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Determination of coordinates of unmanned aircrafts by means of kinematic projection

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A methodology for determining the mechanical trajectory and spatial coordinates of unmanned aircrafts by means of kinematic projection is described. The suggested methodology consists in the formation of two moving independent centers of kinematic projection by launching intercepting drones into space. Electromagnetic radio waves emitted by unmanned aircrafts pierce an unknown flying object and generate two independent projecting rays intersecting at the place of searching a flying object. At that, the instantaneous (at a certain moment) projection of the point of spatial location of the searched object will be located in an imaginary 'picture plane' on the line connecting the projections of the points created by projecting rays of the intercepting drones. Since all the objects of projection are mobile in this case, the whole projection of the trajectory of the searched object will be displayed on the operator's monitor. The formation of a 'more picture-like plan' perpendicular to the main one will allow us to build an axonometric view not only for the projection, but directly for the aircraft spatial movement trajectory. Every point of this trajectory gives us information about 'instant' coordinates of the location of the aircraft in space. Velocity is calculated as the ratio of spatial displacement of a flying object to the duration of movement. The scientific novelty of the method of determining the trajectories, velocities, and coordinates of an unmanned aircraft consists in the combination of radar detection of a moving object with kinematic design used for its implementation in order to calculate instantaneous coordinates of the object. This methodology solves the problem of determining more accurately the coordinates of flying objects that are not clearly and qualitatively displayed on radar monitors due to small size, mass, and specific materials used in manufacture. At the same time, the influence on the accuracy of determining the coordinates of unmanned aircrafts reflected from the fixed obstacles placed on the relief surface of earth was established. The results of this study can be used in practice in military science, for example, during the anti-terrorist operation in the occupied territories of Ukraine, with the aim to detect and neutralize enemy reconnaissance unmanned aircrafts. The use of this method in aerial photography for clear fixation of drone coordinates during aerial surveys for terrain topography is also promising.

Keywords: *unmanned aircraft, drone, mechanical trajectory, coordinates, kinematic projection, mobile projection center, trajectory projection, projecting ray, operator, software.*

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1. Introduction

Rapid development of space technology, means of communication, computer networks, etc. in recent decades inevitably lead to the emergence of new problems and tasks for the engineering science. In particular, they are related not only to increasing the speed of motion of various objects in space, but also to theoretical description and mapping of the coordinate reference of moving objects and devices for monitoring their condition and movement, alongside with the laws of movement of objects. The need for continuous control over moving objects promoted not only the rapid development of modern radio and data transmission devices, but also the development of sciences that study moving objects such as aerial photogeodesy, optical physics, and certain areas of mathematics intended to theoretically describe and explain the laws of movement of such objects.

These problems have also influenced such a branch of mathematics as descriptive geometry. Being focused on the mapping of the arrangement of different objects in space, descriptive geometry faced the task of creating a theory of kinematic mapping of objects. 'Kinematic mapping' means projecting, in which all the elements — center of projection, focal figures of projecting complexes and congruences, projection object (pre-image), and the projection medium ('picture plane') — can perform mutually dependent movements in space and time.

2. Literature overview

One of the first fundamental investigations in the field of kinematic problems of graphical mapping is the work 'Cinematic perspective' [1] written by M. A. Rynin in 1936. The work deals with deformations of cinematographic images of moving objects. The inverse problem — the components of motion of an object are determined using its cinematographic image — is also investigated in the study. Certain partial cases of motion of the original, in particular rectilinear motion, which is perpendicular to the 'picture plane', as well as rotational motion, are also described.

In investigations made by L. M. Likhachev, which are reflected in [2–4], the previous experience of kinematic projection was systematized. In his monograph 'Cinematic perspective' published in 1975, the problems of reconstruction of spatial forms and trajectories of an object were investigated, as well as methods of determining velocities and accelerations. The cinematic perspective introduced by L. M. Likhachev is based on the concept of 'quantum of motion', by which he means a certain abstract quantity

$$\Delta S = \Delta x + \Delta y + \Delta z$$

of spatial displacement of a point in time interval t. The quantum of motion of a figure is determined by the system of quanta of motion of its individual points. After that, perspective images of the quanta of motion of segments and planes (the author calls them qnantagrams) are considered and they are reconstructed and analyzed.

The works [5, 6] by Kul'minskyi are devoted to the problems of cinematic perspective and axonometry as a method of 3-D modelling. The prospects of using cinematic axonometry to designing automobile roads are also considered there.

In invastigations by V. E. Mykhailenko and M. V. Kovtun [7], projecting using the so-called 'flat' beam that moves vertically is considered. In this case, kinematic 'slit' photographing allows us to remove traditional projecting from several centers and therefore the characteristic 'blurring' of image in the picture plane. The picture base is, as a rule, perpendicular to the trajectory of the movement of the centre. The projection of linear and nonlinear (2nd order) objects and the reconstruction of pre-images by their own and cast shadows are studied in the article.

