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Thermomechanical behavior of a solid electroconductive ball under the action of amplitude modulated radioimpulse

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A physical and mathematical model for determining the thermostressed state of an electroconductive solid ball under the action of an amplitude modulated radioimpulse is proposed. The centrally symmetric problem of thermomechanics for the considered ball is formulated. The azimuthal component of the magnetic field strength vector, temperature, and the radial component of the displacement vector were chosen as the determining functions. To construct solutions of the formulated components of the initial-boundary value problems of electrodynamics, heat conductivity, and thermoelasticity, a polynomial approximation of the determining functions over the radial variable is used. As a result, the initial-boundary value problems on the determining functions are reduced to the corresponding Cauchy problems on the integral characteristics of these functions over the radial variable. General solutions of Cauchy problems under homogeneous nonstationary electromagnetic action are obtained. Based on these solutions, the change in time of Joule heat, ponderomotor force, temperature and stresses in the ball under the action of amplitude-modulated radioimpulse depending on its amplitude-frequency characteristics and duration is numerically analyzed.

Keywords: *solid electroconductive ball, radioimpulse, thermal stress state.*

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1. Introduction

Elements of many engineering structures in aviation and ship systems are solid balls made of electroconductive materials. In the process of functioning of such structures, they are affected by various physical factors, in particular, electromagnetic radiation of the radio frequency range. To predict the reliability of operation of structures with electroconductive ball elements, it is necessary to investigate their thermomechanical behavior under the action of unstable electromagnetic fields (EMF), which have the character of amplitude-modulated radioimpulse (AMRI).

Determining and study of the thermostressed state of electroconductive bodies of canonical shape under the action of steady and quasi-steady EMFs are described in [1, 2].

In [3], the optimization of regimes of high-temperature induction processing for nonlinear electroconductive bodies is proposed.

Mathematical modeling of nonlinear behavior of environmental material and three-dimensional internal heat sources in relation to heat conductivity processes is described in [4].

Physical and mathematical model of thermomechanics of non-ferromagnetic electroconductive bodies under the action of impulsed EMFs with amplitude modulation is given in [5]. A monograph [6] is devoted to the study of thermomechanical behavior of non-ferromagnetic electroconductive bodies with plane-parallel boundaries under the action of impulsed EMFs with amplitude modulation.

In [7–9] mathematical models for predicting the temperature of an electroconductive plate element under the action of impulsed electromagnetic radiation of the radio frequency range and calculating the temperature-force regime of operation of cylindrical and ball electroconductive sensors under the action of AMRI and electromagnetic action in the regime of damped sinusoid was considered.

The work [10] is devoted to the prediction of bearing capacity and properties of contact connection of bimetallic hollow balls under the action of electromagnetic impulses.

In [11] the thermomechanical behavior of an electroconductive cylindrical implant under the action of external unstable electromagnetic fields, in particular, radioimpulses, was studied. However, the thermomechanical behavior of electroconductive ball structural elements under the action of impulsed EMFs with amplitude modulation has not been sufficiently investigated.

The aim of this work is to construct a physical and mathematical model for determining the thermostressed state of an electroconductive solid ball under homogeneous nonstationary electromagnetic action and study its thermomechanical behavior under the action of AMRI depending on its amplitude-frequency characteristics and duration.

2. Formulation of the centrally symmetric problem of thermomechanics for an electroconductive ball

Let us consider an electroconductive elastic ball of the radius $r = R$, related to a spherical coordinate system (r, θ, φ), the center of which coincides with the center of the ball. The material of the ball is homogeneous isotropic and non-ferromagnetic, and its physical characteristics are assumed to be constant and equal to their average values in the considered ranges of temperature change. The surface of the ball is insulated or is in conditions of convective heat exchange with the environment.

