MATHEMATICAL MODELING AND COMPUTING, Vol. 9, No. 2, pp. 385–398 (2022)

Study of two species prey–predator model in imprecise environment with harvesting scenario

Vijayalakshmi T., Senthamarai R.*

Department of Mathematics, College of Engineering and Technology,
SRM Institute of Science and Technology,
Kattankulathur – 603 203, Chengulpattu District, Tamilnadu, India
**Corresponding author: senthamr@srmist.edu.in*

(Received 7 October 2021; Accepted 7 February 2022)

This study proposes and explores a prey–predator model that presents a functional response to group behavior of prey–predator harvesting. We study a non-linear model of prey–predator growths in two species. The proposed model is supported by theoretical and numerical results. Some numerical descriptions are provided to help our analytical and theoretical conclusions. For all possible parameter values occurring in a prey–predator system, we solved it by using both VIM (variational iteration method) and HPM (homotopy perturbation method). We also used MATLAB coding to compare our approximate analytical expressions with numerical simulations. We have found that there is no significant difference when comparing analytical and numerical results.

Keywords: *mathematical modeling, harvested prey–predator, precise value, numerical simulation, variational iteration method, homotopy perturbation method.*

2010 MSC: 65L05, 34A34, 93C10 **DOI:** 10.23939/mmc2022.02.385

1. Introduction

Predator–prey interaction and its possible consequences are, possibly, the most studied topics in ecology. The functional response of predators, which is defined as the number of prey eaten by predator per unit time, plays an important role in population ecology because of its profound implications for population dynamics [1]. In theoretical ecology, there are two types of functional responses, namely the prey-dependent functional response and the predator-dependent functional response. The predation rate per capita depends only on the number of prey, the Holling Types III [2] (i.e.,) "the functional response of predators would be a function of the density of the prey in this case number of prey and predators, DeAngelis functional response".

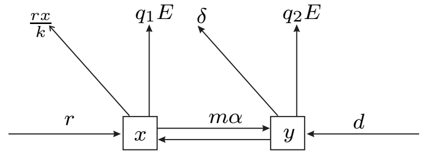


Fig. 1. Schematic diagram of prey–predator model with harvesting.

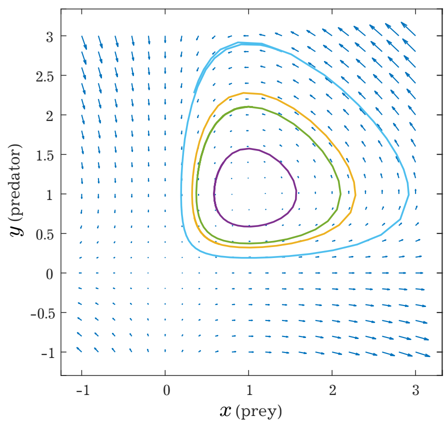


Fig. 2. Phase plane of the system of prey–predator populations.

Recently, G. Liu et. al [3] studied a predator–prey model in which two ecological interacting species are harvested independently with constant rates [4–7]. Harvesting in a ratio-dependent predator–prey model is relatively a new issue and has been studied very recently by some researchers. The complex dynamics of the proposed model system, existence and stability criteria of various equilibrium points, uniform boundedness, including positivity, were discussed. With best of our knowledge, there is no closed form analytical expression of the proposed prey–predator model [8–10].

Sensitivity analysis can be used to look at time-dynamical aspects, which is more suitable for applications that require estimations of model uncertainties, like parameter estimation and model identification, and for the selection of parameters that make a considerable contribution to ecological model output of interest. Local sensitivities, however, are restricted to the analysis around specific parameter values [11]. In this paper, we have derived approximate analytical expressions for prey and predator involved in a non-linear model with numerical simulation by using VIM and HPM. Also, we compared our analytical solutions and found that there is no much more significant difference between our analytical solution and numerical simulation. Our approximate analytical expressions will be useful for the harvesting prey and predator optimizing the parameters on the performances. In Fig. 1, it refers a harvesting prey–predator model for a schematic representation. Harvesting effort E is combined with the catch-ability coefficient q_1 and q_2 of prey and predator species. Phase plane of the prey–predator population system is given in Fig. 2.

2. Mathematical model

Consider a two-species system with harvested, requiring E as the harvesting effort for the two species and q_1 and q_2 as the catch capacity coefficient of the prey and predator species [10]. We can write a system of prey–predator with harvesting as follows:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - \alpha xy - q_1 E\,x, \tag{1}$$

$$\frac{dy}{dt} = m\alpha xy - dy - \delta y^2 - q_2 Ey \tag{2}$$

with the following initial conditions

$$x(t) \geqslant 0, \quad y(t) \geqslant 0. \tag{3}$$

Table 1. Nomenclature.

Parameters	Descriptions	Values
x	prey population	—
y	predator population	—
r	the intrinsic growth rate	$4 - 6$
k	the carrying capacity	$800 - 1000$
m	the conversion factor	$0.6 - 0.8$
d	the natural death rate	$0.10 - 0.20$
α	the predation rate	$0.6 - 0.8$
δ	intra-specific competition rate	$0.0001 - 0.002$
q_1 and q_2	the catch-ability coefficient of the both species	$q_1 = 0.02$, $q_2 = 0.001$
E	harvesting effort for both species	1.4

3. Mathematical analysis

3.1. Approximate analytical expression of Eqs. (1)–(3) using HPM

Several authors using the HPM to different non-linear problems established the effectiveness of the HPM for solving various engineering, physical, chemical and biological problems [12–16]. This method plays a vital role in bio-mathematical sciences. Approximate analytical expressions of prey and predator populations are very useful to understand their interactions. We obtain the following solution for the system of Eqs. (1)–(3) by using HPM (refer to Appendix A)

$$
\begin{aligned}
x(t) &= m_3 e^{(r-q_1E)t} + \left(\frac{rm_3^2}{k(r-q_1E)} - \frac{\alpha m_3 m_4}{d+q_2E}\right) e^{(r-q_1E)t} - \frac{rm_3^2}{k(r-q_1E)} e^{2(r-q_1E)t} + \frac{\alpha m_3 m_4}{d+q_2E} e^{(r-q_1E-d-q_2E)t} \\
&+ \left(\begin{array}{l}
\frac{2rm_3^3}{k(r-q_1E)^2} - \frac{2\alpha m_3^2 m_4}{(d+q_2E)(r-q_1E)} - \frac{rm_3^3}{k(r-q_1E)^2} + \frac{2\alpha m_3^2 m_4}{(d+q_2E)(r-q_1E-d-q_2E)} \\
-\frac{\alpha^2 m m_3^2 m_4}{(r-q_1E)(-d-q_2E)} + \frac{\delta \alpha m_3 m_4^2}{(d+q_2E)^2} + \frac{\alpha^2 m m_3 m_4}{(r-q_1E)(r-q_1E-d-q_2E)} - \frac{\delta \alpha m_3 m_4^2}{2(d+q_2E)^2} \\
+\frac{\alpha m_3^2 m_4}{k(r-q_1E)(d+q_2E)} + \frac{\alpha^2 m_3 m_4^2}{2(d+q_2E)^2} - \frac{\alpha m m_3^2 m_4 r}{k(r-q_1E)(-d-q_2E+r-q_1E)} - \frac{\alpha^2 m_3 m_4^2}{2(d+q_2E)^2}
\end{array}\right) - \frac{2rm_3^3}{k(r-q_1E)^2} e^{2(r-q_1E)t} \\
&+ \frac{2\alpha m_3^2 m_4}{(d+q_2E)(r-q_1E)} e^{2(r-q_1E)t} + \frac{rm_3^3}{k(r-q_1E)^2} e^{3(r-q_1E)t} - \frac{2\alpha m_3^2 m_4}{(d+q_2E)(r-q_1E-d-q_2E)} e^{(2r-2q_1E-d-q_2E)t} \\
&+ \frac{\alpha^2 m m_3^2 m_4}{(r-q_1E)(-d-q_2E)} e^{(r-q_1E-d-q_2E)t} - \frac{\delta \alpha m_3 m_4^2}{(d+q_2E)^2} e^{(r-q_1E-d-q_2E)t} - \frac{\alpha^2 m m_3 m_4}{(r-q_1E)(r-q_1E-d-q_2E)} e^{(2r-2q_1E-d-q_2E)t} \\
&+ \frac{\delta \alpha m_3 m_4^2}{2(d+q_2E)^2} e^{(r-q_1E-2d-2q_2E)t} + \frac{\alpha m_3^2 m_4 r}{k(r-q_1E)(d+q_2E)} e^{(r-q_1E-d-q_2E)t} - \frac{\alpha^2 m_3 m_4^2}{2(d+q_2E)^2} e^{(r-q_1E-d-q_2E)t} \\
&+ \frac{\alpha m m_3^2 m_4 r}{k(r-q_1E)(-d-q_2E+r-q_1E)} e^{(2r-2q_1E-d-q_2E)t} + \frac{\alpha^2 m_3 m_4^2}{2(d+q_2E)^2} e^{(r-q_1E-2d-2q_2E)t}, \qquad (4)
\end{aligned}
$$