In the work by D. I. Tkach [8], central moving projection is investigated, whose projection center trajectory is rectilinear and perpendicular to the plane of projections and whose pre-images are fixed. The existence of a projection space is established. This projection space is in a perspective connection with the original space and there is bijective correspondence between these spaces.

In the work by S. F. Pylypak [9] published in 1991, an algorithmic description of a sequence of determining the points of a solid body moving along a spatial curvelinear trajectoty by projecting from a fixed centre onto a moving plane is given.

A significant contribution in the establishment and development of kinematic projection was made by the scientists of Lviv Polytechnic National University V. M. Glagovs'kyi and I. G. Pul'kevych. In their works [10, 11], the use of linear operators for obtaining gramographic, rotographic, and spinographic representations of objects moving in space was proposed for the first time.

Along with the development of algorithms for solving the direct kinematic projection problem for determining the projections of trajectories of objects moving in space, the authors of [12, 13] also developed and thoroughly investigated algorithms for solving an inverse problem. The inverse problem aims at determining the spatial coordinates of an object when the mechanical trajectory is known.

At the modern stage of development of applied geometry and engineering graphics, investigations of scientists-geometers into the methods and techniques of improving projection mapping in general and the so-called dynamic (or kinematic) projection in particular can be divided into a few directions. First of all, it is improving the practical application of known methods of projection and investigating peculiarities of state-of-art methods of mapping of elements in space headed by professor of "Kyiv Politechnical Institute" National University Vanin V. V. Thus, in [14] he considers the features of structural-parametric geometric modeling which has prospects in manufacturing industry. To develop this theme, Ye. A. Gavrylenko in his work [15] thoroughly researches the so-called variable discrete geometric modeling, which is similar to kinematic projection in terms of technical possibilities. The works [16–18] are also devoted to the improvement of methodological aspects of state-of-art methods of projecting. Besides analyzing generalized laws of of projection relations, these works define specific features of modeling projective n-spaces [17] and describe instruments for such modeling [18]. Investigations into the specificities of application of kinematic projection in cinematography and topography for the so-called 'panoramic shooting' gained further improvement and development. Their results are presented in [19]. Enough attention was paid by geometers to aircraft construction in general and operation of unmanned aircrafts in particular [20]. Along with tracking the aircraft trajectories in space, the latest projection techniques have been applied successfully for optimizing the shape and increasing the durability of aircraft wings [21, 22]. Therefore, it should be noted that geometers are aware of the need to shift the focus of study from the fairly well-studied static projection to kinematic projection.

3. Methods of investigation

The previous successful experience of applying kinematic projection in the field of cinematography opens a real prospect of its application both in the national economy and in the no less important military domain. Automated land cultivation complexes are an example of successful application of kinematic projection in agriculture. Automated agricultural machinery is controlled according to the information generated by kinematic projection. Similarly, in foundry machine building, parts that have complex geometric forms are automatically sized and scanned using the algorithm for solving the 'direct' kinematic projection problem. The modern military domain also sets tasks to kinematic projection. There is a variety of examples starting from the need to adjust the trajectory of artillery shells or missiles fired from moving artillery weapons (tanks, self-propelled artillery units, missile systems, etc.) to hit the targets to detecting the coordinates and trajectories of enemy torpedoes and submarines moving in thick water.

The task of detecting enemy reconnaissance or sabotage unmanned aircrafts should be considered a separate and particularly important problem in the military domain. Invisible in the sky because of insignificant size, practically noiseless due to the use of electric drives, and equipped with modern small cameras and information transmission devices, these aircrafts can cause a big damage due to the collection of important operational information. Moreover, these flying objects are made of durable plastic materials, which absorb but not reflect radio waves, so they are practically invisible for ground radar stations. As a result, air defense equipment aimed at defeating enemy reconnaissance aircrafts are not effective enough without knowing the exact location of the target. Technical means based on using kinematic projection that would be capable of providing the exact coordinates of a moving object would improve the situation considerably.

<u>The aim of the research</u> is to develop basic diagrams and algorithms for solving direct and inverse kinematic projection problems and apply them in detecting the coordinates of unmanned aircrafts.

<u>The tasks of the research are the following:</u>

— to create an algorithm for constructing projections of trajectories of spatial movement of an object of kinematic projection;

— to develop a basic diagram of a search complex for the detection of unmanned aircrafts based on kinematic projection;
— to develop methods of applying the algorithm of the 'inverse problem' of kinematic projection for determining the trajectory and coordinates of spatial motion of aircrafts.