The ball is under the action of nonstationary EMF given on its surface $r = R$ by the values of the azimuthal component H_φ of the magnetic field strength vector $\boldsymbol{H} = \{0; H_\varphi(r,t); 0\}$

$$H_\varphi(R,t) = H_{\varphi 0}(t). \tag{1}$$

Here $H_{0\varphi}(t)$ is a known function that describes the change in time t of the azimuthal component of the vector $\boldsymbol{H}$ on the surface of the ball. In the center of the ball $r = 0$, functions H_φ and $E_\theta = \frac{1}{\sigma}\left(\frac{\partial H_\varphi}{\partial r} + \frac{H_\varphi}{r}\right)$ satisfy the conditions of central symmetry of the electromagnetic field ($H_\varphi(0,t) = 0$, $E_\theta(0,t) = 0$). Here E_θ is the meridian component of the electric field strength vector $\boldsymbol{E} = \{0; 0; E_\theta(r,t)\}$. Hence we obtain the following boundary conditions for the function H_φ in the center of the ball

$$H_\varphi(0,t) = 0, \quad \frac{\partial H_\varphi(0,t)}{\partial r} = 0. \tag{2}$$

At the first stage, we find the function $H_\varphi(r,t)$ from the equation

$$\frac{\partial^2 H_\varphi}{\partial r^2} + \frac{2}{r}\frac{\partial H_\varphi}{\partial r} - \sigma\mu\frac{\partial H_\varphi}{\partial t} = 0, \tag{3}$$

at zero initial condition at the time $t = 0$

$$H_\varphi(r,0) = 0 \tag{4}$$

and boundary conditions (1) on the surface of the ball and conditions (2) in its center. Here σ is the coefficient of electrical conductivity, μ is the magnetic permeability of the ball material.

The specific Joule heat density $Q(r,t)$ and the radial component $F_r(r,t)$ of the ponderomotor force vector $\boldsymbol{F} = \{F_r; 0; 0\}$ in terms of the known function $H_\varphi(r,t)$ are written in the form

$$Q = \frac{1}{\sigma}\left(\frac{\partial H_\varphi}{\partial r} + \frac{H_\varphi}{r}\right)^2, \quad F_r = \mu\left(\frac{\partial H_\varphi}{\partial r} + \frac{H_\varphi}{r}\right)H_\varphi. \tag{5}$$

At the second stage, from the equation of heat conductivity, in which the heat source is Joule heat Q, we find the temperature distribution T in a solid ball

$$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} - \frac{1}{\kappa}\frac{\partial T}{\partial t} = -\frac{Q}{\lambda}. \tag{6}$$

Here, κ, λ are the coefficients of thermal and heat conductivity of the ball material.

The boundary condition on the surface of the ball $r = R$ in the case of its heat insulation is

$$\frac{\partial T(R,t)}{\partial r} = 0. \tag{7}$$

The boundary conditions in the center of the ball $r = 0$ correspond to the conditions of the central symmetry of the temperature field

$$T(0,t) = 0, \quad \frac{\partial T(0,t)}{\partial r} = 0. \tag{8}$$

The initial condition for the temperature at the time $t = 0$ will be

$$T(r,0) = 0. \tag{9}$$

Note that equation (6) can be solved under other than (7) heat conditions on the surface of the ball.

At the third stage, to determine non-zero radial σ_{rr}, azimuthal $\sigma_{\varphi\varphi}$ and meridian $\sigma_{\theta\theta}$ components of the tensor of dynamic stresses $\hat{\sigma}(r,t)$ in the ball, for the initial we choose a system of equations of the centrally symmetric problem of thermoelasticity in displacements [12]. Then the radial component $u_r(r,t)$ of the vector of displacements in the ball is determined from equation [13]

$$\frac{\partial^2 u_r}{\partial r^2} + \frac{2}{r}\frac{\partial u_r}{\partial r} - \frac{2}{r^2}u_r - \frac{1}{c^2}\frac{\partial^2 u_r}{\partial t^2} = \alpha\frac{1+\nu}{1-\nu}\frac{\partial T}{\partial r} - \frac{(1+\nu)(1-2\nu)}{E(1-\nu)}F_r \tag{10}$$

under boundary conditions in the center of the ball and on its outer surface

$$\frac{\partial u_r(0,t)}{\partial r} = 0, \quad \frac{\partial u_r(R,t)}{\partial r} + \frac{\nu}{1-\nu}\frac{2}{R}u_r(R,t) = \alpha\frac{1+\nu}{1-\nu}T(R,t), \tag{11}$$

as well as under zero initial conditions

$$u_r(r,0) = 0, \quad \frac{\partial u_r(r,0)}{\partial r} = 0. \tag{12}$$

Here $c = (E(1-\nu))/(\rho(1+\nu)(1-2\nu))^{-1/2}$ is the velocity of the elastic wave propagation of the elastic wave during isothermal deformation, α is the coefficient of linear heat expansion, ν is Poisson ratio, E is Young modulus, ρ is the density of the ball material.

