock-out power calls.

5. Numerical Results

In this section, a numerical comparison is documented between Crank–Nicolson and the Monte Carlo simulation techniques described earlier. There are three problems that we study: pricing power calls, down-and-out power calls and up-and-out power calls, using Crank–Nicolson and Monte Carlo simulation.

The hypothetical parameters used to price power calls are: $S = 10$, $K = 75$, $\beta = 2$, $r = 0.02$, $\sigma = 0.2$ and $T = 1$. Table 1 and Table 2 document the power call prices obtained via Crank–Nicolson and Monte Carlo simulation, respectively.

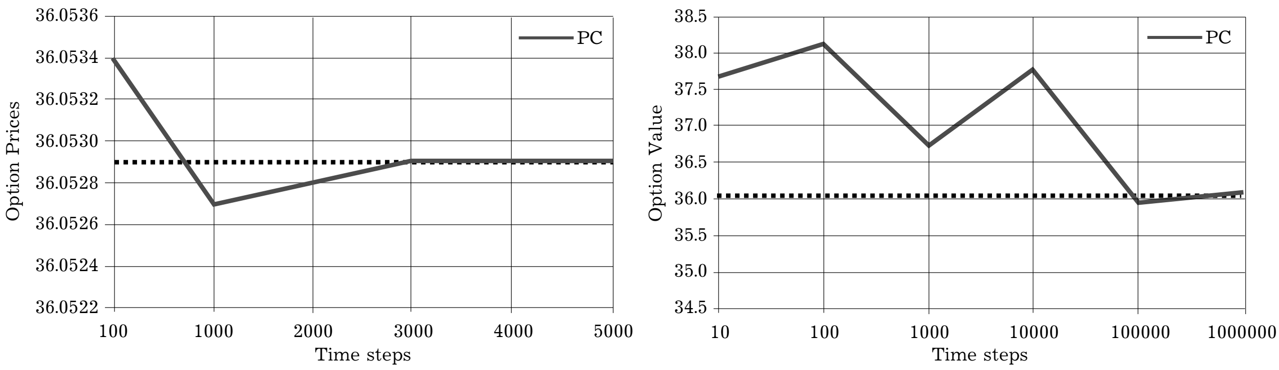
Table 1. Power Call Prices via CN.

$N = M$	$\mathrm{PC}_{\mathrm{CN}}$	Rel Error %
100	36.0775	$6.8233 \cdot 10^{-2}$
1000	36.0534	$1.3869 \cdot 10^{-3}$
2000	36.0527	$5.5474 \cdot 10^{-4}$
3000	36.0528	$2.7737 \cdot 10^{-4}$
4000	36.0529	0

Table 2. Power Call Prices via MCS.

N	$\mathrm{PC}_{\mathrm{MCS}}$	CI	Rel Error %
10^2	38.1229	$(30.2837, 45.9621)$	5.7416
10^3	36.7312	$(34.1087, 39.35760)$	1.8814
10^4	35.7728	$(34.9695, 36.5760)$	0.7769

The hypothetical parameters used to price down-and-out power calls are: $S = 10$, $K = 9$, $\beta = 2$, $r = 0.05$, $\sigma = 0.2$, $T = 1$ and $H_d = 5$. Table 3 and Table 4 document the down-and-out power call prices obtained via Crank—Nicolson and Monte Carlo simulation, respectively.



a Crank–Nicolson. ***b*** Monte Carlo Simulation.

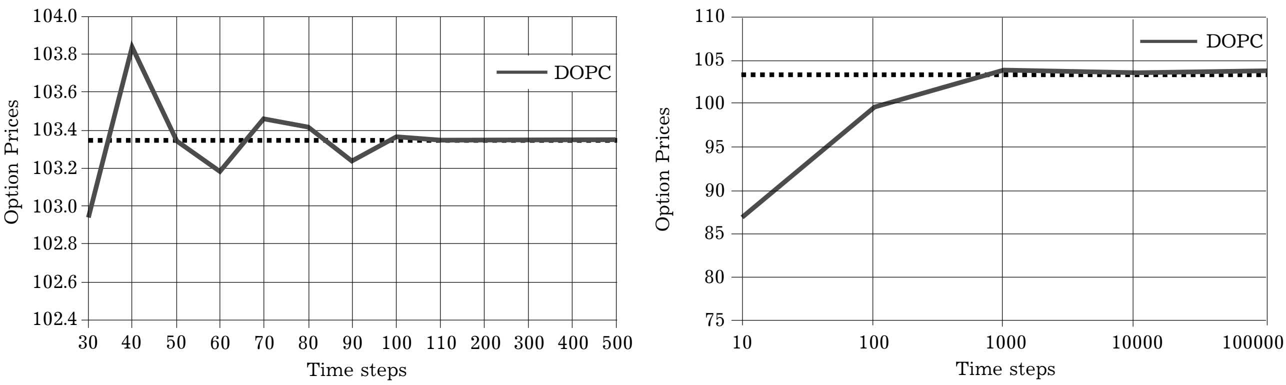
Fig. 1. Power call prices.

Table 3. Down-and-Out Power Call Prices via CN.

$N = M$	DOPC_{CN}	Rel Error %
30	102.9368	$3.9614 \cdot 10^{-1}$
40	103.8386	$4.7646 \cdot 10^{-1}$
50	103.6406	$2.8487 \cdot 10^{-1}$
60	103.1810	$1.5985 \cdot 10^{-1}$
70	103.4614	$1.1147 \cdot 10^{-1}$
80	103.4132	$6.4831 \cdot 10^{-2}$
90	103.2332	$1.0934 \cdot 10^{-1}$
100	103.3666	$1.9739 \cdot 10^{-2}$
110	103.3462	0

Table 4. Down-and-Out Power Call Prices via MCS.

