

3D model and numerical algorithm for gas filtration in porous media

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The article presents a three-dimensional mathematical model of the gas filtration process in porous media and a numerical algorithm for solving the initial-boundary value problem. The developed model is described using the nonlinear differential equation in partial derivatives with the appropriate initial and boundary conditions. The proposed mathematical apparatus makes it possible to carry out hydrodynamic calculations taking into account changes in the main factors affecting the process under consideration: permeability, porosity, and thickness of layers, gas recovery coefficient, viscosity, etc. Computer implementation of the model provides an opportunity to solve practical problems of analysis and forecasting of the gas production process under various conditions of impact on the productive reservoir, as well as making decisions on the development of existing and design of new gas fields.

Keywords: *mathematical model, numerical method, filtration, gas, porous medium, well, permeability, viscosity.*

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1. Introduction

Despite the modern trend of transition to renewable energy sources, their contemporary potential is not enough to actually cover the needs of the real sector of the economy and the population. In fact, the leading role in the energy and raw material balance of any country in the world today continues to belong to hydrocarbon resources. Moreover, change in this situation is unlikely in the coming decades.

Meanwhile, the increase in the capacity of the main industries requires a corresponding increase in energy reserves. The solution to this problem is often associated with the need to accelerate the design time, development, and commissioning of new oil and gas fields, as well as solving the problems of the most complete recovery of products from old oil and gas deposits.

Successful achievement of these goals is impossible without comprehensive research using the entire available spectrum of mathematical modeling and information and communication technologies. Therefore, any hydrogeological study is preceded by the development of an appropriate mathematical apparatus — models, algorithms, and software for computing experiments on a computer.

Numerous researchers carrying out comprehensive studies of the above processes have obtained significant theoretical and applied results by now.

Under certain, physically acceptable assumptions, the three-phase flow of immiscible incompressible fluids can be described by a non-exclusively nonlinear hyperbolic system. Therefore, the authors of [1] combined analytical solutions of the corresponding Riemann problem with an efficient front tracking method for studying Cauchy problems and initial-boundary value problems. The proposed method makes it possible to track individual waves and give a very close (or even accurate) resolution of inhomogeneities. The applicability of the method is shown by the authors using several numerical examples, including modeling the process of water and gas injection into a three-dimensional, inhomogeneous shallow sea formation.

The article [2] simulates the process of gas filtration in a porous medium, taking into account its structure. The presence of pores of different sizes leads to the formation of flow and stagnation zones,

which affect both the pressure distribution in the medium and the active gas mass. The mathematical apparatus developed by the authors makes it possible to determine the proportion of the volume of flowing zones and the exchange coefficient between flowing and stagnant zones in the field of solving the filtration problem.

The authors of [3] presented a more accurate mathematical model instead of the often used Klinkenberg equation to describe the gas flow in a porous medium with low permeability. The developed model makes it possible to analyze the steady flow of gas through a porous medium, including effects not related to Darcy's laws. With the help of the computational algorithm presented in the article, one can quite simply obtain analytical solutions for a two-dimensional gas flow in a porous medium.

Scientific work [4] is devoted to the development of a computational fluid dynamics (CFD) model for performing one- and two-phase fluid flow modeling in two- and three-dimensional perforated porous media with different geometries. The finite volume method was used to solve the system of complete equations of hydrodynamics. Additionally, the volume of liquid (VOF) method was used to accurately determine the volume occupied by each phase. The reliability of the model has been verified by comparing the simulation results with experimental data available in the literature. The model developed by the authors was used to analyze the influence of the geometric parameters of the porous medium perforation, the degree of its heterogeneity, as well as the properties of permeability and thickness on the distribution of pressure and velocity fields. As can be seen from the results presented by the authors, the model successfully predicted the pressure drop and associated temperature changes for the air-water system along clean and gravel-filled perforations under steady conditions. Thus, the model presented by the authors can be used as an effective tool for developing the most appropriate perforation strategy, taking into account the characteristics of the well and the properties of the reservoir.

The authors of [5] carried out a direct numerical simulation of the process of fluid flow and particle dynamics in a structured porous medium. The developed mathematical model is based on the description of the Euler–Lagrange gas flow using one-way phase coupling. To calculate the gas flow, the authors have solved the Navier–Stokes equations for an incompressible liquid using the finite volume discretization method while maintaining symmetry. On the basis of the developed mathematical apparatus, the authors investigated the dependence of the filtration efficiency on the Reynolds number and inertia of particles.

At present, production from a significant part of the largest dry gas deposits is steadily declining. Therefore, in the coming decades, there will be active exploitation of gas condensate fields. Significant differences in the indicators of the design and actual recovery of condensate during the development of reservoirs of this type, as a rule, are caused by insufficient knowledge about the mechanisms of nonequilibrium filtration of gas condensate mixtures in reservoir conditions. The authors of [6] presented a system of differential equations for describing the filtration process of a two-phase multicomponent mixture for one-, two- and three-dimensional cases. The solution of the described system was carried out by the finite element method in the FlexPDE software package. Comparative distributions of velocities, pressures, saturation, and phase composition of the three-component mixture along with the reservoir model and in time for both equilibrium and non-equilibrium filtration processes are obtained. The calculation results showed that the deviation of the system from thermodynamic equilibrium increases the flow rate of the gas phase and decreases the flow rate of the liquid phase during the filtration of the gas-condensate mixture.