4. Results and discussion

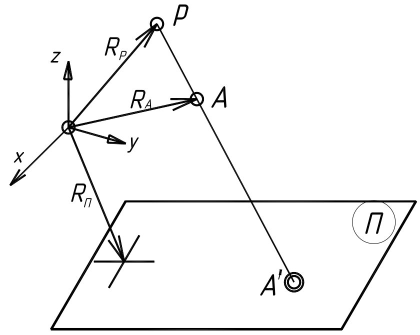

Fig. 1. Basic diagram of gramographic mapping of a point in central kinematic projection.

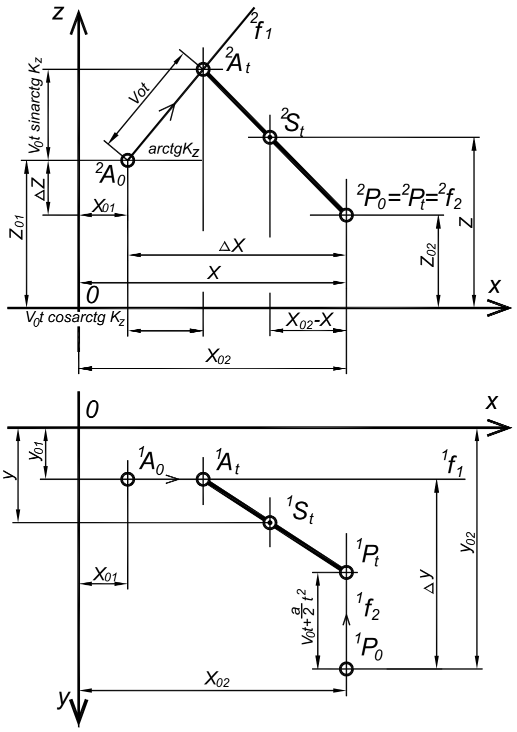

Fig. 2. Gramographic representations of projections of moving object (point A) and of center of projection (point P_0) on the Monge plot projection.

It is reasonable to start research into kinematic projection mapping with the examination of linear operators. In particular, to systematize the combination of basic movement types considered in this work, it is necessary to research gramographic mapping, i.e. such a mapping in which all the elements of the projecting apparatus move rectilinearly (Fig. 1).

Conventionally, kinematic mappings are denoted by linear operators in the following way

$$F\colon A_j \frac{P_V}{(\ldots)} \to A_j \quad (j = 1, 2, \ldots)\,.$$

At that, it should be taken into consideration the projection is carried out at simultaneous and interdependent movements of all projection elements (pre-image A, projection center P, and image carrier π)

$$r_a\colon A_j = A_j(t); \quad r_p\colon P_j = P_j(t); \quad r_\pi\colon \pi_j = \pi_j(t).$$

In parentheses — $(\ldots)$, the types of movements are represented by the following letters: γ — rectilinear, ρ — rotational, σ — helical.

Graphical mapping by linear operators is carried out by means of degenerated complexes of zero curvature, in particular by central projection according to the following algorithm:

$$F\colon A_j \frac{P_V}{(\ldots)} \to A_j \quad (j = 1, 2, \ldots);$$

$$r_a\colon A_j = A_j(t); \quad r_p\colon P_j = P_j(t); \quad r_\pi\colon \pi_j = \pi_j(t);$$

$$P_j \cup A_j = S_j;$$

$$S_j \cap \pi_j = A'_j;$$

$$\{A'_j\} \supset A'_1, A'_2, \ldots.$$

Equations of primary and secondary projections of the trajectories of pre-image A were derived and investigated for the following cases: uniform motion of A and uniformly accelerated motion of the projection centre P; uniform motion of A and P in a special position i.e. when $A_0(x_{01}, 0, Z_{01})$, $P_0(x_{02}, y_{02}, 0)$. Basing ob the derived equations, it is necessary to determine the types of trajectories of the pre-image, i.e. those of the moving object.

Let $f_1(y = y_1; z = k_z; x = b_z)$ and $f_2(x = x_2; z = z_2)$ be rectilinear trajectories of movement of the pre-image (point A) and the center of projection P respectively which begin to move simultaneously: point A moves with the velocity v_0 and acceleration a (Fig. 2).