According to the obtained component $u_r(r,t)$ of the displacement vector $\boldsymbol{u}$, we determine the components σ_{jj} $(j = r, \varphi, \theta)$ of the stress tensor $\hat{\sigma}$ according to the formulas [12, 13]

$$\sigma_{rr} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\frac{\partial u_r}{\partial r} + \nu\frac{2}{r}u_r - \alpha(1+\nu)T\right], \tag{13}$$

$$\sigma_{\varphi\varphi} = \sigma_{\theta\theta} = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\frac{\partial u_r}{\partial r} + \frac{u_r}{r} - \alpha(1+\nu)T\right].$$

In order to estimate the comparative contribution of Joule heat Q and ponderomotor force $\boldsymbol{F}$ to the distribution and magnitude of the components σ_{jj} $(j = r, \varphi, \theta)$ of the dynamic stress tensor $\hat{\sigma}$ when performing numerical studies, we will present these components as the sum of two components [5]

$$\sigma_{jj} = \sigma_{jj}^Q + \sigma_{jj}^F. \tag{14}$$

Here σ_{jj}^Q are components of the stress σ_{jj} caused by Joule heat, and σ_{jj}^F are components due to ponderomotor force.

3. Methodology of construction of solutions of centrally symmetric initial-boundary value problems under homogeneous nonstationary electromagnetic action

To construct solutions of the centrally symmetric initial-boundary value problems formulated above, which describe the electromagnetic and temperature fields and the thermostressed state in a continuous electroconductive ball, we will find for key functions $\Phi(r,t) = \{H_\varphi, T, u_r\}$ in the form of [14]

$$\Phi(r,t) = \sum_{i=1}^{4} a_{i-1}^{\Phi}(t) r^{i-1}. \tag{15}$$

The coefficients of the approximation polynomials (15) are determined by the given boundary values of the functions $\Phi(r,t)$ on the surface of the ball $r = R$ and integral characteristics $\Phi_s(t)$ of these functions with respect to the radial variable,

$$\Phi_s(t) = \frac{s+1}{R^{s+1}} \int_0^R \Phi(r,t) r^{s+1} dr, \quad s = 1, 2. \tag{16}$$

As a result, we obtain the following representations:
– azimuthal component $H_\varphi(r_*, t)$ of the vector $\boldsymbol{H}$

$$H_\varphi(r_*,t) = H_{\varphi 1}(t)\left(630r_*^2 - 1470r_*^3 + 840r_*^4\right) + H_{\varphi 2}(t)\left(-840r_*^2 + 2016r_*^3 - 1176r_*^4\right) + H_{\varphi 0}(t)\left(15r_*^2 - 42r_*^3 + 28r_*^4\right), \tag{17}$$

– temperature $T(r_*, t)$ under conditions of heat insulation of the ball surface

$$T(r_*,t) = T_1(t)\left(16 - 60r_*^2 + 40r_*^3\right) + T_2(t)\left(-21 + 90r_*^2 - 60r_*^3\right), \tag{18}$$

– radial component $u_r(r_*, t)$ of the displacement vector

$$u_r(r_*,t) = \sum_{i=0}^{3} \left[a_{i1}u_{r1}(t) + a_{i2}u_{r2}(t) + a_{i3}T(R,t)\right] r_*^i. \tag{19}$$

Here $r_* = r/R$ is dimensionless radial coordinate, a_{is} $\left(s = \overline{1,3}\right)$ are numerical coefficients, which are determined by the physical and mechanical characteristics of the material of the ball and its radius R.