$$
\begin{aligned}
y(t) &= m_4 e^{-(d-q_2E)t} + \left(\frac{-\alpha m m_3 m_4}{(r-q_1E)} - \frac{\delta m_4^2}{(d+q_2E)}\right) e^{-(d-q_2E)t} + \frac{\alpha m m_3 m_4}{(r-q_1E)} e^{(r-q_1E-d-q_2E)t} \\
&+ \frac{\delta m_4^2}{(d+q_2E)} e^{-2(d-q_2E)t} + \left(\begin{array}{l}
\frac{m^2\alpha^2 m_3^3 m_4}{k(r-q_1E)^2} - \frac{\alpha\delta m_3 m_4^2}{(d+q_2E)(r-q_1E)} - \frac{m^2\alpha^2 m_3^3 m_4}{2(r-q_1E)^2} - \frac{\delta\alpha m m_3 m_4^2}{(d+q_2E)(r-q_1E-d-q_2E)} \\
-\frac{\alpha r m m_3^2 m_4}{k(r-q_1E)^2} + \frac{\alpha^2 m m_3 m_4^2}{(d+q_2E)(r-q_1E)} + \frac{\alpha r m m_3 m_4^2}{2k(r+q_1E)^2} + \frac{\alpha^2 m m_3 m_4}{(d+q_2E)(r-q_1E-d-q_2E)} \\
-\frac{2\delta\alpha m m_3 m_4^2}{(-d-q_2E)(r-q_1E)} + \frac{2\delta^2 m_4^3}{2(d+q_2E)^2} - \frac{2\delta\alpha m m_3 m_4^2}{(r-q_1E)(-d-q_2E+r-q_1E)} - \frac{\delta^2 m_4^2}{(d+q_2E)^2}
\end{array}\right) e^{-(d+q_2E)t} \\
&- \frac{m^2\alpha^2 m_3^3 m_4}{(r-q_1E)^2} e^{(r-q_1E-d-q_2E)t} + \frac{\alpha\delta m m_3 m_4^2}{(d+q_2E)(r-q_1E)} e^{(r-q_1E-d-q_2E)t} + \frac{m^2\alpha^2 m_3^3 m_4}{2(r-q_1E)^2} e^{(2r-2q_1E-d-q_2E)t} \\
&- \frac{\delta\alpha m m_3 m_4^2}{(d+q_2E)(r-q_1E-d-q_2E)} e^{(r-q_1E-2d-2q_2E)t} + \frac{\alpha r m m_3^2 m_4}{k(r-q_1E)^2} e^{(r-q_1E-d-q_2E)t} - \frac{\alpha^2 m m_3 m_4^2}{(d+q_2E)(r-q_1E)} e^{(r-q_1E-d-q_2E)t} \\
&- \frac{\alpha r m m_3^2 m_4}{(r-q_1E)(r-q_1E-d-q_2E)} e^{(2r-2q_1E-d-q_2E)t} + \frac{\delta\alpha m_3 m_4^2}{2(d+q_2E)^2} e^{(r-q_1E-2d-2q_2E)t} + \frac{\alpha m_3^2 m_4 r}{2k(r-q_1E)^2} e^{(-d-q_2E+2r-2q_1E)t} \\
&- \frac{m\alpha^2 m_3 m_4}{(d+q_2E)(r-q_1E-d-q_2E)} e^{(-2d-2q_2E+r-q_1E)t} + \frac{2\delta\alpha m m_3 m_4^2}{(-d-q_2E)(r-q_1E)} e^{-2(d+q_2E)t} + \frac{2\delta^2 m_4^3}{2(d+q_2E)^2} e^{(-2d-2q_2E+r-q_1E)t} \\
&- \frac{2\delta\alpha m m_3 m_4^2}{(r-q_1E-d-q_2E)(r-q_1E)} e^{(-2d-2q_2E+r-q_1E)t} + \frac{2\delta^2 m_4^3}{2(d+q_2E)^2} e^{-3(d+q_2E)t}. \qquad (5)
\end{aligned}
$$

3.2. Approximate analytical expression of Eqs. (1)–(3) using VIM

The variational iteration method (VIM) established by Ji-Huan He is now used to handle a wide variety of linear and nonlinear problems. This method provides rapidly convergent successive approximations of the exact solution [17–20]. By using VIM (refer to Appendix B), we obtain the following solution for the system of Eqs. (1)–(3),

$$
\begin{aligned}
x(t) &= c_1 e^{-(r+q_1E)t} + \frac{c_1(r+q_1E)}{2r}\left(e^{-(r+q_1E)t} - e^{-(r-q_1E)t}\right) - \frac{c_1}{2}\left(e^{-(r+q_1E)t} - e^{-(r-q_1E)t}\right) \\
&+ \frac{rc_1^2}{k(q_1E+3r)}\left(e^{-2(r+q_1E)t} - e^{-(r-q_1E)t}\right) + \frac{q_1Ec_1}{2r}\left(e^{-(r+q_1E)t} - e^{-(r-q_1E)t}\right) \\
&+ \frac{\alpha c_1 c_2}{q_1E+d+2r}\left(e^{-(r+q_1E+q_2E+d)t} - e^{(r-q_1E)t}\right), \qquad (6)
\end{aligned}
$$

$$
\begin{aligned}
y(t) &= c_2 e^{-(d+q_2E)t} + c_2 t(d+q_2E) - \frac{m\alpha c_1 c_2}{(r+q_1E)}\left(e^{-(r+q_1E+d+q_2E)t} - e^{(-q_1E+r)t}\right) \\
&- dc_2 t(e^{-(q_2E+d)t} + \frac{\delta c_2^2}{(q_2E+d)}(e^{-2(d+q_2E)t} - e^{-(d+q_2E)t}) - q_2Ec_2 e^{-(d+q_2E)t}. \qquad (7)
\end{aligned}
$$

4. Numerical simulation

Very new and closed expressions for the harvesting prey and predator populations are presented in Eqs. (4)–(7). We have compared our analytical solutions obtained by HPM and VIM with the numerical simulation obtained by MATLAB software. To test the accuracy of our approximate analytical expressions, the system (1)–(3) is also solved numerically for all possible values of parameters. The MATLAB program is also given in Appendix C. When we compared our analytical results with numerical simulations, we found that the overall average error does not exceed 0.6%.

5. Discussion

Equations (4)–(7) are very new expressions for harvesting prey–predator populations. They are satisfying initial conditions. In Figs. 3–5 and Tables 2, 3, we have presented and compared the harvesting prey populations for fixed values of imprecise parameters r, k, α, d, δ, m, q_1, q_2 and E given in Table 1 by Homotopy perturbation method & Variational iteration method with numerical simulation using MATLAB software. In Fig. 3a, when r increases, the populations of prey also increases. In Figs. 4, 5, when k and α increase, there is no variation in the prey population. In Figs. 6–9 and Tables 4–9, the harvesting predator population is obtained by using both methods are compared with numerical simulation using MATLAB software. In Fig. 6, when d increases, the predator population is decreasing. In Fig. 7, when the parameter d increases, the population of predator also increases. In Fig. 8, when m increases, the population of predator also increases. In Fig. 9, when the parameter δ increases, there is no variation in the population of the predator.