The article [7] considers the problem of gas filtration in a porous medium using a mathematical model described by a nonlinear partial differential equation and the corresponding boundary and internal conditions. The authors present the main stages of constructing a mathematical model of the gas filtration process in porous media taking into account changes in hydrodynamic parameters. To solve the problem, the author's used methods such as: locally one-dimensional schemes and schemes of longitudinal-transverse direction. Also, to solve the problem of nonlinear gas filtration in a porous medium, the authors tested several methods for constructing an iterative process. To study the responses of the main process parameters, a series of computational experiments, analysis of the results, and conclusions are given.

The article [8] discusses the problem of developing gas fields with a closed water loop to increase oil recovery and determining the main reservoir parameters, as well as the position of moving boundaries. In order to study the process under consideration, the authors have developed a computer model described by a differential equation with the corresponding initial and boundary conditions. An algorithm for solving the problem using the methods of the longitudinal-transverse scheme and the flow version of the sweep method is developed. The adequacy of the model has been verified on the example of a real operating object.

As follows from the analysis of the above-considered works of the authors, in the overwhelming majority, they used one- and two-dimensional mathematical models to solve the problems of oil and gas filtration. This is primarily due to the complexity of the numerical schemes. Meanwhile, the use of three-dimensional models makes it possible to more accurately describe the processes occurring in porous reservoir media. Therefore, the purpose of this study is to develop a three-dimensional mathematical model of gas filtration in porous media, as well as a numerical algorithm for solving the problem.

2. The statement of the problem

In order to investigate and more adequately describe the process of gas filtration in porous media as well as determine the main indicators used in the development of oil and gas fields, we introduce a 3D mathematical model described by the equation

$$\frac{\partial}{\partial x}(\nu_x \rho) + \frac{\partial}{\partial y}(\nu_y \rho) + \frac{\partial}{\partial z}(\nu_z \rho) = -\frac{\partial}{\partial t}(\rho m), \quad (1)$$

and in special nodes (wells)

$$\frac{\partial}{\partial x}(\nu_x \rho) + \frac{\partial}{\partial y}(\nu_y \rho) + \frac{\partial}{\partial z}(\nu_z \rho) = -\frac{\partial}{\partial t}(m \rho) - F_Q. \quad (2)$$

In the equations (1)–(2)

$$\nu_x = -\frac{K}{\mu} \frac{\partial P}{\partial x}, \quad \nu_y = -\frac{K}{\mu} \frac{\partial P}{\partial y}, \quad \nu_z = -\frac{K_z}{\mu} \frac{\partial P}{\partial z}. \quad (3)$$

Substituting equation (3) in (2) and taking into account the variability of the reservoir capacity, we obtain [9–11, 15]:

$$\frac{\partial}{\partial x} \left(\frac{K}{\mu} b \rho \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{K}{\mu} b \rho \frac{\partial P}{\partial y} \right) + \frac{\partial}{\partial z} \left(\frac{K_z}{\mu} b \rho \frac{\partial P}{\partial z} \right) = \frac{\partial}{\partial t} (m \rho) \tilde{b} - F_Q, \quad (4)$$

where

$$F_Q = \frac{\rho Q P_{at}}{P \Delta z \Delta x \Delta y} \cdot \frac{2\mu}{b \cdot K} \delta(x, y, z), \quad \delta(x, y, z) = \begin{cases} 1, & (x, y, z) \in \gamma_\nu; \\ 0, & (x, y, z) \notin \gamma_\nu. \end{cases}$$

Here Q is a volumetric flow rate (at atmospheric pressure) in the wells, $Q\rho$ is a mass flow, P is pressure; P_{at} is atmospheric pressure, ρ is density, b is a power of the stratum, \tilde{b} is the average power value in the grid “square”, Δx , Δy , Δz are steps by x , y and z coordinates; m is reservoir porosity; K , μ are the filtration coefficient and viscosity of the gas, respectively, $K_z = f(m, g)$, γ_ν are the set of points of the domain G , in which wells may be present.

Let us assume that the gas is ideal and we obtain

$$\frac{\partial}{\partial x} \left(\frac{K}{\mu} b P \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{K}{\mu} b P \frac{\partial P}{\partial y} \right) + \frac{\partial}{\partial z} \left(\frac{K_z}{\mu} b P \frac{\partial P}{\partial z} \right) = \frac{\partial}{\partial t} (m P) \tilde{b} - F_Q. \quad (5)$$

Equation (5) is valid for any law of filtration and any dependence on the density of the pressure.

If in the equation (5) all coefficients are constant, i.e., $K = K_z = \mu = b = m = \text{const}$, then we obtain

$$\frac{\partial^2 P^2}{\partial x^2} + \frac{\partial^2 P^2}{\partial y^2} + \frac{\partial^2 P^2}{\partial z^2} = \frac{2m\mu}{K} \frac{\partial P}{\partial t} - \frac{Q P_{at}}{\Delta x \Delta y \Delta z} \cdot \frac{2\mu}{b \cdot K} \delta(x, y, z) \quad (6)$$

with the corresponding boundary conditions:

$$P(x, y, z, t)|_{t=0} = P_0, \quad (7)$$

$$\frac{\partial P}{\partial n} \Big|_{\Gamma} = 0, \quad \oint \frac{K}{\mu} b \cdot \frac{\partial P}{\partial n} ds = C Q_{\nu}, \quad (8)$$

$$\frac{\partial P}{\partial z} \Big|_{z=0} = 0, \quad \frac{\partial P}{\partial z} \Big|_{z=H} = 0. \quad (9)$$

Here P_0 is initial pressure; $Q_{\nu} = Q$; C is some constant value to bring to dimension; Γ is border of the area G .