To obtain the equations for the integral characteristics $\Phi_s(t)$ of the required functions $\Phi(r,t)$, equations (3), (6) and (10) are integrated over the radial variable r_* according to formula (16). At transformations we use representations (17)–(19). Then we obtain the following systems of equations to determine the integral characteristics $H_{\varphi s}(t)$, $T_s(t)$ and $u_{rs}(t)$ of functions $H_\varphi(r_*,t)$, $T(r_*,t)$ and $u_r(r_*,t)$:

$$\begin{cases} \dfrac{dH_{\varphi 1}(t)}{dt} - d_1 H_{\varphi 1}(t) - d_2 H_{\varphi 2}(t) = d_3 H_{\varphi 0}(t), \\ \dfrac{dH_{\varphi 2}(t)}{dt} - d_4 H_{\varphi 1}(t) - d_5 H_{\varphi 2}(t) = d_6 H_{\varphi 0}(t), \end{cases} \tag{20}$$

$$\begin{cases} \dfrac{dT_1}{dt} + d_1^T T_1 + d_2^T T_2 = W_1^Q(t), \\ \dfrac{dT_2}{dt} + d_3^T T_1 + d_4^T T_2 = W_2^Q(t). \end{cases} \tag{21}$$

$$\begin{cases} \dfrac{du_{r1}}{dt} + d_1^u u_{r1} + d_2^u u_{r2} = W_1^u(t), \\ \dfrac{du_{r2}}{dt} + d_3^u u_{r1} + d_4^u u_{r2} = W_2^u(t). \end{cases} \tag{22}$$

Here: coefficients $d_{1\div 6}$, $d_{1\div 4}^T$, $d_{1\div 4}^u$ are given in terms of the radius R of the ball and the physical and mechanical characteristics of its material,

$$W_s^Q(t) = \frac{\kappa}{\lambda} \int_0^R Q(r_*,t) r_*^s dr_*,$$

$$W_s^u(t) = \int_0^R \left[\alpha \frac{1+\nu}{1-\nu} \frac{\partial T(r_*, t)}{\partial r} - \frac{(1+\nu)(1-2\nu)}{E(1-\nu)} F_r(r_*, t)\right] r_*^s dr_* \quad (s = 1, 2).$$

Systems of equations (20)–(22) according to conditions (4), (9), (12) are solved under zero initial conditions on integral characteristics $H_{\varphi s}(t)$, $T_s(t)$ and $u_{rs}(t)$ of key functions.

Solutions of Cauchy problems (20)–(22) with respect to the integral characteristics of key functions are found using the Laplace integral transform with respect to the time variable t. They have a form of convolutions of functions that describe given boundary conditions for key functions and homogeneous solutions of Cauchy problems.

The expressions of the azimuthal component $H_\varphi(r_*, t)$ of the vector $\boldsymbol{H}$

$$H_\varphi(r_*, t) = \sum_{i=0}^{3} \left\{ \sum_{k=1}^{2} a_{ik} \int_0^t A_k(p_k) H_{\varphi 0}(\tau) e^{p_k(t-\tau)} d\tau + a_{i3} H_{\varphi 0}(t) \right\} r_*^i, \tag{23}$$

temperature

$$T(r_*, t) = \sum_{j=0}^{3} \sum_{m=1}^{2} \left(b_{jm} \int_0^t \left[B_{m1}(p_m) W_1^Q(\tau) + B_{m2}(p_m) W_2^Q(\tau) \right] e^{p_m(t-\tau)} d\tau \right) r_*^j \tag{24}$$

and the radial component $u_r(r_*, t)$ of the displacement vector

$$u_r(r_*, t) = \sum_{\alpha=0}^{3} \sum_{n=1}^{2} \left(c_{\alpha n} \int_0^t \left[B_{n1}(p_n) W_1^u(\tau) + B_{n2}(p_n) W_2^u(\tau) \right] e^{p_n(t-\tau)} d\tau \right) r_*^\alpha \tag{25}$$

are obtained.

Here a_{ik}, $A_k(p_k)$, b_{jm}, $B_{ms}(p_m)$, $c_{\alpha n}$, $B_{ns}(p_n)$ are expressions that depend on the radius of the ball and on the roots p_k, p_m, p_n $\left(k, m, n = \overline{1, 2}\right)$ of the characteristic equations, which correspond to the homogeneous solutions of Cauchy problems (20)–(22) for determining the integral characteristics of the functions H_φ, T and u_r.