From the linear congruence $s^2(f_1; f_2)$ with focal figures f_1 and f_2, the set conditions of movement single out a ray-generated surface $\sigma \supset A$, P as a continuous set of projecting rays $\{s\}(s_i \supset A_i; P_i)$ that correspondent to instantaneous positions of the points A and P. For an arbitrary point $s(x, y, z) \in \sigma$ lying on the ray $s_i \subset A_i, P_i$ at the moment of time t, we have the following relations:

$$\frac{\Delta y - \left(v_0 \cdot t + \frac{a \cdot t^2}{2}\right)}{\Delta x + v_0 \cdot t \cdot \cos \operatorname{arctg} k_z} = \frac{y - y_{01}}{x - x_{01} + v_0 \cdot t \cdot \cos \operatorname{arctg} k_z} \tag{1}$$

$$\frac{\Delta z - v_0 \cdot t \cdot \sin \operatorname{arctg} k_z}{\Delta x - v_0 \cdot t \cdot \cos \operatorname{arctg} k_z} = \frac{z - z_{02}}{z_{02} - x} \tag{2}$$

Here

$$(\Delta x = |x_{02} - x_{01}|, \quad \Delta y = |y_{02} - y_{01}|, \quad \Delta z = |z_{02} - z_{01}|)\,.$$

Substituting

$$t = \varphi(x, z) = \frac{\Delta z(x_{02} - x) - \Delta x(z - z_{02})}{v_0\left[(z - z_{02}) \cdot \cos \operatorname{arctg} k_z + (x_{02} - \sin \operatorname{arctg} k_z)\right]}$$

into (1), we obtain the equation

$$\begin{aligned} \Delta y - \left(v_0 \cdot \varphi(x, z) + \frac{a}{2}(x, z))^2\right) \cdot (x - x_{01} + v_0 \cdot \varphi(x, z) \cdot \cos \operatorname{arctg} k_z) - \\ - (\Delta x + v_0 \cdot \varphi(x, z) \cdot \cos \operatorname{arctg} k_z) \cdot (y - y_{01}) = 0, \end{aligned} \tag{3}$$

which defines the third-order surface.

If the pre-image A and the projection centre P perform uniform rectlinear motion with the velocities v_1 and v_2 respectively, then the surface is a quadric (single-sheet hyperboloid or, in the boundary case, a hyperbolic paraboloid), then equation (1) takes the following form

$$\begin{aligned} (\Delta y - v_2 \cdot \varphi(x, z)) \cdot (x - x_{01} + v_1 \cdot \varphi(x, z) \cdot \cos \operatorname{arctg} k_z) - \\ - (\Delta x + v_1 \cdot \varphi(x, z) \cdot \cos \operatorname{arctg} k_z) \cdot (y - y_{01}) = 0, \end{aligned} \tag{4}$$

where

$$\varphi(x, z) = \frac{\Delta x \cdot (z - z_{02}) - \Delta z \cdot (x_{02} - x)}{v_1 \cdot \left[(z - z_{02}) \cdot \cos \operatorname{arctg} k_z - (x_{02} - x) \cdot \sin \operatorname{arctg} k_z\right]}.$$

In general, when the path travelled by A or P in time t is equal to $v_0 \cdot t + \frac{a}{2} \cdot t^n$, the surface is of the $n + 1$ order. Now let us consider the plane of projections π with its fixed coordinate system $\eta\varsigma$. Let us set the initial position of $A_0(x_{01}, y_{01}, z_{01})$, $P_0(x_{02}, y_{02}, z_{02})$ and $\pi_0(x = 0, \eta = y, \varsigma = z)$ elements of progection and the rectilinear trajectories

$$\begin{aligned} f_1\left(y = k_{1v} \cdot x + b_{1v} z = k_{1/} \cdot x + b_{1/}\right), \\ f_2\left(y = k_{2v} \cdot x + b_{2v} z = k_{2/} \cdot x + b_{2/}\right) \end{aligned}$$

of the points A and P respectively. Let us also assume that in the process of the motion of the plane π, the x-axis remains its guiding vector.

We assume that the elements of mapping start their movements simuitaneously: the pre-image A and the plane π move with their constant vilocities at a speed of v_0, and the motion of the center P is uniformly accelerated with the initial speed v_0 and acceleration a. Then, at every moment of time t_1, the projection f_1' of f_1 onto the plane π in position $\pi_1(x = v_0 \cdot t_1)$, which corresponds to this moment of time, is the intersection of the ray-generated surface with the plane π.

Parametric equations $y = y(t)$, $z = z(t)$ of the infinite sets

$$^1A', {}^1A'_2, \ldots ; {}^2A'_1, {}^2A'_2, \ldots$$

of secondary projections (re-projected from the moving plane π onto the fixed projection planes xy and xz) of point A in its sequential positions in f_1 are the third-order parabolic branches

$$y = d_1 \cdot x^3 + d_2 \cdot x^2 + d_3 \cdot x + d_4$$

according to the Newtonian classification, namely:

$$(y_{02} - I_2 - y) \cdot (x_{01} - m_1 - v_0 \cdot t) = (x_{02} - m_2 - v_0 \cdot t) \cdot (y_{01} + I_1 - y)\,, \tag{5}$$