As a result, the final form of the equation is obtained which gives us possibility to study the filtration process of any considered component in a porous medium in order to determine the main parameters of the development and design of oil and gas fields.

3. The solution of the problem

To solve the problem (6)–(9), using the relations

$$x = x^*L, \quad y = y^*L, \quad z = z^*H, \quad P = P^*P_0, \quad Q^* = \frac{2\mu P_{at}}{bK P_0^2}Q, \quad \Delta t^* = \frac{KP_0}{2m\mu L^2}\Delta t,$$

we bring equation (6) to a dimensionless form:

$$\frac{\partial^2 P^2}{\partial x^2} + \frac{\partial^2 P^2}{\partial y^2} + \frac{\partial^2 P^2}{\partial z^2} = \frac{1}{2\tilde{P}} \frac{\partial P^2}{\partial t} - F_q, \quad (10)$$

where

$$F_q = \delta(x, y, z) Q \frac{1}{\Delta x \Delta y \Delta z}.$$

To linearize equation (10) by Newton's method, we use the expression $P^2 = 2P \cdot \tilde{P} - (\tilde{P})^2$, and we obtain the following

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = \frac{1}{2\tilde{P}} \frac{\partial P}{\partial t} - \frac{\delta(x, y, z)Q}{2\tilde{P}\Delta x \Delta y \Delta z}. \quad (10^*)$$

Since the problem (7)–(10) is described by nonlinear partial differential equation with the corresponding internal and boundary conditions, it is complicated to obtain an analytical solution.

One of the main methods that allow us to determine the patterns of change in filtration variables is the method of approximating differential operators by the finite-difference conservative scheme, which could be effectively implemented on a computer.

Using the finite-difference approximation of the problem, we obtain a system of algebraic equations, solving which by the sweep method, we determine the desired parameters of the object and the acceptable ranges of their variation, both in time and in the spatial variable.

Based on the above, to integrate problem (7)–(10), we introduce a uniform mesh in x , y , z and t [9–15]:

$$\omega_{\Delta x, \Delta y, \Delta z, \Delta t} = \left\{ \begin{array}{l} x_i = i\Delta x, \quad i = 0, 1, 2, \dots, N_x, \quad \Delta x = \frac{L_x}{N_x}; \\ y_j = j\Delta y, \quad j = 0, 1, 2, \dots, N_y, \quad \Delta y = \frac{L_y}{N_y}; \\ z_k = k\Delta z, \quad k = 0, 1, 2, \dots, N_z, \quad \Delta z = \frac{L_z}{N_z}; \\ t_l = l\Delta t, \quad l = 0, 1, 2, \dots, N_t, \quad \Delta t = \frac{T}{N_t}. \end{array} \right.$$

Replacing the differential operators in equation (10*) with finite-difference operators and using the scheme of longitudinal-transverse direction along Ox , we obtain [9–15]:

$$\frac{P_{i-1,j,k}^{n+\frac{1}{3}} - 2P_{i,j,k}^{n+\frac{1}{3}} + P_{i+1,j,k}^{n+\frac{1}{3}}}{\Delta x^2} + \frac{P_{i,j-1,k}^n - 2P_{i,j,k}^n + P_{i,j+1,k}^n}{\Delta y^2}$$

$$+ \frac{P_{i,j,k-1}^n - 2P_{i,j,k}^n + P_{i,j,k+1}^n}{\Delta z^2} = \frac{1}{2\tilde{P}_{i,j,k}} \frac{P_{i,j,k}^{n+\frac{1}{3}} - P_{i,j,k}^n}{\Delta t/3} - \frac{Q\delta_{i,j,k}}{6\tilde{P}_{i,j,k}\Delta x\Delta y\Delta z}. \quad (11)$$

We transform the equation (11) to the following form:

$$\begin{aligned} & \frac{1}{\Delta x^2} P_{i-1,j,k}^{n+\frac{1}{3}} - \frac{2}{\Delta x^2} P_{i,j,k}^{n+\frac{1}{3}} + \frac{1}{\Delta x^2} P_{i+1,j,k}^{n+\frac{1}{3}} + \frac{1}{\Delta y^2} P_{i,j+1,k}^n - \frac{2}{\Delta y^2} P_{i,j,k}^n \\ & + \frac{1}{\Delta y^2} P_{i,j-1,k}^n + \frac{1}{\Delta z^2} P_{i,j,k+1}^n - \frac{2}{\Delta z^2} P_{i,j,k}^n + \frac{1}{\Delta z^2} P_{i,j,k-1}^n \\ & = \frac{3}{2\tilde{P}_{i,j,k}\Delta t} P_{i,j,k}^{n+\frac{1}{3}} - \frac{3}{2\tilde{P}_{i,j,k}\Delta t} P_{i,j,k}^n - \frac{Q\delta_{i,j,k}}{6\tilde{P}_{i,j,k}\Delta x\Delta y\Delta z}, \end{aligned} \quad (12)$$