In Figs. 5a, 5b, we present the fixed values of parameters r, k, d, δ, m, q_1, q_2 and E for the harvesting prey and predator populations. In Fig. 5a, we have compared our approximate analytical expressions obtained by HPM with numerical simulation and in Fig. 5b we have compared our approximate analytical expressions obtained by VIM with numerical simulation. In both figures, our approximate analytical expressions accurately match the numerical simulation. From Fig. 6, it is inferred that when r varies from 4 to 6, the analytical results of prey obtained by the Homotopy perturbation method & Variational iteration method slightly deviate from the numerical solution, and their errors by VIM are smaller than by HPM, as shown in Table 5. In Fig. 7, when d varies from 0.10 to 0.20, α varies from 0.6 to 0.8 and δ varies from 0.001 to 0.002, the analytical results of the predator population are that the differences among VIM, HPM and the numerical solution are negligible and their errors are shown in Tables 6–8. In Fig. 8, the α parameters r, k and α against time t increase as the population of prey also increases. In Fig. 9a, the parameter d against time t increases as the population of predator decreases. In Figs. 9b–9d, parameter α, m and δ against time t increase as the population of predator also increases. Finally, these methods can be easily applied to obtain the approximate analytical solution of the nonlinear equation. From all the above figures and tables, we can observe that our approximate analytical expressions are more useful to analyze the harvesting prey–predator populations in this descriptive model and we can apply the same procedure to obtain approximate analytical solution for other nonlinear systems also.

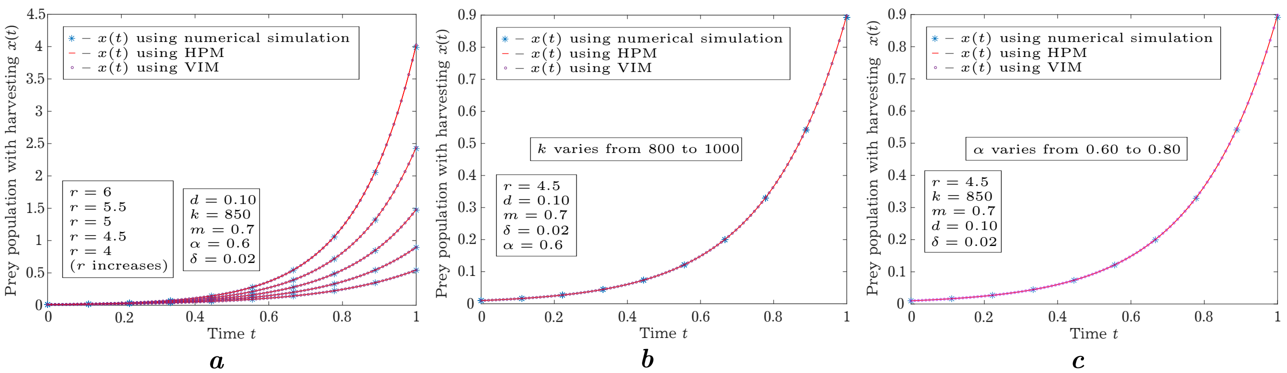


Fig. 3. Harvesting prey population series using various values of the parameters: (***a***) when $r = 4$ to 6 and other parameters k, α, d, δ, m, q_1, q_2 and E are fixed, (***b***) when $k = 800$ to 1000 and other parameters r, α, d, δ, m, q_1, q_2, E are fixed and (***c***) when $\alpha = 0.50$ to 0.75 and other parameters r, k, d, δ, m, q_1, q_2, E are fixed.

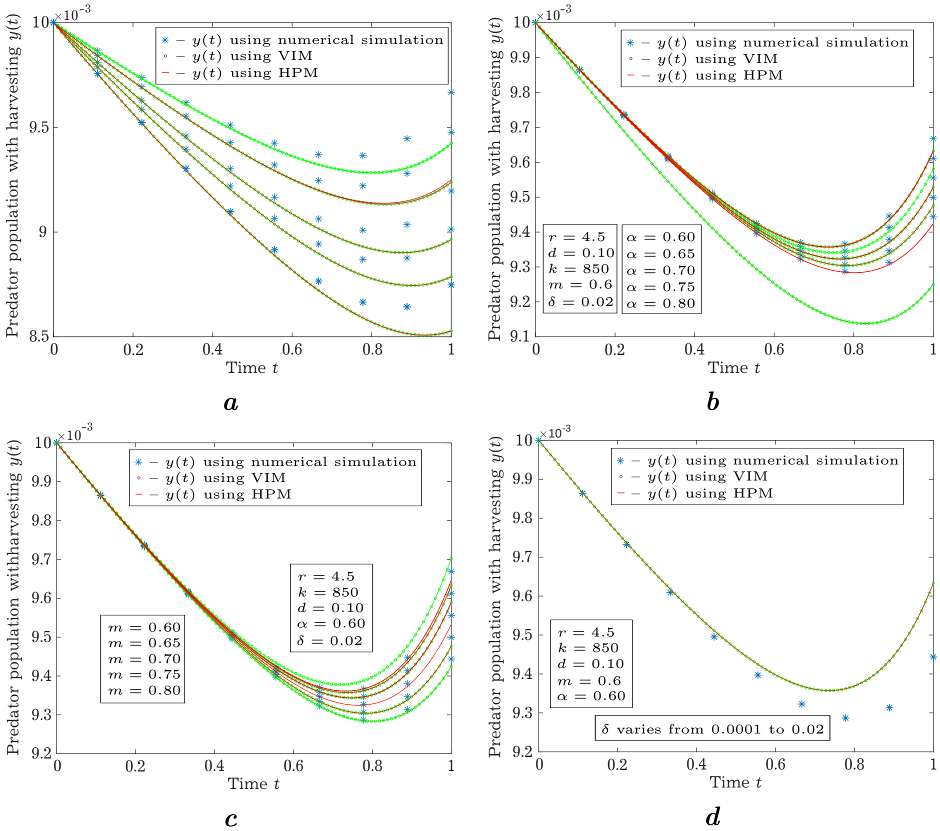


Fig. 4. Harvesting predator population series using various values of the parameters: (***a***) when $d = 0.10$ to 0.20 and other parameters k, α, r, δ, m, q_1, q_2, E are fixed, (***b***) when $\alpha = 0.50$ to 0.75 and other parameters r, k, d, δ, m, q_1, q_2, E are fixed, (***c***) when $m = 0.60$ to 0.80 and other parameters r, α, d, δ, m, q_1, q_2, E are fixed and (***d***) when $\delta = 0.0001$ to 0.002 and other parameters r, k, d, α, m, q_1, q_2, E are fixed.

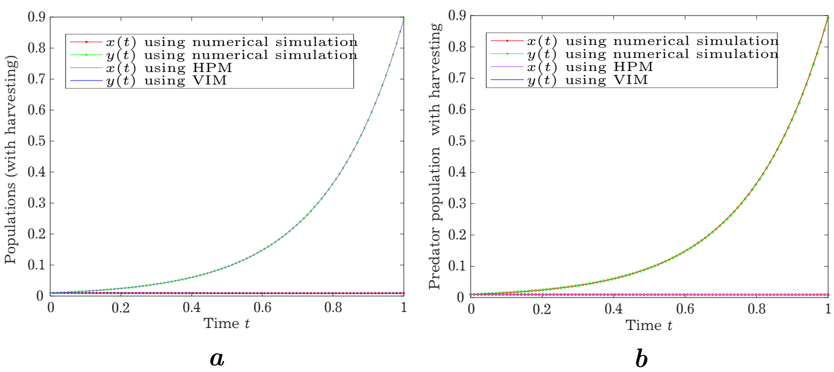


Fig. 5. Analytical expression and numerical simulation results are compared; (***a***) Harvesting prey population and (***b***) Harvesting predator population.

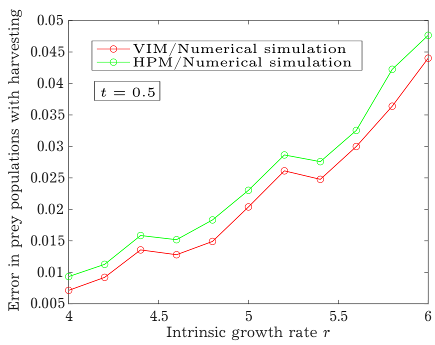


Fig. 6. Comparison of error in the harvesting prey population obtained by HPM and VIM with the numerical simulation at $t = 0.5$.