4. Finding a solution of the thermomechanical problem for a ball under the action of amplitude-modulated radioimpulse

Amplitude-modulated radioimpulse (AMRI) is obtained using generators of high-frequency electromagnetic oscillations [15].

The electromagnetic action of the AMRI form is mathematically described by the expression of function $H_{\varphi 0}(t)$ that characterizes the change in time of the azimuthal component $H_\varphi(r, t)$ of the vector $\boldsymbol{H}$ on the surface $r = R$ of the ball, in the form [6, 16]

$$H_{\varphi 0}(t) = kH_0 \left(\exp(-\beta_1 t) - \exp(-\beta_2 t)\right) \cos \omega t. \tag{26}$$

Here H_0 is the amplitude of sinusoidal carrier electromagnetic frequency oscillations ω; β_1, β_2 are parameters characterizing the times of the rise and fall fronts of the impulse signal $\varphi(t) = \exp(-\beta_1 t) - \exp(-\beta_2 t)$ that modulate the carrier sinusoidal electromagnetic oscillations.

Substituting expression (26) into the relations (23), (5), (24), (25), (13), we obtain the expressions of the azimuthal component $H_\varphi(r, t)$ of the vector $\boldsymbol{H}$, the specific Joule heat densities Q and the radial component F_r of the ponderomotor force vector $\boldsymbol{F}$, temperature T and component σ_{jj} $(j = r, \varphi, \theta)$ of the dynamic stress tensor $\hat{\sigma}$ in the ball.

5. Investigation of the thermomechanical behavior of a ball under the action of amplitude-modulated radioimpulse

Calculations were performed for a non-ferromagnetic ball of radius $R = 0.01\,\mathrm{m}$ made of stainless steel. The parameters of AMRI: duration $t_i = 100\,\mu\mathrm{s}$, frequency of electromagnetic oscillations $\omega = 6.28 \cdot 10^5\,\mathrm{rad/s}$ (beyond the resonant frequencies ω_{rk}, where $k-$ is the resonant frequency number) and $\omega = \omega_{r1}$ ($\omega_{r1} = 1.255 \cdot 10^6\,\mathrm{rad/s}$ is the first resonant EMF frequency for the ball). Note that the resonant frequencies of the EMF are $\omega_{rk} \approx \omega_{nk}/2$, where ω_{nk} are the eigenfrequencies of mechanical oscillations of a given ball [1, 6].

Figures 1–3 show the change in time of the values Q, F_r, T when the frequency of the carrier signal is $\omega = 6.28 \cdot 10^5\,\mathrm{rad/s}$ for the duration of the electromagnetic action $t_i = 100\,\mu\mathrm{s}$. At this duration, there are 10 periods $f = 2\pi/\omega$ of electromagnetic oscillations of this frequency.

Curves 1, 2 in Figs. 1–3 correspond to the values of the radial coordinate $r = R$ and $0.5R$. It is obtained that the radial component F_r of the ponderomotor force $\boldsymbol{F}$ in the ball has an oscillating compressive nature and, like Joule heat Q, reaches its maximum value on the surface of the ball $r = R$ at time $t \approx 0.1t_i$ and temperature – at time $t \approx 0.5t_i$. It is obtained that the nature of the distribution of all these physical quantities is close to near-surface distribution.

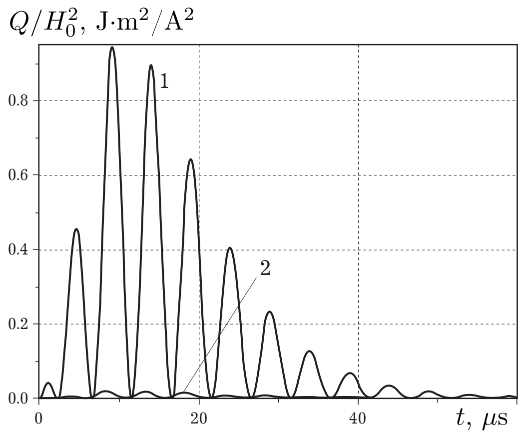

Fig. 1. Change in time of Joule heat emissions in a solid ball for frequency $\omega = 6.28 \cdot 10^5\,\mathrm{rad/s}$ at $r = R$ and $r = 0.5R$ (lines 1, 2).