then, grouping similar terms, equation (12) becomes the following system of three diagonal algebraic equations:

$$a_{i,j,k} P_{i-1,j,k}^{n+\frac{1}{3}} - b_{i,j,k} P_{i,j,k}^{n+\frac{1}{3}} + c_{i,j,k} P_{i+1,j,k}^{n+\frac{1}{3}} = -d_{i,j,k}, \quad (13)$$

where

$$\begin{aligned} a_{i,j,k} &= \frac{1}{\Delta x^2}, \quad b_{i,j,k} = \frac{2}{\Delta x^2} + \frac{3}{2\tilde{P}_{i,j,k}\Delta t}, \quad c_{i,j,k} = \frac{1}{\Delta x^2}, \\ d_{i,j,k} &= \left(\frac{3}{2\tilde{P}_{i,j,k}\Delta t} - \frac{2}{\Delta y^2} - \frac{2}{\Delta z^2} \right) P_{i,j,k}^n + \frac{1}{\Delta y^2} P_{i,j-1,k}^n + \frac{1}{\Delta y^2} P_{i,j+1,k}^n \\ & + \frac{1}{\Delta z^2} P_{i,j,k+1}^n + \frac{1}{\Delta z^2} P_{i,j,k-1}^n - \frac{Q\delta_{i,j,k}}{6\tilde{P}_{i,j,k}\Delta x\Delta y\Delta z}, \end{aligned}$$

Approximating (10*) by finite-difference scheme in Oy , we obtain [9–15]:

$$\begin{aligned} & \frac{P_{i-1,j,k}^{n+\frac{1}{3}} - 2P_{i,j,k}^{n+\frac{1}{3}} + P_{i+1,j,k}^{n+\frac{1}{3}}}{\Delta x^2} + \frac{P_{i,j-1,k}^{n+\frac{2}{3}} - 2P_{i,j,k}^{n+\frac{2}{3}} + P_{i,j+1,k}^{n+\frac{2}{3}}}{\Delta y^2} \\ & + \frac{P_{i,j,k-1}^{n+\frac{1}{3}} - 2P_{i,j,k}^{n+\frac{1}{3}} + P_{i,j,k+1}^{n+\frac{1}{3}}}{\Delta z^2} = \frac{1}{2\tilde{P}_{i,j,k}} \frac{P_{i,j,k}^{n+\frac{2}{3}} - P_{i,j,k}^{n+\frac{1}{3}}}{\Delta t/3} - \frac{Q\delta_{i,j,k}}{6\tilde{P}_{i,j,k}\Delta x\Delta y\Delta z}, \end{aligned} \quad (14)$$

Then, transforming the equation (14) to

$$\begin{aligned} & \frac{1}{\Delta x^2} P_{i-1,j,k}^{n+\frac{1}{3}} - \frac{2}{\Delta x^2} P_{i,j,k}^{n+\frac{1}{3}} + \frac{1}{\Delta x^2} P_{i+1,j,k}^{n+\frac{1}{3}} + \frac{1}{\Delta y^2} P_{i,j+1,k}^{n+\frac{2}{3}} - \frac{2}{\Delta y^2} P_{i,j,k}^{n+\frac{2}{3}} \\ & + \frac{1}{\Delta y^2} P_{i,j-1,k}^{n+\frac{2}{3}} + \frac{1}{\Delta z^2} P_{i,j,k+1}^{n+\frac{1}{3}} - \frac{2}{\Delta z^2} P_{i,j,k}^{n+\frac{1}{3}} + \frac{1}{\Delta z^2} P_{i,j,k-1}^{n+\frac{1}{3}} \\ & = \frac{3}{2\tilde{P}_{i,j,k}\Delta t} P_{i,j,k}^{n+\frac{2}{3}} - \frac{3}{2\tilde{P}_{i,j,k}\Delta t} P_{i,j,k}^{n+\frac{1}{3}} - \frac{Q\delta_{i,j,k}}{6\tilde{P}_{i,j,k}\Delta x\Delta y\Delta z}, \end{aligned} \quad (15)$$

and grouping similar terms, equation (15) is reduced to a system of linear algebraic equations (SLAE):

$$\bar{a}_{i,j,k} P_{i,j-1,k}^{n+\frac{2}{3}} - \bar{b}_{i,j,k} P_{i,j,k}^{n+\frac{2}{3}} + \bar{c}_{i,j,k} P_{i,j+1,k}^{n+\frac{2}{3}} = -\bar{d}_{i,j,k}, \quad (16)$$

where

$$\begin{aligned} \bar{a}_{i,j,k} &= \frac{1}{\Delta y^2}, \quad \bar{b}_{i,j,k} = \frac{2}{\Delta y^2} + \frac{3}{2\tilde{P}_{i,j,k}\Delta t}, \quad \bar{c}_{i,j,k} = \frac{1}{\Delta y^2}, \\ \bar{d}_{i,j,k} &= \left(\frac{3}{2\tilde{P}_{i,j,k}\Delta t} - \frac{2}{\Delta x^2} - \frac{2}{\Delta z^2} \right) P_{i,j,k}^{n+\frac{1}{3}} + \frac{1}{\Delta x^2} P_{i-1,j,k}^{n+\frac{1}{3}} + \frac{1}{\Delta x^2} P_{i+1,j,k}^{n+\frac{1}{3}} \\ & + \frac{1}{\Delta z^2} P_{i,j,k-1}^{n+\frac{1}{3}} + \frac{1}{\Delta z^2} P_{i,j,k+1}^{n+\frac{1}{3}} - \frac{Q\delta_{i,j,k}}{6\tilde{P}_{i,j,k}\Delta x\Delta y\Delta z}. \end{aligned}$$