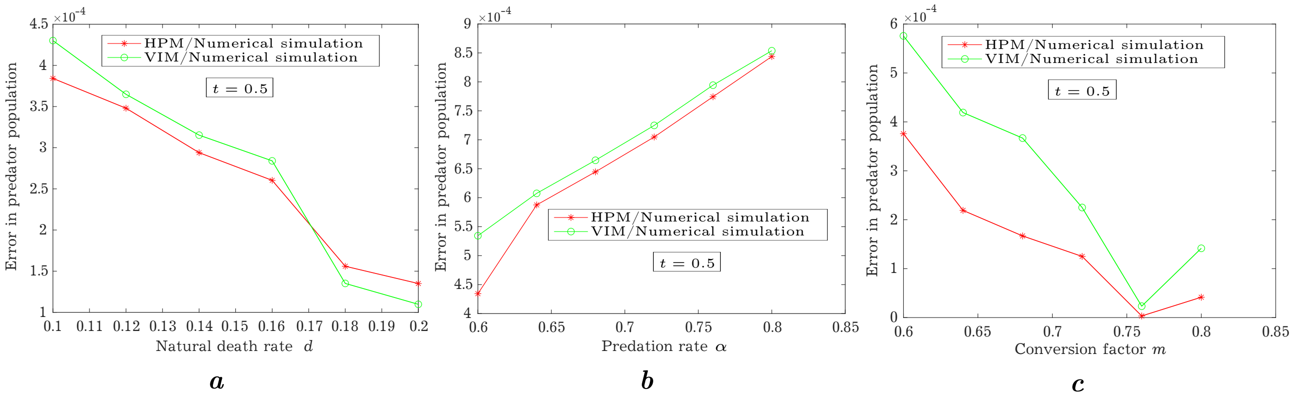


Fig. 7. Comparison of error in the harvesting predator population obtained by HPM and VIM with the numerical simulation at $t = 0.5$.

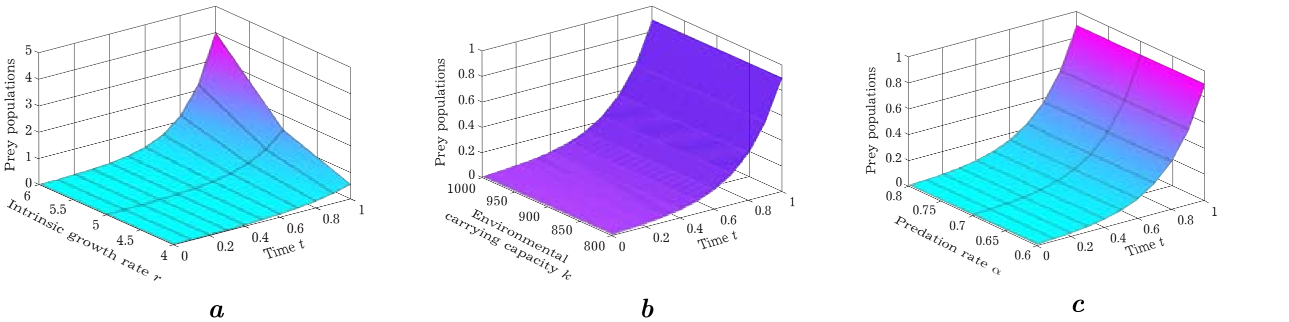


Fig. 8. Prey populations series using various values of the parameters r, k and α.

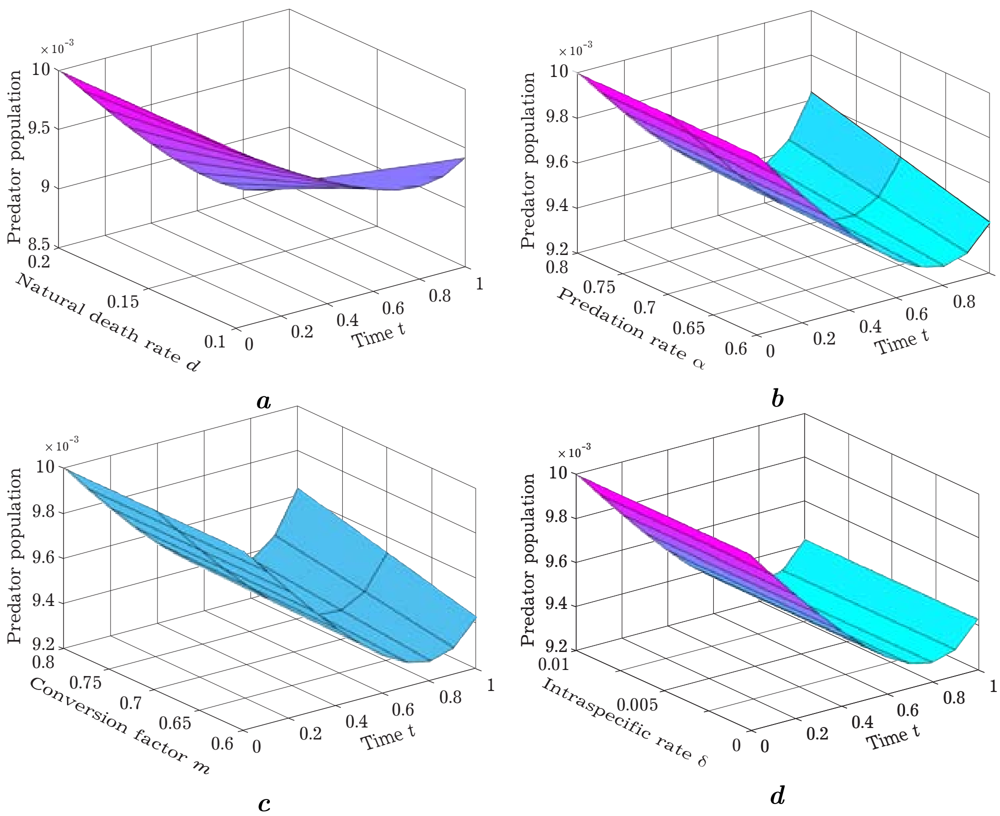
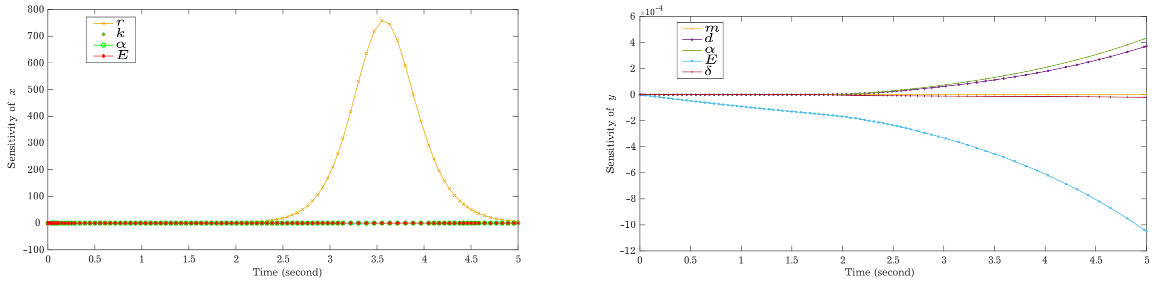


Fig. 9. Predator population series using various values of the parameters d, α, m and δ.