$$(z - z_{02} - n_2) \cdot (\Delta x + m_1 - m_2) = (x_{02} - m_2 - v_0 \cdot t) \cdot (\Delta z - n_1 - n_2)\,, \tag{6}$$

where

$$\Delta x = |x_1 - x_2|\,; \quad \Delta z = |z_1 - z_2|\,;$$

$$m_1 = v_0 \cdot t \cdot \cos\gamma_1 \cdot \cos\beta_1 : m_2 = \left(v_0 \cdot t + \frac{a \cdot t^2}{2}\right) \cdot \cos\gamma_2 \cdot \cos\beta_2;$$

$$n_1 = v_0 \cdot t \cdot \cos\gamma_1 \cdot \sin\beta_1 : n_2 = \left(v_0 \cdot t + \frac{a \cdot t^2}{2}\right) \cdot \cos\gamma_2 \cdot \operatorname{tg}\alpha_2 \cdot \cos\beta_2;$$

$$I_1 = v_0 \cdot t \cdot \cos\gamma_1 \cdot \operatorname{tg}\alpha_1 \cdot \cos\beta_1 : I_2 = \left(v_0 \cdot t + \frac{a \cdot t^2}{2}\right) \cdot \cos\gamma_2 \cdot \operatorname{tg}\alpha_2 \cdot \cos\beta_2.$$

Here

$$\alpha_1 = \operatorname{arctg} k_{1v}; \quad \beta_1 = \operatorname{arctg} k_{1/}; \quad \gamma_1 = \operatorname{arctg}(\operatorname{tg}\alpha_1 \cdot \cos\beta_1)\,; \quad j = 1, 2.$$

In case of uniform motion of all elements of the projection, the equations (5) and (6) determine quadratic parabolae.

In the boundary-particular position of the trajectory of the pre-image $A_0(x_{01}, O_1, z_{01})$ and the centre $P_0(x_{02}, y_{02}, 0)$

$$f_1(x = x_1, y = 0), \quad f_2(x = x_2, z = 0)$$

and uniform motion of all the elements with the same unit-value speed ($v = 1$), equations $y = y(t)$ and $z = z(t)$ are the following

$$y = \frac{1}{\Delta x} \cdot (y_2 - t) \cdot (x_1 - t), \tag{7}$$

$$z = \frac{1}{\Delta x} \cdot (z_2 - t) \cdot (x_1 - t), \tag{8}$$

or, in standart form,

$$\left[t - \frac{1}{2} \cdot (x_1 + y_2)\right]^2 = \Delta x \cdot \left[y + \frac{1}{\Delta x} \cdot \left(\frac{1}{4} \cdot (x_1 + y_2)^2 - x_1 \cdot y_2\right)\right], \tag{9}$$

$$\left[t - \frac{1}{2} \cdot (x_1 + z_1)\right]^2 = \Delta x \cdot \left[z + \frac{1}{\Delta x} \cdot \left(\frac{1}{4} \cdot (x_1 + z_1)^2 - x_1 \cdot z_1\right)\right]. \tag{10}$$

Thus in this case, the projections of the trajectory of the pre-image A are congruent quadratic parabolae (9), (10) with the parameter is $\frac{1}{2} \cdot \Delta x$.

The epure in Fig. 2 is an illustration of gramographic mapping. Primary (in the moving plane π) f'_1 and secondary (re-projected from π onto the fixed planes of coordinates xy, xz, yz) projections ${}^1f'_1$, ${}^2f'_1$, ${}^3f'_1$ of the rectlinear trajectory f_1 of the pre-image (point A) in $\gamma\gamma\gamma$-napping are presented in this epure.

A_0, P_0, π_0 are initial locations of the pre-image A, the projection centre P, and plane of projections π respectively. f_2 is the trajectory of movement of P, f_3 is the guide-vector of the movement of plane π. All the movements are uniform, the velocities of the projection elements are equal, the movements are simultaneous. Such a regime of motion is called standart regime.

As shown earlier, all the projections of the f_1-trajectory are quadratic parabolae. As mentioned above, the inverse (location) problem of kinematic projection mapping is to determine the position of pre-image A from its image A', which means

$$F^1 \colon A' \subset \pi \to A \subset \Pi$$

at any moment of time t (Fig. 3).

It is obvious that for injectivity of location it is necessary to have the two components A'_1, A'_2, of the image A' which are obtained by projecting the image A from two centers P_1, P_2 (linear mappings) [12], or from two congruences (non-linear mappings) [13].

$$F' \colon A'_1, A'_2 \frac{P_1, P_2}{(\ldots)} \to A;$$

$$F' \colon A'_1, A'_2 \frac{S_1^2\left(\varphi_{1_1}, \varphi_{1_2}\right) \cdot S_2^2\left(\varphi_{2_1}, \varphi_{2_2}\right)}{(\ldots)} \to A.$$

A strictly graphical (by means of compasses) solution of the problem is possible in a homogeneous space, because it is ultimately implemented as an intersection of two rectilinear projecting rays S_2^0 ($A = S_1^0 \cap S_2^0$) [13].