We approximate (10*) by finite-difference scheme in Oz , we obtain [9–15]:

$$\frac{P_{i-1,j,k}^{n+\frac{2}{3}} - 2P_{i,j,k}^{n+\frac{2}{3}} + P_{i+1,j,k}^{n+\frac{2}{3}}}{\Delta x^2} + \frac{P_{i,j-1,k}^{n+\frac{2}{3}} - 2P_{i,j,k}^{n+\frac{2}{3}} + P_{i,j+1,k}^{n+\frac{2}{3}}}{\Delta y^2} + \frac{P_{i,j,k-1}^{n+1} - 2P_{i,j,k}^{n+1} + P_{i,j,k+1}^{n+1}}{\Delta z^2} = \frac{1}{2\tilde{P}_{i,j,k}} \frac{P_{i,j,k}^{n+1} - P_{i,j,k}^{n+\frac{2}{3}}}{\Delta t/3} - \frac{Q\delta_{i,j,k}}{6\tilde{P}_{i,j,k}\Delta x\Delta y\Delta z}. \tag{17}$$

We transform the equation (17) to the following form:

$$\begin{aligned} &\frac{1}{\Delta x^2}P_{i-1,j,k}^{n+\frac{2}{3}} - \frac{2}{\Delta x^2}P_{i,j,k}^{n+\frac{2}{3}} + \frac{1}{\Delta x^2}P_{i+1,j,k}^{n+\frac{2}{3}} + \frac{1}{\Delta y^2}P_{i,j-1,k}^{n+\frac{2}{3}} - \frac{2}{\Delta y^2}P_{i,j,k}^{n+\frac{2}{3}} \\ &+ \frac{1}{\Delta y^2}P_{i,j+1,k}^{n+\frac{2}{3}} + \frac{1}{\Delta z^2}P_{i,j,k-1}^{n+1} - \frac{2}{\Delta z^2}P_{i,j,k}^{n+1} + \frac{1}{\Delta z^2}P_{i,j,k+1}^{n+1} \\ &= \frac{3}{2\tilde{P}_{i,j,k}\Delta t}P_{i,j,k}^{n+1} - \frac{3}{2\tilde{P}_{i,j,k}\Delta t}P_{i,j,k}^{n+\frac{2}{3}} - \frac{Q\delta_{i,j,k}}{6\tilde{P}_{i,j,k}\Delta x\Delta y\Delta z}. \end{aligned} \tag{18}$$

Grouping similar terms, equation (18) becomes SLAE:

$$\bar{a}_{i,j,k}P_{i,j,k-1}^{n+1} - \bar{b}_{i,j,k}P_{i,j,k}^{n+1} + \bar{c}_{i,j,k}P_{i,j,k+1}^{n+1} = -\bar{d}_{i,j,k}, \tag{19}$$

where

$$\begin{aligned} \bar{a}_{i,j,k} &= \frac{1}{\Delta z^2}, \quad \bar{b}_{i,j,k} = \frac{2}{\Delta z^2} + \frac{3}{2\tilde{P}_{i,j,k}\Delta t}, \quad \bar{c}_{i,j,k} = \frac{1}{\Delta z^2}, \\ \bar{d}_{i,j,k} &= \left(\frac{3}{2\tilde{P}_{i,j,k}\Delta t} - \frac{2}{\Delta x^2} - \frac{2}{\Delta y^2} \right) P_{i,j,k}^{n+\frac{2}{3}} + \frac{1}{\Delta x^2}P_{i-1,j,k}^{n+\frac{2}{3}} + \frac{1}{\Delta x^2}P_{i+1,j,k}^{n+\frac{2}{3}} \\ &+ \frac{1}{\Delta y^2}P_{i,j-1,k}^{n+\frac{2}{3}} + \frac{1}{\Delta y^2}P_{i,j+1,k}^{n+\frac{2}{3}} - \frac{Q\delta_{i,j,k}}{6\tilde{P}_{i,j,k}\Delta x\Delta y\Delta z}. \end{aligned}$$

We solve the equations (13), (16) and (19) by the sweep method:

by Ox

$$P_{i,j,k}^{n+\frac{1}{3}} = \alpha_{i,j,k}P_{i+1,j,k}^{n+\frac{1}{3}} + \beta_{i,j,k}, \quad i = \overline{1, N_x - 1}, \tag{20}$$

by Oy

$$P_{i,j,k}^{n+\frac{2}{3}} = \bar{\alpha}_{i,j,k}P_{i,j+1,k}^{n+\frac{2}{3}} + \bar{\beta}_{i,j,k}, \quad j = \overline{1, N_y - 1}, \tag{21}$$

by Oz

$$P_{i,j,k}^{n+1} = \bar{\bar{\alpha}}_{i,j,k}P_{i,j,k+1}^{n+1} + \bar{\bar{\beta}}_{i,j,k}, \quad k = \overline{1, N_z - 1}. \tag{22}$$