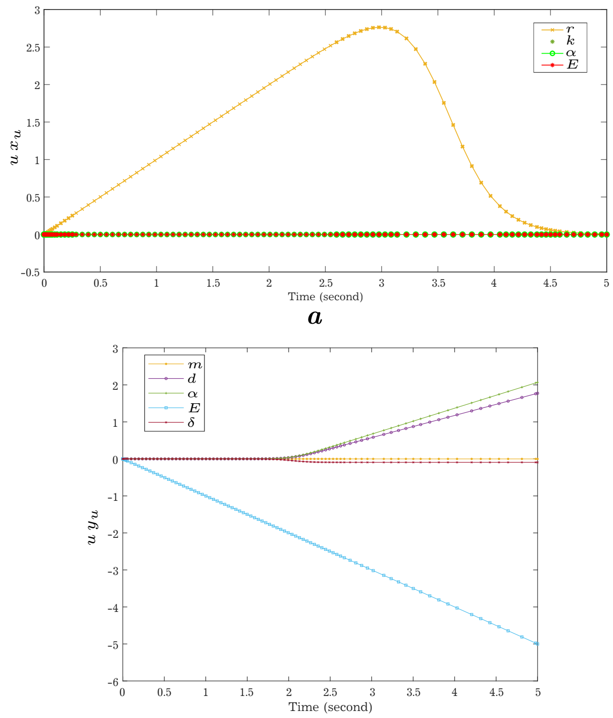
6. Sensitivity analysis

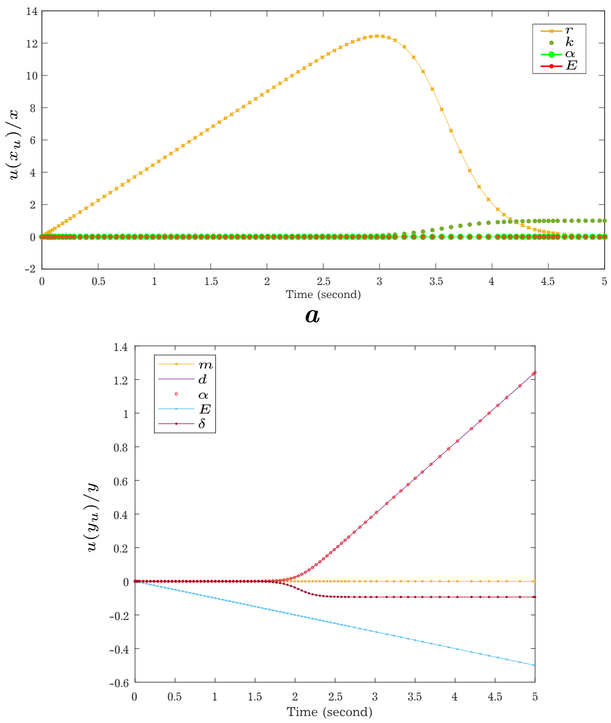
For Eqs. (1) and (2), sensitivity index solutions determine the changes that doubling the parameters yields in the value of the state variable. In Fig. 10, it is seen that doubling the intrinsic growth rate r of prey species, the population of prey increases and attains its maximum of 600 in 3 days and decreases to 0.01 mgl^{-1} at the end of 5 days. Prey population increases by 10 mgl^{-1} on doubling the carrying capacity k, and it shows a slight increase on doubling the predation rate α and harvesting effort E. Further, doubling the parameter d increases, decreasing the predator population by 0.001 in 5 days. The intra specific competition rate δ, predation rate α and harvesting effort E yields a negligible decrease in the predator population.



a ***b***

Fig. 10. Sensitivity analysis of (***a***) Prey population and (***b***) Predator population.





b ***b***

Fig. 11. Semi-relative sensitivity of (***a***) Prey population and (***b***) Predator population.

Fig. 12. Logarithmic sensitivity of (***a***) Prey population and (***b***) Predator population.

The semi-relative sensitivity as well as logarithmic solutions (i.e., $u\,x_u(t,x)$) determines the changes that doubling a parameters yields in the value of a state variable. From Fig. 11, it is doubling the parameter r is increases the prey population also increases by 2.5 in 3 days and again decreases to by 0.0001 in end of 5 days. Also, on doubling the parameter k, α and E, the prey population increases slightly. The predator population shows a slight change on doubling the parameter δ, d and α. However, the predator population decreases by 5 in 5 days on doubling the parameter E and increases by 3 in 5 days for the parameter m.

Moreover, logarithmic sensitivity solutions (i.e., $\frac{w}{X(t,w)} X_w(t,w)$, calculate the percentage change in the value of a state variable induced by doubling a parameter. In Fig.10, it is seen that doubling the parameter r of prey species, prey population increases to a maximum of 1000% in 3 days and decreases to 0.01% at the end of 5 days. Also, the population increases by 100% in 5 days by doubling the parameter k. The predation rate α and harvesting effort E shows a slight increase in the prey population. Further, doubling the parameter d, the predator population decreases by 50% in 5 days and the parameters δ, α and E, yields a slight decrease in the predator population.

7. Error approximations

Table 2. Error estimates for the harvesting prey population $x(t)$ for various values of r versus time t.

t	$r = 4.0$				
	HPM	VIM	Numerical	Error in HPM	Error in VIM
0	0.00963800	0.00963200	0.00968100	0.44416899	0.50614605
0.2	0.02203278	0.02204409	0.02205314	0.09232245	0.04103724
0.4	0.04888202	0.04896137	0.04915428	0.55388869	0.38944726
0.6	0.10807047	0.10836992	0.10822989	0.14729757	0.12938200
0.8	0.23259898	0.23255146	0.23233271	0.11460719	0.09415376
1.0	0.52706814	0.52991798	0.52835514	0.24358788	0.29579344
Mean Error				0.26597879	0.24265995
t	$r = 6.0$				
	HPM	VIM	Numerical	Error in HPM	Error in VIM
0	0.00239300	0.00239400	0.00239020	0.11714500	0.15898251
0.2	0.03638201	0.03631418	0.03640420	0.18632465	0.24727916
0.4	0.11755129	0.11753120	0.11752845	0.01943359	0.23398589
0.6	0.34589648	0.34821684	0.34852847	0.75517216	0.08941306
0.8	1.20094192	1.20089372	1.20092048	0.17852972	0.00222829
1.0	3.90862867	3.90642882	3.91300209	0.11176635	0.16798534
Mean Error				0.228061912	0.14997904

Table 3. The results of intrinsic growth rate r versus harvesting prey population $x(t)$ at $t = 0.5$.

S.No	r	HPM	VIM	Numerical	Error in HPM	Error in VIM
1.	4.0	0.07280212	0.07296312	0.07348725	0.009323114	0.007132258
2.	4.4	0.08907775	0.08928450	0.09051175	0.015843247	0.013559013
3.	4.8	0.10910625	0.10948575	0.11114225	0.018318866	0.014904323
4.	5.2	0.13385775	0.13420550	0.13780500	0.028643736	0.026120242
5.	5.6	0.16461675	0.16505150	0.17015350	0.032539736	0.02998469
6.	6.0	0.20301800	0.20379301	0.21317700	0.047655235	0.044019758
Mean Error					0.025387322	0.022620047

Table 4. Error estimates for the harvesting predator population $y(t)$ for various values of d versus time t.

t	$d = 0.10$				
	HPM	VIM	Numerical	Error in HPM	Error in VIM
0	0.01000000	0.01000400	0.01000500	0.04997501	0.00999500
0.2	0.00981249	0.00981156	0.00981074	0.01783750	0.00868906
0.4	0.00964738	0.00964908	0.00965037	0.01328446	0.01336736
0.6	0.00953183	0.00952852	0.00952995	0.01972722	0.01500532
0.8	0.00953062	0.00952245	0.00953067	0.00524622	0.08624787
1.0	0.00981398	0.00976408	0.00980880	0.05280972	0.04559103
Mean Error				0.02648002	0.02977279
t	$d = 0.20$				
	HPM	VIM	Numerical	Error in HPM	Error in VIM
0	0.01000000	0.01000000	0.01000000	0.00000000	0.00000000
0.2	0.00961815	0.00962170	0.00961816	0.03680537	0.00044707
0.4	0.00926896	0.00927549	0.00926901	0.06991037	0.00051785
0.6	0.00897505	0.00898500	0.00897649	0.01604190	0.01604190
0.8	0.00878970	0.00880531	0.00879751	0.00886614	0.08877511
1.0	0.00883986	0.00886822	0.00887966	0.12883376	0.44821535
Mean Error				0.04340959	0.09233288

Table 5. The results of natural death rate d versus harvesting predator population $y(t)$ at $t = 0.5$.

S.No	d	HPM	VIM	Numerical	Error in HPM	Error in VIM
1.	0.10	0.00957803	0.00958045	0.00947176	0.011219668	0.011475164
2.	0.12	0.00944520	0.00948522	0.00937966	0.01114746	0.011254139
3.	0.14	0.00937867	0.0093881	0.00928551	0.01072836	0.011048397
4.	0.16	0.00929685	0.00929765	0.00919816	0.01068932	0.010816294
5.	0.18	0.00920431	0.00920529	0.00911250	0.01047517	0.010182716
6.	0.20	0.00954467	0.00954615	0.00901896	0.01008960	0.058453525
Mean Error					0.01072493	0.018871706

Table 6. Error estimates for the harvesting predator population $y(t)$ for various values of α versus time t.

t	$\alpha = 0.60$				
	HPM	VIM	Numerical	Error in HPM	Error in VIM
0	0.01000000	0.01000000	0.01000000	0.00000000	0.00000000
0.2	0.00981600	0.00981244	0.00981060	0.05504250	0.01875522
0.4	0.00965381	0.00964718	0.00964096	0.13328548	0.06451639
0.6	0.00954023	0.00952982	0.00951427	0.27285330	0.16343870
0.8	0.00953813	0.00952060	0.00948571	0.55262073	0.36781643
1.0	0.00970014	0.00976515	0.00969882	0.01360990	0.68389762
Mean Error				0.17123531	0.21640406
t	$\alpha = 0.80$				
	HPM	VIM	Numerical	Error in HPM	Error in VIM
0	0.00999900	0.01000000	0.01000000	0.01000000	0.00000000
0.2	0.00981676	0.00978234	0.00981427	0.02537121	0.32534258
0.4	0.00966241	0.00963441	0.00964689	0.12978275	0.12936811
0.6	0.00957045	0.00956789	0.00957092	0.00491094	0.03165839
0.8	0.00912200	0.00917853	0.00913477	0.13979552	0.47904873
1.0	0.01001322	0.01001278	0.01002484	0.12044606	0.011591207
Mean Error				0.07171774	0.162834830

Table 7. The results of predation rate α versus harvesting predator population $y(t)$ at $t = 0.5$.