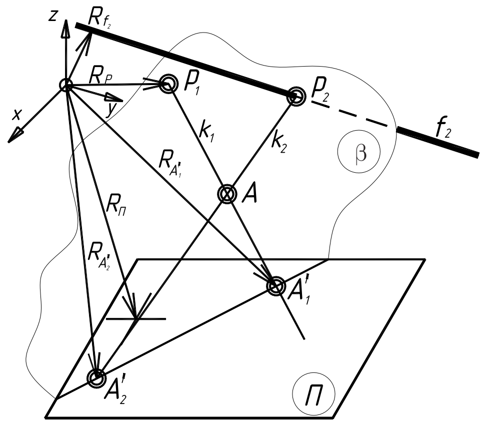

Fig. 3. Gramographic representation of solution of 'inverse problem' of kinematic projection for searching an object.

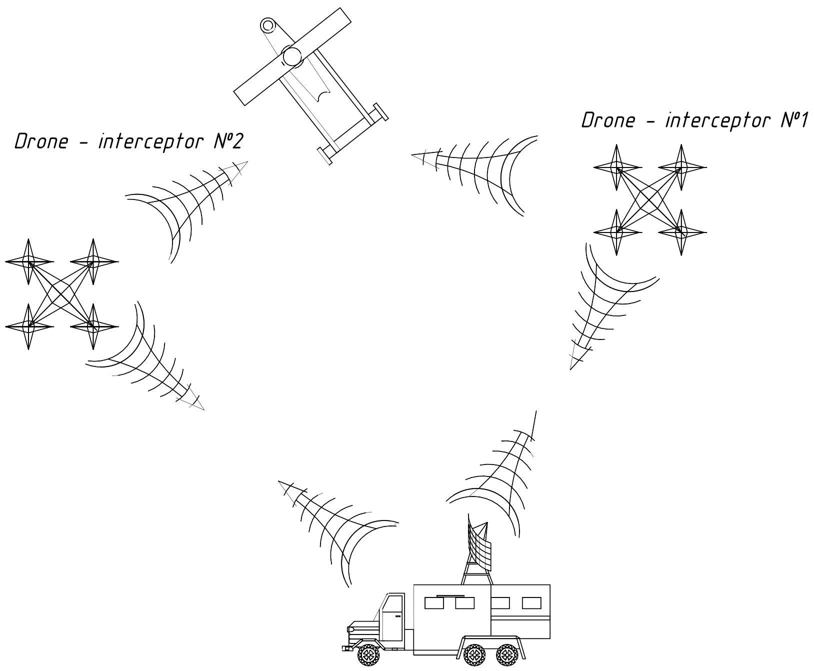

Fig. 4. Basic diagram of using kinematic projection to determine the coordinates and construct the trajectory of movement of unknown unmanned aircrafts.

As an example, in Fig. 4 one of the basic diagrams of solution of the 'inverse' kinematic projection problem that can be applied in practice is presented. It can be used to find spatial coordinates of an unknown unmanned aircraft by projections of the trajectory of its spacial movement. As shown in the diagram of the drone A, the technical devices used to find the spatial coordinates of an unknown object, may be two searching drones No.1 and No.2 equipped with radio transmitters. These drones play the role of projecting centers P_1 and P_2. The radio contact between the searching drones is done through a ground station (radar) equipped with appropriate hardware and software.

The search of coordinates of an unknown object is carried out in the following sequence (Fig. 5). Coordinates of an imaginary spatial centre (point O) of the reference system, i.e. of a Cartesian system of coordinates with axes x, y, z are set by specialized software. Having set the coordinates of three arbitrary points, for example L, M, N, spatial location of the 'picture plane' α is appointed, onto which the aircraft trajectory projection points will be projected.

A command is given to the searching drones No.1 and No.2 which serve as centeres of projection and direct projecting rays at an unknown aircraft with small time intervals (in this case it is drone A). The computer calculates the points of intersection of the projecting rays P_1A and P_2A with the 'picture plane' α, i.e. the points A_1' and A_2', forms an imaginary cutting plane β that is set by two projecting rays P_1A and P_2A as a plane set by intersecting straight lines. The computer program calculates the location of the line k as the line of intersection of the known planes α and β that goes through the already known points A_1' and A_2'.