To find $\alpha_i, \beta_i, \bar{\alpha}_j, \bar{\beta}_j$ and $\bar{\bar{\alpha}}_k, \bar{\bar{\beta}}_k$, respectively, in (20), (21) and (22), replace i with $i - 1, j$ with $j - 1, k$ with $k - 1$, and obtain

$$P_{i-1,j,k}^{n+\frac{1}{3}} = \alpha_{i-1}P_{i,j,k}^{n+\frac{1}{3}} + \beta_{i-1}, \quad P_{i,j-1,k}^{n+\frac{2}{3}} = \bar{\alpha}_{j-1}P_{i,j,k}^{n+\frac{2}{3}} + \bar{\beta}_{j-1}, \quad P_{i,j,k-1}^{n+1} = \bar{\bar{\alpha}}_{k-1}P_{i,j,k}^{n+1} + \bar{\bar{\beta}}_{k-1},$$

where the coefficients $\alpha_i, \beta_i, \bar{\alpha}_j, \bar{\beta}_j, \bar{\bar{\alpha}}_k, \bar{\bar{\beta}}_k$ are unknown.

Substituting the latter, respectively (13), (16) and (19), we exclude $P_{i-1,j,k}^{n+\frac{1}{3}}, P_{i,j-1,k}^{n+\frac{2}{3}}$ and $P_{i,j,k-1}^{n+1}$. After some simple calculations, we obtain formulas for calculating the sweep coefficients in the directions Ox, Oy, Oz :

$$\alpha_{i,j,k} = \frac{a_{i,j,k}}{b_{i,j,k} - c_{i,j,k}\alpha_{i-1,j,k}}, \quad \beta_{i,j,k} = \frac{c_{i,j,k}\beta_{i-1,j,k} + d_{i,j,k}}{b_{i,j,k} - c_{i,j,k}\alpha_{i-1,j,k}}, \tag{23}$$

$$\bar{\alpha}_{i,j,k} = \frac{\bar{a}_{i,j,k}}{\bar{b}_{i,j,k} - \bar{c}_{i,j,k}\bar{\alpha}_{i,j-1,k}}, \quad \bar{\beta}_{i,j,k} = \frac{\bar{c}_{i,j,k}\bar{\beta}_{i,j-1,k} + \bar{d}_{i,j,k}}{\bar{b}_{i,j,k} - \bar{c}_{i,j,k}\bar{\alpha}_{i,j-1,k}}, \tag{24}$$

$$\bar{\alpha}_{i,j,k} = \frac{\bar{a}_{i,j,k}}{\bar{b}_{i,j,k} - \bar{c}_{i,j,k}\bar{\alpha}_{i,j,k-1}}, \quad \bar{\beta}_{i,j,k} = \frac{\bar{c}_{i,j,k}\bar{\beta}_{i,j,k-1} + \bar{d}_{i,j,k}}{\bar{b}_{i,j,k} - \bar{c}_{i,j,k}\bar{\alpha}_{i,j,k-1}}. \quad (25)$$

These formulas are used to calculate sweep coefficient values. In formulas (23), (24) and (25) it is necessary, to start the calculations, to find α_0 and β_0 . We determine them from the boundary conditions.

If the first boundary condition on G is given, then

$$P(x, y, z, t)|_{\Gamma} = P_0, \quad \text{then} \quad \alpha_0 = 0, \quad \beta_0 = P_0. \quad (26)$$

If $\frac{\partial P}{\partial n}|_{\Gamma} = 0$ is given, using fictitious filtering region, in each direction we approximate the boundary condition, i.e.,

by Ox direction

$$\frac{\partial P}{\partial x_i} \Big|_{x_i=0} = \frac{P_{1,j,k} - P_{0,j,k} - 0.5(P_{2,j,k} - 2P_{1,j,k} - P_{0,j,k})}{\Delta x} = 0, \quad (27)$$

$$\frac{\partial P}{\partial x_i} \Big|_{x_i=1} = \frac{P_{N_x,j,k} - P_{N_x-1,j,k} - 0.5(P_{N_x+1,j,k} - 2P_{N_x,j,k} - P_{N_x-1,j,k})}{\Delta x} = 0, \quad (28)$$

by Oy

$$\frac{\partial P}{\partial y_j} \Big|_{y_j=0} = \frac{P_{i,1,k} - P_{i,0,k} - 0.5(P_{i,2,k} - 2P_{i,1,k} - P_{i,0,k})}{\Delta y} = 0, \quad (29)$$

$$\frac{\partial P}{\partial y_j} \Big|_{y_j=1} = \frac{P_{i,N_y,k} - P_{i,N_y-1,k} - 0.5(P_{i,N_y+1,k} - 2P_{i,N_y,k} - P_{i,N_y-1,k})}{\Delta y} = 0, \quad (30)$$

by Oz

$$\frac{\partial P}{\partial z_k} \Big|_{z_k=0} = \frac{P_{i,j,1} - P_{i,j,0} - 0.5(P_{i,j,2} - 2P_{i,j,1} - P_{i,j,0})}{\Delta z} = 0, \quad (31)$$

$$\frac{\partial P}{\partial z_k} \Big|_{z_k=1} = \frac{P_{i,j,N_z} - P_{i,j,N_z-1} - 0.5(P_{i,j,N_z+1} - 2P_{i,j,N_z} - P_{i,j,N_z-1})}{\Delta z} = 0. \quad (32)$$