S.No	α	HPM	VIM	Numerical	Error in HPM	Error in VIM
1.	0.60	0.00957807	0.00958334	0.009581763	0.00038542	0.000164583
2.	0.64	0.00958227	0.00958686	0.009587456	0.000540915	0.00012590
3.	0.68	0.00958641	0.00959035	0.009592138	0.000597156	0.000186403
4.	0.72	0.00959062	0.00959382	0.009597888	0.000757250	0.000423843
5.	0.76	0.00959570	0.00959823	0.009602613	0.000719908	0.000456438
6.	0.80	0.00959999	0.00969156	0.009607263	0.000757031	0.008774299
Mean Error					0.00062628	0.001688578

Table 8. Error estimates for the harvesting predator population $y(t)$ for various values of m versus time t.

t	$m = 0.6$				
	HPM	VIM	Numerical	Error in HPM	Error in VIM
0	0.00999910	0.01000000	0.01000000	0.01000000	0.00000000
0.2	0.00981797	0.00981113	0.00981060	0.00540231	0.07512282
0.4	0.00966074	0.00964164	0.00964096	0.00705323	0.20516629
0.6	0.00955818	0.00951366	0.00951427	0.00641142	0.46151728
0.8	0.00958213	0.00947934	0.00948571	0.06715364	0.01016476
1.0	0.00960700	0.00966475	0.00969882	0.35127984	0.94671310
Mean Error				0.07288340	0.28461404
t	$m = 0.8$				
	HPM	VIM	Numerical	Error in HPM	Error in VIM
0	0.00999910	0.00999930	0.01000000	0.00900000	0.00700000
0.2	0.00981421	0.00981386	0.00981438	0.00529834	0.00173215
0.4	0.00965380	0.00964732	0.00965378	0.00020717	0.06691679
0.6	0.00954744	0.00952274	0.00954934	0.01989666	0.27855328
0.8	0.00956464	0.00949397	0.00957539	0.11226696	0.85030479
1.0	0.00987355	0.00989174	0.00992926	0.56106900	0.37787307
Mean Error				0.11795635	0.26373001

Table 9. The results of predation rate α versus harvesting predator population $y(t)$ at $t = 0.5$.

S.No	m	HPM	VIM	Numerical	Error in HPM	Error in VIM
1.	0.60	0.00956074	0.009569346	0.009570594	0.001029612	0.000130399
2.	0.64	0.009564473	0.009574386	0.009575675	0.001169839	0.000134612
3.	0.68	0.00956413	0.009578443	0.009575764	0.001214942	0.000279769
4.	0.72	0.00957172	0.009582495	0.009583775	0.001257855	0.000133559
5.	0.76	0.00957527	0.009586548	0.009588831	0.00141425	0.00023809
6.	0.80	0.009578930	0.009591600	0.009592844	0.001450456	0.00012968
Mean Error					0.001256159	0.000174352

8. Conclusion

In this paper, we have derived a simple and closed form of approximate analytical expressions to the nonlinear predator–prey system for all possible values of the parameters involved in the system. The proposed model is analyzed for both HPM and VIM. Both methods are widely used for solving nonlinear differential equations. Our logical outcomes are compared to the numerical results and found to be in excellent agreement. The estimated harvesting prey–predator population calculations for all the values of parameters r, k, α, d, δ, m, q_1, q_2 and E are found numerically and analytically. Only limited iterations of both methods give excellent results. Therefore, these are dynamic and efficient techniques for determining the nonlinear phenomena.

Appendix A

The analytical solutions of the system of Eqs. (1)–(3) using Homotopy perturbation method. A homotopy is constructed as follows,

$$(1-p)\left[\tfrac{dx}{dt}-(r-q_1E)x\right]+p\left[\tfrac{dx}{dt}-rx+\tfrac{rx^2}{k}+\alpha xy\right]=0,$$

$$(1-p)\left[\tfrac{dy}{dt}+(d+q_2E)y\right]+p\left[\tfrac{dy}{dt}+(d+q_2E)y-m\alpha xy+\delta y^2+q_1Ey\right]=0,$$

and the initial conditions are as follows,

$$x_0(0)=m_3,\quad y_0(0)=m_4$$

$$x=x_0+px_1+p^2x_2+\ldots,\quad y=y_0+py_1+p^2y_2+\ldots,$$

$$\begin{aligned}
p^0&\colon \frac{dx_0}{dt}-(r-q_1E)x_0=0,\\
p^1&\colon \frac{dx_1}{dt}-(r-q_1E)x_1+\frac{rx_0^2}{k}+\alpha x_0y_0=0,\\
p^2&\colon \frac{dx_2}{dt}-(r-q_1E)x_2+\frac{2rx_0x_1}{k}+\alpha x_0y_1+\alpha x_1y_0=0,\\
p^0&\colon \frac{dy_0}{dt}+(d+q_2E)y_0=0,\\
p^1&\colon \frac{dy_1}{dt}+(d+q_2E)y_1+m\alpha x_0y_0+\delta y_0^2=0,\\
p^2&\colon \frac{dy_2}{dt}+(d+q_2E)y_2-m\alpha x_0y_1-m\alpha x_1y_0+2\delta y_0y_1=0,
\end{aligned}$$

$$x_0(t)=m_3e^{(r-q_1E)t},\quad y_0(t)=m_4e^{-(d+q_2E)t},$$

$$x_1(t)=\left(\tfrac{rm_3^2}{k(r-q_1E)}-\tfrac{\alpha m_3m_4}{d+q_2E}\right)e^{(r-q_1E)t}-\tfrac{rm_3^2}{k(r-q_1E)}e^{2(r-q_1E)t}+\tfrac{\alpha m_3m_4}{d+q_2E}e^{(r-q_1E-d-q_2E)t},$$

$$y_1(t)=\left(\tfrac{-\alpha mm_3m_4}{(r-q_1E)}-\tfrac{\delta m_4^2}{(d+q_2E)}\right)e^{-(d+q_2E)t}+\tfrac{\alpha mm_3m_4}{(r-q_1E)}e^{(r-q_1E-d-q_2E)t}+\tfrac{\delta m_4^2}{(d+q_2E)}e^{-2(d+q_2E)t},$$