M	DOPC_{MCS}	CI	Rel Error %
10^2	99.8234	$(88.4492, 111.1975)$	3.4087
10^3	101.8983	$(98.3480, 105.4487)$	1.4010
10^4	103.6511	$(102.4793, 104.8228)$	$2.9503 \cdot 10^{-1}$



a Crank–Nicolson. ***b*** Monte Carlo Simulation.

Fig. 2. Down-and-out power call prices.

The hypothetical parameters used to price up-and-out power calls are: $S = 10$, $K = 50$, $\beta = 2$, $r = 0.05$, $\sigma = 0.2$, $T = 1$ and $H_u = 110$. Table 5 and Table 6 document the down-and-out power call prices obtained via Crank–Nicolson and Monte Carlo simulation, respectively.

The relative errors between the closed-form solutions and the other two applied methods are computed as follows:

$$\varepsilon \approx \frac{|P_T - P_A|}{P_T} \cdot 100\%, \tag{20}$$

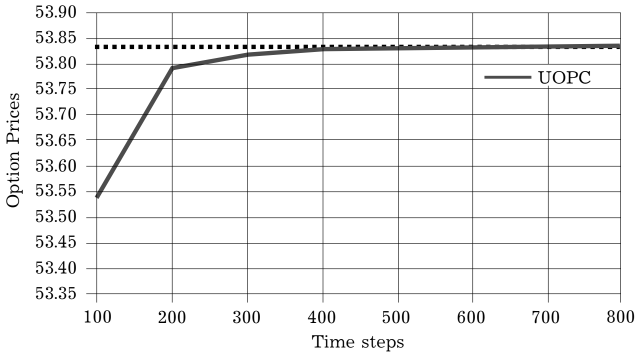
where P_T is the price obtained from the closed-form formula, and P_A is the price obtained from Crank–Nicolson and Monte Carlo simulation. It can be seen from the tables, the relative errors are small, which implies accurate prices are produced by the both methods.

Table 5. Up-and-Out Power Call Prices via CN.

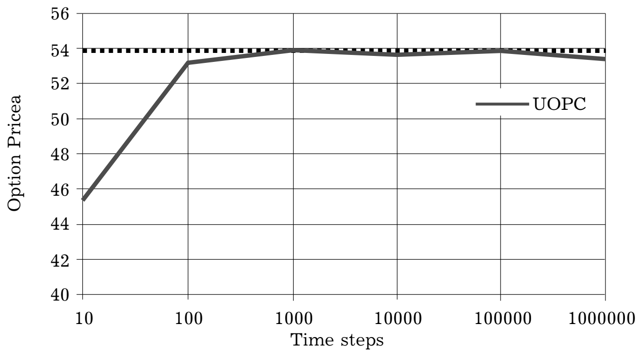
$N = M$	$\mathrm{UOPC_{CN}}$	Rel Error %	Time (seconds)
100	53.6563	$3.2768 \cdot 10^{-1}$	0.0158
200	53.7913	$7.6905 \cdot 10^{-1}$	0.0283
300	53.8166	$2.9907 \cdot 10^{-2}$	0.0726
400	53.8256	$1.3189 \cdot 10^{-2}$	0.1807
500	53.8298	$5.3871 \cdot 10^{-3}$	0.4725
600	53.8322	$9.2880 \cdot 10^{-4}$	0.8224
620	53.8326	$1.8576 \cdot 10^{-4}$	0.8960
621	53.8327	0	0.9187

Table 6. Up-and-Out Power Call Prices via MCS.

M	$\mathrm{UOPC_{MCS}}$	CI	Time (seconds)	Rel Error %
10^2	52.1308	$(43.3074, 60.9543)$	0.7269	2.8928
10^3	51.7587	$(48.8824, 54.6351)$	1.2459	3.8527
10^4	52.9001	$(51.9597, 53.8405)$	13.8557	1.7324



a Crank–Nicolson.



b Monte Carlo Simulation.

Fig. 3. Up-and-out power call prices.

6. Conclusion

This paper develops a pricing framework for power calls and knock-out power calls under Black and Scholes [2] which utilizes Crank–Nicolson technique and Monte Carlo simulation.

Monte Carlo simulation is more straightforward to implement than Crank—Nicolson one; nevertheless the latter produces more accurate prices than the former. Crank–Nicolson is also computationally efficient than Monte Carlo simulation.

Pricing power knock-out options with numerical methods can be difficult and certain methods can be slow to converge. Future research in this area would focus on developing other models to produce computational efficient option prices, such as the fast Fourier transform (FFT) which has been used to study the valuation of various types of options, given the characteristic functions are available in closed-form [13, 14].

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Оцінка степеневих кол опціонів-нокаут за описом Блека–Шоулза

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Степеневі кол опціони-нокаут — це опціони, які включають бар'єри для оцінки опціонів. Введення бар'єрів для опціонів зменшує витрати на утримання опціонів, які, як відомо, мають більший важіль, ніж стандартні ванільні опціони. У цій статті оцінюються степеневі кол опціони-нокаут за допомогою моделювання Кранка–Ніколсона та Монте–Карло в описі Блека–Шоулза. Результати показують, що моделювання Кранка–Ніколсона є більш точним і ефективним, ніж моделювання Монте–Карло, для визначення ціни на степеневі кол опціони-нокаут.

Ключові слова: *степеневі кол опціони, степеневі кол опціони-нокаут, моделювання Блека–Шоулза, Кранка–Ніколсона, Монте–Карло.*