As noted above, the problem is described using nonlinear partial differential equations, which can be solved using an iterative method. The conditions for the convergence of the iterative process are as follows:

$$|(P_{i,j,k}^n)^{(s+1)} - (P_{i,j,k}^n)^{(s)}| \leq \varepsilon,$$

where ε is the given value (10^{-4}).

4. Results and discussion

On the basis of the developed mathematical apparatus, a software package has been created and there were carried out series of computational experiments for various parameters and conditions of the process of gas filtration in porous media.

According to the analysis of the obtained results, the main factors affecting the process of gas filtration in porous media are: coefficients of permeability and porosity of the medium, heterogeneity and arbitrary shape of the filtration area, reservoir thickness, as well as gas recovery and viscosity coefficients.

As it follows from the numerical calculations, the value of the pressure around the wells gradually decreases when the filtration coefficient is constant, and the pressure decreasing propagates within the considered area by time passing.

As it follows from Figs. 1–4, with an increase of the values of thickness and porosity of the reservoir, the pressure drop in filtration area slows down.

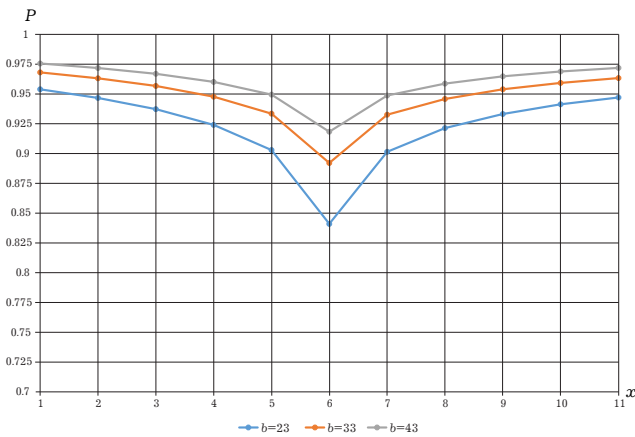


Fig. 1. The dynamics of pressure redistribution in the reservoir at different values of the reservoir thickness ($\tau = 365$ days).

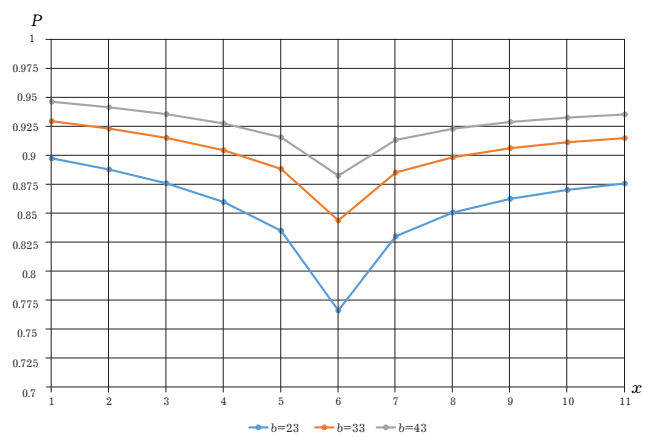


Fig. 2. The dynamics of pressure redistribution in the reservoir at different values of the reservoir thickness ($\tau = 1825$ days).

It can be seen from the computational experiments (Figs.1–4) that the smaller the value of the thickness and porosity of the reservoir is — the faster the gas pressure drop, and vice versa — the greater the value of the thickness and porosity of the reservoir is — the slower the gas pressure drop.

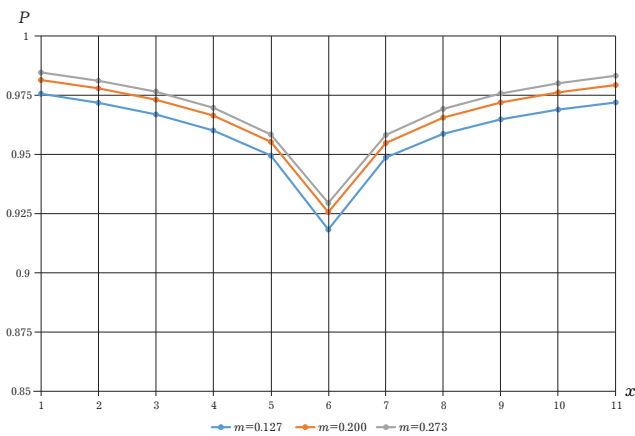


Fig. 3. Changing of the pressure along the length of the reservoir depending on its porosity ($\tau = 365$ days).