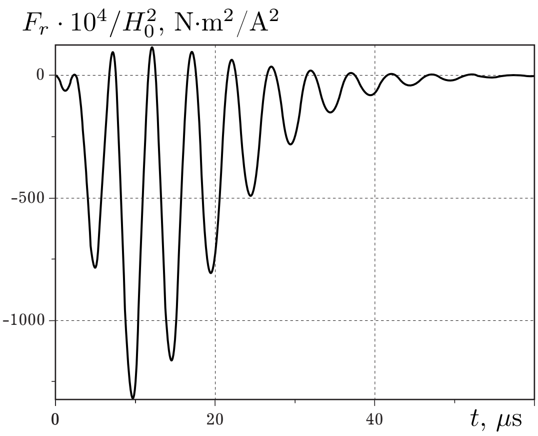

Fig. 2. Change in time of ponderomotor force in a solid ball for frequency $\omega = 6.28 \cdot 10^5\,\mathrm{rad/s}$ at $r = R$.

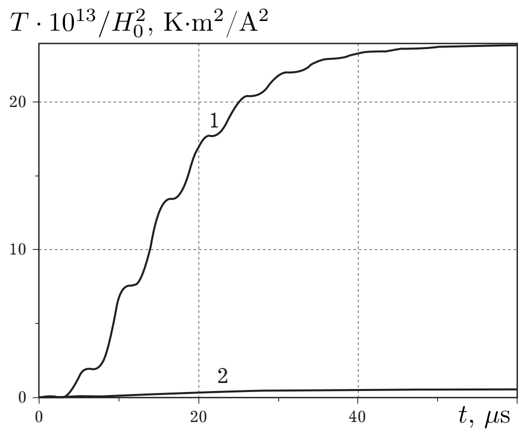

Fig. 3. Change in time of temperature in a solid ball for frequency $\omega = 6.28 \cdot 10^5\,\mathrm{rad/s}$ at $r = R$ and $r = 0.5R$ (lines 1, 2).

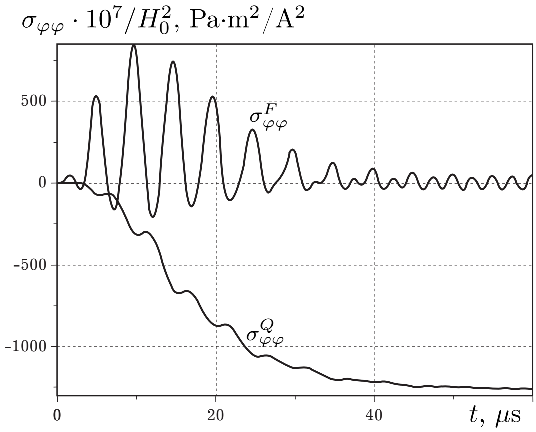

Fig. 4. Change in time of azimuthal stresses in a solid ball at $r = R$.

According to expresion (14) we present numerical studies of each component $\sigma_{\varphi\varphi}^Q$ and $\sigma_{\varphi\varphi}^F$ of azimuthal stresses separately. Figure 4 shows the change in time of the components $\sigma_{\varphi\varphi}^Q$ and $\sigma_{\varphi\varphi}^F$ of azimuthal stresses at frequencies $\omega = 6.28 \cdot 10^5$ rad/s on the surface of the ball $r = R$, where they reach maximum values.

At a given frequency $\omega \neq \omega_{r1}$, the components $\sigma^Q_{\varphi\varphi}$ and $\sigma^F_{\varphi\varphi}$ of azimuthal stresses are values of the same order. Accordingly, the component $\sigma^F_{\varphi\varphi}$ has a tensile oscillating nature and reaches maximum values over the time $t \approx 0.1t_i$. The component $\sigma^Q_{\varphi\varphi}$ has a compressive nature and reaches maximum values over the time $t \approx 0.5t_i$.