$$x_2(t) = \begin{pmatrix} \frac{2rm_3^3}{k(r-q_1E)^2} - \frac{2\alpha m_3^2 m_4}{(d+q_2E)(r-q_1E)} - \frac{rm_3^3}{k(r-q_1E)^2} + \frac{2\alpha m_3^2 m_4}{(d+q_2E)(r-q_1E-d-q_2E)} \\ -\frac{\alpha^2 m m_3^2 m_4}{(r-q_1E)(-d-q_2E)} + \frac{\delta\alpha m_3 m_4^2}{(d+q_2E)^2} + \frac{\alpha^2 m m_3 m_4}{(r-q_1E)(r-q_1E-d-q_2E)} - \frac{\delta\alpha m_3 m_4^2}{2(d+q_2E)^2} \\ +\frac{\alpha m_3^2 m_4}{k(r-q_1E)(d+q_2E)} + \frac{\alpha^2 m_3 m_4^2}{(d+q_2E)^2} - \frac{\alpha m m_3^2 m_4 r}{k(r-q_1E)(-d-q_2E+r-q_1E)} - \frac{\alpha^2 m_3 m_4^2}{2(d+q_2E)^2} \end{pmatrix}$$
$$- \frac{2rm_3^3}{k(r-q_1E)^2} e^{2(r-q_1E)t} + \frac{2\alpha m_3^2 m_4}{(d+q_2E)(r-q_1E)} e^{2(r-q_1E)t} + \frac{rm_3^3}{k(r-q_1E)^2} e^{3(r-q_1E)t}$$
$$- \frac{2\alpha m_3^2 m_4}{(d+q_2E)(r-q_1E-d-q_2E)} e^{(2r-2q_1E-d-q_2E)t} + \frac{\alpha^2 m m_3^2 m_4}{(r-q_1E)(-d-q_2E)} e^{(r-q_1E-d-q_2E)t}$$
$$- \frac{\delta\alpha m_3 m_4^2}{(d+q_2E)^2} e^{(r-q_1E-d-q_2E)t} - \frac{\alpha^2 m m_3 m_4}{(r-q_1E)(r-q_1E-d-q_2E)} e^{(2r-2q_1E-d-q_2E)t}$$
$$+ \frac{\delta\alpha m_3 m_4^2}{2(d+q_2E)^2} e^{(r-q_1E-2d-2q_2E)t} + \frac{\alpha m_3^2 m_4 r}{k(r-q_1E)(d+q_2E)} e^{(r-q_1E-d-q_2E)t} - \frac{\alpha^2 m_3 m_4^2}{2(d+q_2E)^2} e^{(r-q_1E-d-q_2E)t}$$
$$+ \frac{\alpha m m_3^2 m_4 r}{k(r-q_1E)(-d-q_2E+r-q_1E)} e^{(2r-2q_1E-d-q_2E)t} + \frac{\alpha^2 m_3 m_4^2}{2(d+q_2E)^2} e^{(r-q_1E-2d-2q_2E)t},$$

$$y_2(t) = \begin{pmatrix} \frac{m^2\alpha^2 m_3^3 m_4}{k(r-q_1E)^2} - \frac{\alpha\delta m_3 m_4^2}{(d+q_2E)(r-q_1E)} - \frac{m^2\alpha^2 m_3^3 m_4}{2(r-q_1E)^2} - \frac{\delta\alpha m m_3 m_4^2}{(d+q_2E)(r-q_1E-d-q_2E)} \\ -\frac{\alpha r m m_3^2 m_4}{k(r-q_1E)^2} + \frac{\alpha^2 m m_3 m_4^2}{(d+q_2E)(r-q_1E)} + \frac{\alpha r m m_3 m_4^2}{2k(r+q_1E)^2} + \frac{\alpha^2 m m_3 m_4}{(d+q_2E)(r-q_1E-d-q_2E)} \\ -\frac{2\delta\alpha m m_3 m_4^2}{(-d-q_2E)(r-q_1E)} + \frac{2\delta^2 m_4^3}{2(d+q_2E)^2} - \frac{2\delta\alpha m m_3 m_4^2}{(r-q_1E)(-d-q_2E+r-q_1E)} - \frac{\delta^2 m_4^2}{(d+q_2E)^2} \end{pmatrix} e^{-(d+q_2E)t}$$
$$- \frac{m^2\alpha^2 m_3^3 m_4}{(r-q_1E)^2} e^{(r-q_1E-d-q_2E)t} + \frac{\alpha\delta m m_3 m_4^2}{(d+q_2E)(r-q_1E)} e^{(r-q_1E-d-q_2E)t} + \frac{m^2\alpha^2 m_3^3 m_4}{2(r-q_1E)^2} e^{(2r-2q_1E-d-q_2E)t}$$
$$- \frac{\delta\alpha m m_3 m_4^2}{(d+q_2E)(r-q_1E-d-q_2E)} e^{(r-q_1E-2d-2q_2E)t} + \frac{\alpha r m m_3^2 m_4}{k(r-q_1E)^2} e^{(r-q_1E-d-q_2E)t}$$
$$- \frac{\alpha^2 m m_3 m_4^2}{(d+q_2E)(r-q_1E)} e^{(r-q_1E-d-q_2E)t} - \frac{\alpha r m m_3^2 m_4}{(r-q_1E)(r-q_1E-d-q_2E)} e^{(2r-2q_1E-d-q_2E)t}$$
$$+ \frac{\delta\alpha m_3 m_4^2}{2(d+q_2E)^2} e^{(r-q_1E-2d-2q_2E)t} + \frac{\alpha m_3^2 m_4 r}{2k(r-q_1E)^2} e^{(-d-q_2E+2r-2q_1E)t}$$
$$- \frac{m\alpha^2 m_3 m_4}{(d+q_2E)(r-q_1E-d-q_2E)} e^{(-2d-2q_2E+r-q_1E)t} + \frac{2\delta\alpha m m_3 m_4^2}{(-d-q_2E)(r-q_1E)} e^{-2(d+q_2E)t}$$
$$+ \frac{2\delta^2 m_4^3}{2(d+q_2E)^2} e^{(-2d-2q_2E+r-q_1E)t} - \frac{2\delta\alpha m m_3 m_4^2}{(r-q_1E-d-q_2E)(r-q_1E)} e^{(-2d-2q_2E+r-q_1E)t} + \frac{2\delta^2 m_4^3}{2(d+q_2E)^2} e^{-3(d+q_2E)t},$$

$$x(t) = \lim_{p\to 1} x(t) = x_0 + x_1 + x_2 + \ldots, \quad y(t) = \lim_{p\to 1} y(t) = y_0 + y_1 + y_2 + \ldots.$$

Final solution is given in the text as equations (4) and (5).

Appendix B

The approximate analytical solutions of the system of Eqs. (1)–(3) using Variational iteration method. We are given the following non-linear differential equation

$$L[x(t)] + N[x(t)] = g(t).$$

Here $L(t)$ is linear operator, $N(t)$ is non-linear operator, and $g(t)$ is a given function. According to the Variational iteration method can be established and analyzed using a correct functional as follows

$$x_{n+1}(t) = x_n(t) + \int_0^t \phi\{L[x_n(\tau)] + n[\tilde{x}_n(\tau)] - g[\tau]\}d\tau,$$

where ϕ is a general Lagrange multiplier which can be identified optimally via variational theory, u_n is the n^{th} order approximate solution, and $\tilde{u}_n$ denotes a restricted variation, i.e., $\psi u_n = 0$.

$$x_{n+1}(t) = x_n(t) + \int_0^t \phi_3 \left\{x_n'(\chi) - r x_n(\chi) + \frac{r}{k}[x_n^2(\chi)] + [\alpha x_n(\chi) y_n(\chi)] + q_1 E x_n(\chi)\right\} d\chi,$$

$$\psi x_{n+1}(t)=\psi x_n(t)+\psi\int_0^t \phi_3\{x_n'(\chi)-rx_n(\chi)+\frac{r}{k}[x_n^2(\chi)]+[\alpha x_n(\chi)y_n(\chi)]+q_1Ex_n(\chi)\}d\chi,$$
$$\psi y_{n+1}(t)=\psi y_n(t)+\psi\int_0^t \phi_4\{y_n'(\chi)-[m\alpha x_n(\chi)y_n(\chi)]+dy_n(\chi)+[\delta y_n^2(\chi)]+q_2Ey_n(\chi)\}d\chi,$$

where ϕ_3 and ϕ_4 are General Lagrange multiplier, x_0 and y_0 are initial approximation function $x_n^2(\chi)$, $y_n^2(\chi)$ and $m\alpha x_n(\chi)y_n(\chi)$ are restricted variation $\psi\tilde{x}_n=0$, $\psi\tilde{y}_n=0$ and $\psi\tilde{x}_n\tilde{y}_n=0$, $\psi x_n(0)=0$, $\psi y_n(0)=0$ and $\psi x_n(0)y_n(0)=0$. $\psi x_n\colon 1+\phi_3(\chi)|_{\phi=t}=0$, $\psi y_n\colon 1+\phi_4(\chi)|_{\phi=t}=0$,

$$\psi x_n\colon -\phi_3'(\chi)-r\phi_3(\chi)+q_1E\phi_3(\chi)|_{\phi=t}=0,\quad \psi y_n\colon -\phi_4'(\chi)+d\phi_4(\chi)+q_2E\phi_4(\chi)|_{\phi=t}=0.$$