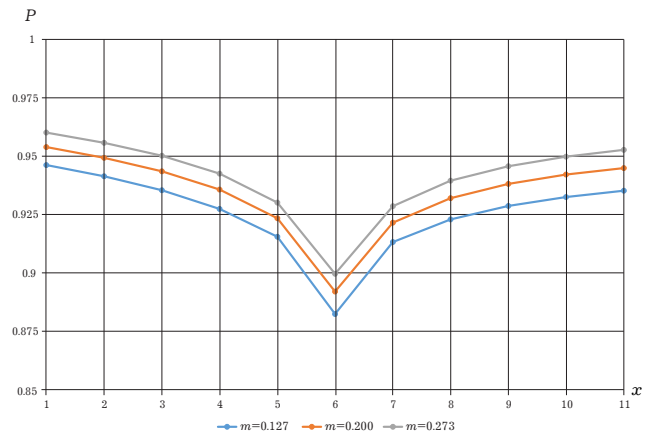


Fig. 4. Changing of the pressure along the length of the reservoir depending on its porosity ($\tau = 1825$ days).

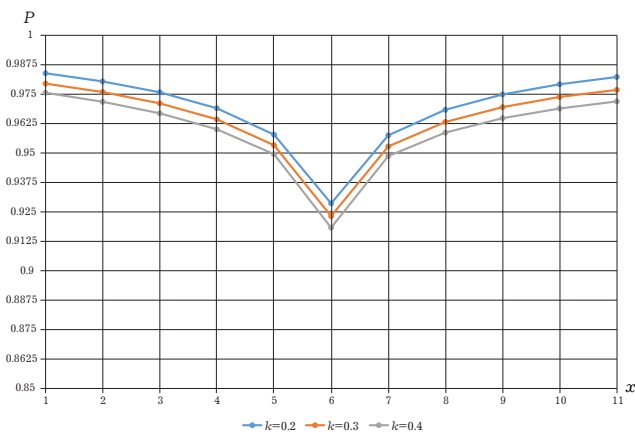


Fig. 5. Changing the pressure along the length of the reservoir depending on its permeability ($\tau = 365$ days).

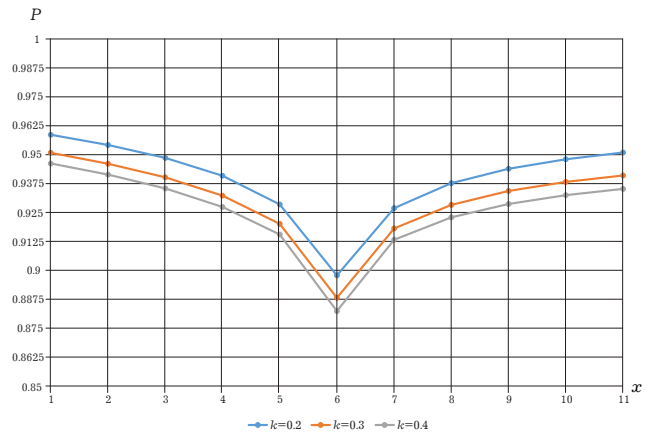


Fig. 6. Changing the pressure along the length of the reservoir depending on its permeability ($\tau = 1825$ days).

The analysis of the performed numerical calculations (Figs. 5, 4) showed that with an increase in the permeability coefficient, changes of pressure in the filtration area significantly decreases. This is especially noticeable at the points where the wells are located and in the area around them. With increasing the permeability coefficient, the area of influence of wells on the considered process increases significantly with time. As the value of the permeability coefficient increases, the gas flow through the rock from adjacent areas improves within the well circumference. Thus, there is not a sharp drop in gas pressure at the well locations but a uniform pressure distribution around the well locations occurs.

5. Conclusion

A three-dimensional mathematical model of the gas filtration process in porous media described using the nonlinear differential equation has been developed. This model serves as one of the means for solving the problem of hydrodynamic calculations in porous media, taking into account changes in the most critical indicators and parameters of the object (coefficients of permeability, porosity, and reservoir thickness, coefficients of oil and gas recovery, viscosity, etc.).

A numerical algorithm for the process of gas filtration in porous media has been developed. On the basis of the stated numerical algorithm, the software tool has been created for analyzing the functioning and control of the gas production process from reservoir systems.

The developed mathematical software can be used in order to design, forecast, and refine design solutions for gas fields based on the analysis of gas pressure in the filtration area in order to increase well flow rates and gas recovery in the field region.

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3D-модель та чисельний алгоритм фільтрації газу в пористих середовищах

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У статті представлена тривимірна математична модель процесу фільтрації газу в пористих середовищах та чисельний алгоритм вирішення початково-крайової задачі. Розроблена модель описується з використанням нелінійного диференціального рівняння в часткових похідних з відповідними початковими та граничними умовами. Запропонований математичний апарат дозволяє проводити гідродинамічні розрахунки з урахуванням змін основних факторів, що впливають на процес, що розглядається: проникність, пористість і товщина шарів, коефіцієнт відновлення газу, в'язкість тощо. Комп'ютерна реалізація моделі забезпечує можливість вирішити практичні завдання аналізу та прогнозування процесу видобутку газу в різних умовах впливу на продуктивний пласт, а також прийняття рішень щодо розробки існуючих та проектування нових родовищ газу.

Ключові слова: *математична модель, числовий метод, фільтрація, газ, пористе середовище, свердловина, проникність, в'язкість.*