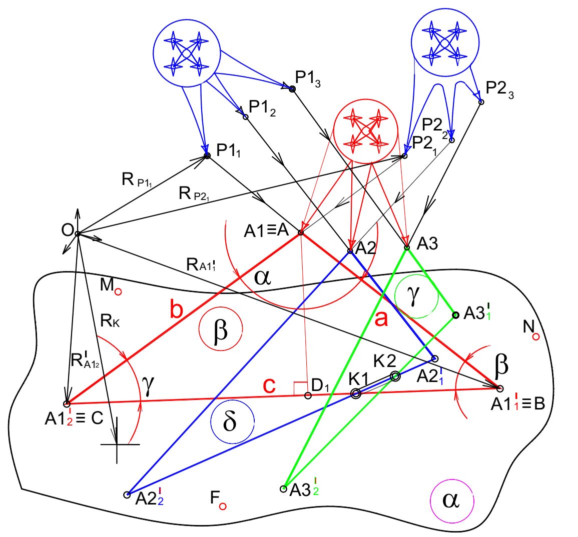

Fig. 5. Determining coordinates of an unknown object and its trajectory by solving the 'inverse problem' of kinematic projection.

In a short period of time Δt, all the three drones moving at different speeds change their location in space. The stages of kinematic projection described above are repeated and on the basis of signals reflected from the searched object by radio waves or laser beam, the computer registers and displays changes of coordinates of the searching drones, which serve as centres of projection, as well the changes of their projection coordinates on the 'picture plane'. At the same time (if necessary), a new line l will be built and displayed to show the intersection between plane γ formed by the new projecting rays and the 'picture plane' α, i.e. $l = \gamma \cap \alpha$. In its turn, the intersection of the lines l and k in the 'picture plane' forms the point $K = l \cap k$, which belongs to the spatial trajectory of the unknown object (drone A) in the 'picture plane'.

Everything repeats over and over again after the period Δt. In the 'picture plane', another line $n = \nu \cap \alpha$ will be displayed as an intersection of another plane $\nu = P_1A_1 \cap P_2A_2$ formed by another intersection of another pair of projecting rays. The intersection of this line n with the previously built line l gives a new point K, i.e. $K = n \cap l$. The in-series connection of the points K_0', K_{01}', K_{02}'. K_{0n}' forms the searched projection of the spatial trajectory of the unknown flying object (in this case drone A).

Since flying objects mostly move in three-dimensional space with certain altitude values in Z-coordinate, it is relevant to set at least one more 'picture plane' perpendicular to α, for example ω, simultaneously with setting a horizontal 'picture plane' α. At the same time, by analogy, a projection of the trajectory of movement of object α should be built on this 'picture plane' as well.

Then, having drawn connecting lines from the corresponding points of horizontal plane and the auxiliary plane ω perpendicular to it, the computer program finds the axonometric projection of the spatial trajectory of the flying object (drone A) and coordinates of any point of the trajectory. This is the ultimate result of the solution of the problem.

This method for searching coordinates and trajectories of unknown flying objects is quite universal and can be used both in the air, on land, on water, and in the underwater space.

The method for determining coordinates, velocities, and spatial movement of unmanned aircrafts is not only original, but also new. The kinematic scheme of determining the coordinates of unmanned aircraft described in this article is published for the first time.

As compared to existing methods of detecting and neutralizing enemy reconnaissance unmanned aircrafts based on finding and detecting them as 'material bodies of a certain mass', the kinematic projection method mathematically calculates the coordinates of objects as the coordinates of the points of intersection of projecting rays, which is easier and much more accurate.

The advantages of kinematic projection are the following:
– the possibility to identify and show an object on computer display (screen) not only in its plane view, but also taking into account its spatial coordinate by altitude, for example distance from the horizon. This allows observing the movements of the object not only in the time interval, but also in three-dimensional space.

5. Conclusions

1. Rapid development of computer technologies and software has enabled us to widely apply kinematic projection in practice. Kinematic projection allows imaging in projection relation classic elements of projection (projection centre, object, projecting ray, and projection onto the 'picture plane') moving in space at mutually independent velocities.
2. The solution of the so-called 'direct problem' of kinematic projection involves the possibility of projecting the mechanical trajectory of a projected object in the 'picture plane' both as a planar and axonometric mapping. In practice, it enables us to set, track, and adjust mechanical trajectories of vehicles, mechanized tillage equipment, automated tillage complexes, etc.
3. The solution of the so-called 'inverse problem' of kinematic projection allows finding the coordinates of the moving trajectory and spatial location of a projected object, provided its mechanical trajectory is known. For this purpose, we can use two moving independent centres of projecting, the intersection of projecting rays of which reflects the coordinates of an unidentified object. The application of solutions of the 'inverse problem' of kinematic projection is effective in searching aircrafts and unidentified flying objects as well as detecting the trajectories of movement of underwater vessels (torpedos, submarines, urberwater hydrocycles).