The above equations are called Lagrange Euler equations. The Lagrange multipliers, can be identified as

$$\phi_3(\chi)=-e^{(q_1E-r)(\chi-t)},\quad \phi_4(\chi)=-e^{(q_2E+d)(\chi-t)},$$

$$x_1(t)=x_0(t)-\int_0^t e^{-r(\chi-t)}\left[x_0'(\chi)-rx_0(\chi)+\frac{r}{k}x_0^2(\chi)+\alpha x_0(\chi)y_0(\chi)+q_1Ex_0(\chi)\right]d\chi,$$
$$y_1(t)=y_0(t)-\int_0^t e^{d(\chi-t)}\left[y_0'(\chi)-m\alpha x_0(\chi)y_0(\chi)+dy_0+\delta y_0^2(\chi)+q_2Ey_0(\chi)\right]d\chi,$$

where

$$\begin{aligned}x_1(t)=c_1e^{(r+q_1E)t}-\tfrac{c_1(r+q_1E)}{2r}(e^{-(q_1E+r)t}-e^{(r-q_1E)t})-\tfrac{c_1}{2}(e^{-(q_1E+r)t}-e^{(r-q_1E)t})\\+\tfrac{rc_1^2}{k(q_1E+3r)}(e^{-2(q_1E+r)t}-e^{(r-q_1E)t})+\tfrac{q_1Ec_1}{2r}(e^{-(q_1E+r)t}-e^{(r-q_1E)t})\\+\tfrac{\alpha c_1c_2}{(q_1E+d+2r)}(e^{-(r+q_1E+q_2E+d)t}-e^{(-q_1E+r)t}),\end{aligned}$$

$$\begin{aligned}y_2(t)=c_2e^{(q_2E+d)t}+c_2t(q_2E+d)e^{-(q_2E+d)t}-\tfrac{m\alpha c_1c_2}{(q_1E+r)}(e^{-(r+q_1E+q_2E+d)t}-e^{-(q_2E+d)t})\\-dc_2te^{-(q_2E+d)t}+\tfrac{\delta c_2^2}{(q_2E+d)}(e^{-2(q_2E+d)t}-e^{-(q_2E+d)t})-q_2Ec_2te^{-(q_2E+d)t}.\end{aligned}$$

Final solution is given in the text as equations (6) and (7).

Appendix C

MATLAB Program to find the solution of the Eqs. (1)–(3)

```
function main
options = odeset('RelTol', 1e-6, 'Stats', 'on');
Xo = [0.01;0.01];
tspan = [0,1];
tic
[t,X] = ode45(@TestFunction,tspan,Xo,options);
toc
figure
hold on
plot(t, X(:,1), 'red')
plot(t, X(:,2), 'black')
legend('x1','x2')
ylabel('x - Population')
```

```
xlabel('t - Time')
return
function [dx_dt]= TestFunction(~,x)
r=4.5; k=900; alpha=0.60; d=0.15; delta=0.0019; m=0.7; q1=0.02; q2=0.001; E=1.4;
dx_dt(1)=((r*x(1))*(1-(x(1)/k)))-(alpha*x(1)*x(2))-q1*E*x(1);
dx_dt(2) =(m*alpha*x(1)*x(2))-d*x(2)-(delta*(x(2)^2))- q2*E*x(2);
dx_dt = dx_dt';
return
```

---

[1] Liu L., Meng X. Optimal harvesting control and dynamics of two-species stochastic model with delays. Advances in Difference Equations. **2017**, Article number: 18 (2017).

[2] Souna F., Lakmechea A., Djilali S. Spatiotemporal patterns in a diffusive predator-prey model with protection zone and predator harvesting. Chaos, Solitons & Fractals. **140**, 110180 (2020).

[3] Liu G., Wang X., Meng X., Gao S. Extinction and persistence in mean of a novel delay impulsive stochastic infected predator-prey system with jumps. Complexity. **2017**, 1950970 (2017).

[4] Bian F., Zhao W., Song Y., Yue R. Dynamical analysis of a class of prey-predator model with Beddington-Deangelis functional response, stochastic perturbation, and impulsive toxicant input. Complexity. **2017**, 3742197 (2017).

[5] Sahoo B., Das B., Samanta S. Dynamics of harvested-predator–prey model: role of alternative resources. Modeling Earth Systems and Environment. **2**, 140 (2016).

[6] Vijayalakshmi T., Senthamarai R. An analytical approach to the density dependent prey-predator system with Beddington — deangelies functional response. AIP Conference Proceedings. **2112**, 020077 (2019).

[7] Meng X.-Y., Qin N.-N., Huo H.-F. Dynamics analysis of a predator–prey system with harvesting prey and disease in prey species. Journal of Biological Dynamics. **12**, 342–374 (2018).

[8] Das K. A study of harvesting in a predator–prey model with disease in both populations. Mathematical Methods in the Applied Sciences. **39**, 2853–2870 (2016).

[9] Senthamarai R., Vijayalakshmi T. An analytical approach to top predator interference on the dynamics of a food chain model. Journal of Physics: Conference Series. **1000**, 012139 (2018).

[10] Das A., Pal M. Theoretical analysis of an imprecise prey-predator model with harvesting and optimal control. Journal of Optimization. **2019**, 9512879 (2019).

[11] Van Voorn G. A. K., Kooi B. W. Combining bifurcation and sensitivity analysis for ecological models. The European Physical Journal Special Topics. **226**, 2101–2118 (2017).

[12] Yu D.-N., He J.-H., Garcia A. G. Homotopy perturbation method with an auxiliary parameter for nonlinear oscillators. Journal of Low Frequency Noise, Vibration and Active Control. **38** (3–4), 1540–1554 (2019).

[13] Shirejini S. Z., Fattahi M. Mathematical modeling and analytical solution of two-phase flow transport in an immobilized-cell photo bioreactor using the homotopy perturbation method (HPM). International Journal of Hydrogen Energy. **41** (41), 18405–18417 (2016).

[14] Sivakumar M., Senthamarai R. Mathematical Model of Epidemics: SEIR Model by using Homotopy Perturbation Method. AIP Conference Proceedings. **2112**, 020080 (2019).

[15] Nivethitha M., Senthamarai R. Analytical approach to a steady-state predator-prey system of Lotka–Volterra model. AIP Conference Proceedings. **2277**, 210005 (2020).

[16] Sivakumar M., Senthamarai R. Mathematical model of epidemics: Analytical approach to SIRW model using homotopy perturbation method. AIP Conference Proceedings. **2277**, 1–8 (2020).

[17] Vijayalakshmi T., Senthamarai R. Application of homotopy perturbation and variational iteration methods for nonlinear imprecise prey–predator model with stability analysis. The Journal of Supercomputing. **78**, 2477–2502 (2021).

[18] Ghorbani A. Approximate solution of delay differential equations via variational iteration method. Nonlinear science letters. A, Mathematics, physics and mechanics. **8** (2), 236–239 (2017).

[19] Anjum N., He J.-H. Laplace transform: Making the variational iteration method easier. Applied Mathematics Letters. **92**, 134–138 (2019).

[20] Senthamarai R., Saibavani T. N. Substrate mass transfer: analytical approach for immobilized enzyme reactions. Journal of Physics: Conference Series. **1000**, 012146 (2018).

Дослідження моделі "жертва–хижак" двох видів у неточному середовищі зі сценарієм здобування

Віджаялакшмі Т., Сентхамарай Р.

Кафедра математики, Інженерно-технологічний коледж,
Інститут науки і техніки SRM,
Каттанкулатур – 603 203, район Ченгулпатту, Тамілнаду, Індія

У цьому дослідженні пропонується та досліджується модель "хижак–жертва", в якій є функціональна реакція здобування на групову поведінку "жертва–хижак". Досліджено нелінійну модель росту "жертва–хижак" двох видів. Запропонована модель підтверджена теоретичними та чисельними результатами. Деякі числові описи подані для пояснення отримання аналітичних та теоретичних висновків. Для всіх можливих значень параметрів, які з'являються в системі "жертва–хижак", розв'язано представлену модель як за допомогою варіаційного ітераційного методу (ВІМ) так і методом гомотопних збурень (МГЗ). Також використано кодування MATLAB, щоб порівняти отримані наближені аналітичні вирази з результатами комп'ютерного моделювання. Виявлено, що суттєвої різниці між аналітичними та чисельними результатами немає.

Ключові слова: *математичне моделювання, здобута здобич – хижак, точне значення, чисельне моделювання, варіаційний ітераційний метод, метод гомотопних збурень.*