Figures 5–8 show the change in time of the values Q, F_r, T and $\sigma^Q_{\varphi\varphi}$, $\sigma^F_{\varphi\varphi}$ at the frequency of the carrier electromagnetic oscillations, equal to the first resonant frequency of the AMRI $\omega = \omega_{r1} = 1.255 \cdot 10^6$ rad/s.

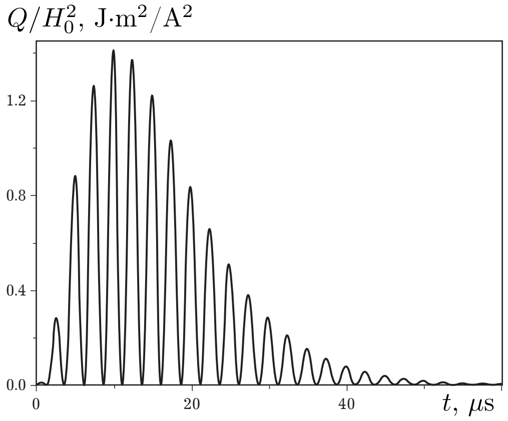

Fig. 5. Change in time of Joule heat emissions in a solid ball for frequency $\omega = \omega_{r1} = 1.255 \cdot 10^6$ rad/s at $r = R$.

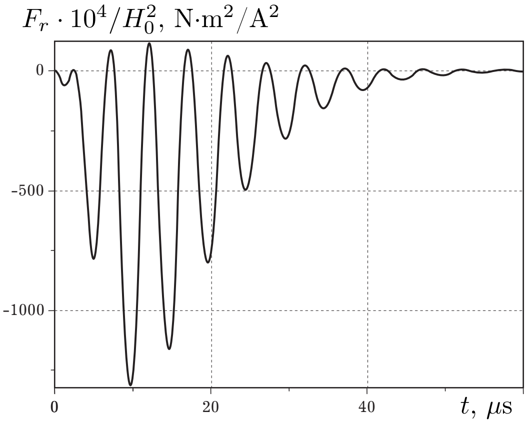

Fig. 6. Change in time of ponderomotor force in a solid ball for frequency $\omega = \omega_{r1} = 1.255 \cdot 10^6$ rad/s at $r = R$.

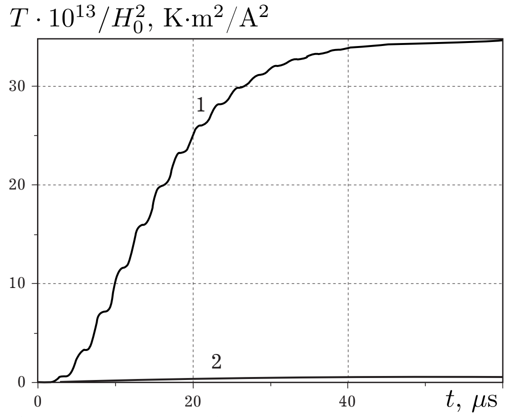

Fig. 7. Change in time of temperature in a solid ball for frequency $\omega = \omega_{r1} = 1.255 \cdot 10^6$ rad/s at $r = R$ and $r = 0.5R$ (lines 1, 2).

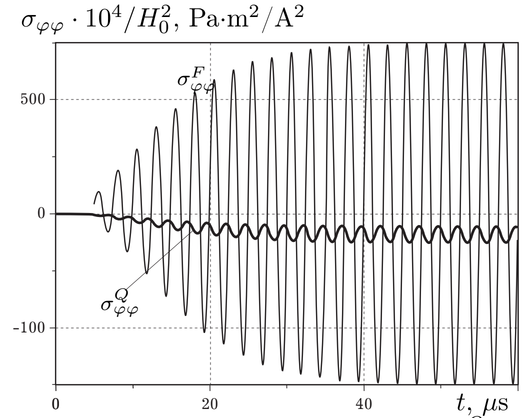

Fig. 8. Change in time of components $\sigma^Q_{\varphi\varphi}$ and $\sigma^F_{\varphi\varphi}$ of azimuthal stresses in a solid ball for frequency $\omega = \omega_{r1} = 1.255 \cdot 10^6$ rad/s at $r = R$.