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Визначення координат безпілотних літальних апаратів засобами кінематичного проеціювання

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Описано методику визначення механічної траєкторії та координат просторового розташування за допомогою кінематичного проєктування стосовно безпілотних літальних апаратів. Запропонована методика полягає в утворенні двох рухомих і незалежних центрів кінематографічної проекції, за допомогою запущених в простір дронів–перехоплювачів. Електромагнітні радіохвилі, що випромінюються безпілотниками, пронизують невідомий літаючий об'єкт і генерують два незалежних проектуючих промені, що перетинаються в місці розшуку літаючого об'єкта. При цьому миттєва (у певний момент часу) проєкція точки розташування об'єкта пошуку буде розташована в уявній "картинній площині" на лінії, що з'єднує проекції точок, створених проектуючими променями дронів–перехоплювачів. Оскільки всі об'єкти проекції в даному випадку мобільні, то вся проекція траєкторії шуканого об'єкта буде виведена на монітор оператора. Формування "більш картинного плану" перпендикулярно головному дозволить побудувати аксонометрію не лише для проекції, а й безпосередньо для траєкторії просторового руху повітряного простору. Кожна точка цієї траєкторії дає нам інформацію про "миттєві" координати просторового розташування літака. Швидкість обчислюється як відношення просторового переміщення літаючого об'єкта до тривалості руху. Наукова новизна методу визначення траєкторій, швидкостей і координат БПЛА полягає у вперше запропонованому поєднанні радіолокаційного виявлення рухомого об'єкта з використанням для його реалізації кінематичного проектування з метою обчислення його миттєвих координат. Задача, що вирішується цим методом, полягає у підвищенні точності визначення координат переміщення повітряних об'єктів, які, у зв'язку з невеликими розмірами і масою, а також специфічними матеріалами їх виготовлення, не чітко і якісно відображаються на радарних моніторах. Разом з тим, встановлено вплив на точність визначення координат БПЛА, відбитого від фіксованих перешкод, розташованих на рельєфній поверхні Землі. Практичне використання результатів дослідження вбачається насамперед у військовій справі, наприклад, під час проведення антитерористичної операції на окупованих територіях України з метою виявлення та знешкодження розвідувальних БПЛА противника. Досить перспективним вважається використання цього методу в аерофотозйомці для чіткого розподілу координат дрона під час критичних зйомок для топографії місцевості.

Ключові слова: *безпілотний літальний апарат, дрон, траєкторія руху, координати, кінематичне проеціювання, рухомий центр проеціювання, проекція траєкторії, проектуючий промінь, оператор, програмне забезпечення.*

Personalia

On April 8, 2021, the head of the Department of Mathematical Problems of Mechanics of Heterogeneous Solids of Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of the National Academy of Sciences of Ukraine, laureate of the prize named after M. M. Krylov of the Presidium of the National Academy of Sciences of Ukraine, Doctor of Physical and Mathematical Sciences, Professor

Vasyl Feodosiyovych CHEKURIN

unexpectedly passed away.

Chekurin V. F. was born on January 14, 1951, in the village of Mankivka, Bershad district, Vinnytsia region. He graduated from the Faculty of Electrophysics of Lviv Polytechnic Institute (1974), after which he worked in a research laboratory. From 1977 to 1981, he took a correspondence postgraduate course (supervisor Professor Burak Y. Y.) at the Institute for Applied Problems of Mechanics and Mathematics named after Ya.S.Pidstryhach of the National Academy of Sciences of Ukraine, where he moved to work in 1984 and was bound up with it all his life. He started working there in scientific positions, and in 1998, he headed the department of mathematical problems of mechanics of heterogeneous solids.

Professor Chekurin V. F. obtained significant results in the field of mechanics of interconnected fields and physical materials science. He is the author and co-author of more than 300 scientific papers, including four monographs, a reference manual, nineteen inventions and patents. He developed a macroscopic theory of deformation of semiconductor bodies and structures taking into account locally nonequilibrium states, structural defects, specific surface properties, and material separation limits under mechanical, thermal, and electromagnetic stationary and nonstationary loads. His scientific results in the theory of the interaction of polarized light and ultrasound with inhomogeneously deformed bodies and their application to create methods for non-destructive determination of the stress-strain state and structure parameters of heterogeneous solids have been widely recognized.

Under his supervision, two doctoral theses and five candidate dissertations were defended.

Sudden illness and death caught him in the implementation of far-reaching plans and intentions, and his pass was a significant loss for the entire scientific community of Ukraine. Friends, colleagues, and students of Vasyl Feodosiyovych remember his bright personality and appreciate his considerable contribution to the achievements of modern mechanics with deep gratitude.

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