For frequency $\omega = \omega_{r1} = 1.255 \cdot 10^6$ rad/s, the maximum values of the component $\sigma^F_{\varphi\varphi}$ of azimuthal stresses are approximately ten times greater than the maximum values of the component $\sigma^Q_{\varphi\varphi}$. Thus, the stress state of the ball at $\omega = \omega_{r1}$ is mainly determined by the stresses due to the action of ponderomotor force $\boldsymbol{F}$.

Note that the azimuthal stresses $\sigma^F_{\varphi\varphi}$ at frequency $\omega = \omega_{r1}$ are two orders of magnitude greater than the azimuthal stresses $\sigma^Q_{\varphi\varphi}$ at $\omega = 6.28 \cdot 10^5$ rad/s.

The components σ^Q_{rr} and σ^F_{rr} of radial stresses at both frequencies are about two orders of magnitude smaller than the components $\sigma^Q_{\varphi\varphi}$ and $\sigma^F_{\varphi\varphi}$ of azimuthal stresses. These stresses, as well as the meridian stresses $\sigma^Q_{\theta\theta}$ and $\sigma^F_{\theta\theta}$, according to formula (13) give a determining contribution to the stress state of the considered solid ball.

6. Conclusions

At the frequency of the carrier electromagnetic oscillations of AMRI $\omega \neq \omega_{r1}$, components $\sigma^{Q}_{\varphi\varphi}$ and $\sigma^{F}_{\varphi\varphi}$ of azimuthal stresses are values of the same order. It is obtained that the component $\sigma^{F}_{\varphi\varphi}$, which has a stretching oscillating character, is determining at times $t < 0.2t_i$. Accordingly, the component $\sigma^{Q}_{\varphi\varphi}$, which has a compressive nature, is determining at times $t > 0.2t_i$ and takes maximum values that are approximately 1.5 times greater than the maximum stretching values of the component $\sigma^{F}_{\varphi\varphi}$.

At the AMRI carrier electromagnetic oscillation frequencies $\omega = \omega_{r1}$, the maximum values of the azimuthal stress component $\sigma^{F}_{\varphi\varphi}$ are two orders of magnitude greater than the same values of the component $\sigma^{Q}_{\varphi\varphi}$ and make a decisive contribution to the stress state of the ball.

It is obtained that the maximum values of the component $\sigma^{Q}_{\varphi\varphi}$ that are determining at the frequency $\omega \neq \omega_{r1}$ are two orders of magnitude smaller than the same values of the component $\sigma^{F}_{\varphi\varphi}$ that is determining at the frequency $\omega = \omega_{r1}$.

The revealed regularities of the thermomechanical behavior of an electroconductive solid ball under the action of AMRI can be a theoretical basis for predicting the optimal regimes of electromagnetic treatment of electroconductive balls with the help of radioimpulses.

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Термомеханічна поведінка суцільної електропровідної кулі за дії амплітудно модульованого радіоімпульсу

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Запропоновано фізико-математичну модель визначення термонапруженого стану електропровідної суцільної кулі за дії амплітудно модульованого радіоімпульсу. Сформульовано центрально-симетричну задачу термомеханіки для розглядуваної кулі. За визначальні функції вибрано азимутальну компоненту вектора напруженості магнітного поля, температуру та радіальну компоненту вектора переміщень. Для побудови розв'язків сформульованих складових початково-крайових задач електродинаміки, теплопровідності і термопружності використано поліноміальну апроксимацію визначальних функцій за радіальною змінною. У результаті вихідні початково-крайові задачі на визначальні функції зведено до відповідних задач Коші на інтегральні за радіальною змінною характеристики цих функцій. Отримано загальні розв'язки задач Коші за однорідної нестаціонарної електромагнітної дії. На основі цих розв'язків чисельно проаналізовано зміну в часі тепла Джоуля, пондеромоторної сили, температури і напружень у кулі за дії амплітудно модульованого радіоімпульсу залежно від його амплітудно-частотних характеристик і тривалості.

Ключові слова: *суцільна електропровідна куля, радіоімпульс, термонапружений стан.*