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Pricing equity warrants with jumps, stochastic volatility, and stochastic interest rates

Sawal A. S.[1], Ibrahim S. N. I.[1,2,*], Roslan T. R. N.[3]

[1] *Institute for Mathematical Research, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, Malaysia*
[2] *Department of Mathematics and Statistics, Faculty of Science, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, Malaysia*
[3] *Othman Yeop Abdullah Graduate School of Business, Universiti Utara Malaysia, 50300 Kuala Lumpur, Malaysia*
[*] *Corresponding Author: iqmal@upm.edu.my*

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A warrant is a derivative that gives the right, but not the obligation, to buy or sell a security at a certain price before the expiration. The warrant valuation method was inspired by option valuation because of the certain similarities between these two derivatives. The warrant price formula under the Black–Scholes is available in the literature. However, the Black–Scholes formula is known to have a number of flaws; hence, this study aims to develop a pricing formula for warrants by incorporating jumps, stochastic volatility, and stochastic interest rates into the Black–Scholes model. The closed-form pricing formula is presented in this study, where the derivation involves stochastic differential equations (SDE), which include the Cauchy problem and heat equation.

Keywords: *equity warrant, jump, stochastic volatility, stochastic interest rate.*

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1. Introduction

A warrant is a financial security that permits the holder to purchase the issuing company's underlying assets at a strike price until the maturity date. The call warrants give the holder a right, but not the obligation, to buy the underlying asset at a predetermined exercise price on or before the predetermined expiry date. In contrast, the put warrants give the holder a right, but not the obligation, to sell the underlying asset at a predetermined exercise price on or before the predetermined expiry date. The underlying asset could be stocks, indices, or commodities. Warrants traded on the Bursa Malaysia market are securities that can be traded like stocks. In the market, two types of warrants are available: call and put warrants, but Bursa Malaysia trades only call warrants.

While options traded on exchanges are issued by exchanges, warrants are issued by companies. In addition, warrants generally have longer expiration dates than options, and warrants cause dilution because a company is obligated to issue new shares when the warrant is exercised. Warrants are further classified as covered warrants and equity warrants. The former operates like options but with a longer time frame and are typically issued by traders and financial sectors. Additionally, they are for investors who do not want to sell their shares after expiration. While the latter is only to be issued by the listed companies, and the underlying assets are the issued stock of that company. Moreover, the company is required to issue new stocks when an equity warrant is exercised; hence dilution

Warrant pricing is important because it helps traders to value their products accurately to avoid losses. Basically, the classic methodology for warrant pricing is based on the Black–Scholes model [1], in

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which the underlying asset price follows a geometric Brownian motion (GBM). However, many studies have revealed the drawbacks of the Black–Scholes model, such as volatility smile [2, 3]. Therefore, extension and development of new underlying asset price processes have been proposed to better fit the empirical facts, for instance, jump diffusion [4], stochastic volatility [5], and stochastic interest rates [6]. In [9], the prices of equity warrants were modelled with stochastic volatility and stochastic interest rate, and showed improvement in the estimation of the prices. Hence, to better fit the stylized facts exhibited by underlying asset prices [7], we develop a model that extends the model of [9] by including jumps to the asset price dynamics. Thus, our model corresponds better with the real market.

The aim of this paper is modeling the underlying asset price with a more general model that includes jumps, stochastic volatility, and stochastic interest rate. This paper is organized as follows. In Section 2, we provide the process of the underlying asset that connects jumps, stochastic volatility, and stochastic interest rate, and continue with the derivation of the pricing formula. In Section 3, we simulate the stochastic processes involved via Euler discretization to enable us to price the equity warrants using Monte Carlo simulation technique. Section 4 documents the numerical experiment, providing a setup to price equity warrants using Monte Carlo simulation method. Section 5 concludes this paper.

2. The pricing formula

The Black–Scholes model assumed constant volatility and constant interest rates, which shows inconsistency with the evidence of volatility smile and stochastic interest rates, proven by empirical stylized facts. This section provides the stochastic processes that are used in our model where the dynamics of the asset price is assumed to incorporate jumps [4], stochastic volatility [5], and stochastic interest rates [8] (JSVSR). The JSVSR model is defined as follows:

$$dS(t) = r(t)S(t)dt + \sqrt{v(t)}S(t)dW_1(t) + S(t)(Y(t) - 1)dN(t), \tag{1}$$

$$dv(t) = \kappa(\theta - v(t))dt + \sigma_v\sqrt{v(t)}dW_2(t), \tag{2}$$

$$dr(t) = \alpha(\beta - r(t))dt + \eta\sqrt{r(t)}dW_3(t). \tag{3}$$

Equation (1) models the underlying asset price process where $S(t)$ is the underlying asset, $r(t)$ is the interest rate, and $v(t)$ is the volatility. Moreover, it has a jump component depicted by the jump size, $Y(t) > 0$ with $(1 + Y(t)) \sim \text{lognormal}(\mu, \delta^2)$, where $k_j = e^{\mu+\frac{\delta^2}{2}}$, and a Poisson process, $N(t)$ with intensity λ. Following that, Equation (2) describes the process of its volatility, where κ is the rate of mean reversion, θ is the long-term variance, and σ_v is the volatility of the volatility. Equation (3) describes the process of its interest rate, where α is the speed of the mean-reversion, β is the long-term interest rate, and η is the volatility of the interest rate. The Wiener processes $W_1(t)$, $W_2(t)$ and $W_3(t)$ are correlated such that $\langle W_1(t), W_2(t)\rangle = \rho dt$, where $\rho \in (-1, 1)$, and $\langle W_1(t), W_3(t)\rangle = \langle W_2(t), W_3(t)\rangle = 0$ for $t \in [0, T]$.

On the account of independence of the jumps and the Poisson process, Equation (1) can be represented as follows:

$$dS(t) = [r(t) - \lambda k_j]S(t)dt + \sqrt{v(t)}S(t)dW_1(t) + S(t)(Y(t) - 1)dN(t), \tag{4}$$

$$dv(t) = \kappa(\theta - v(t))dt + \sigma_v\sqrt{v(t)}dW_2(t), \tag{5}$$

$$dr(t) = \alpha(\beta - r(t))dt + \eta\sqrt{r(t)}dW_3(t). \tag{6}$$

In addition, following [10], we let the price of a zero-coupon bond $P(r, t, T)$ with maturity T at time $t \in [0, T]$ is described by:

$$P(r, t, T) = A(t, T)\exp\{-B(t, T)r(t)\},$$

where

$$A(t,T)=\left[\frac{2\exp\left[\left(\alpha+\sqrt{\alpha^2+2\eta^2}\right)\frac{T-t}{2}\right]\sqrt{\alpha^2+2\eta^2}}{2\sqrt{\alpha^2+2\eta^2}+\left(\alpha+\sqrt{\alpha^2+2\eta^2}\right)\left(\exp\left[\sqrt{\alpha^2+2\eta^2}(T-t)\right]-1\right)}\right]^{\frac{2\alpha\beta}{\eta^2}},$$

$$B(t,T)=\frac{2\left(\exp\left[\sqrt{\alpha^2+2\eta^2}(T-t)\right]-1\right)}{2\sqrt{\alpha^2+2\eta^2}+\left(\alpha+\sqrt{\alpha^2+2\eta^2}\right)\left(\exp\left[\sqrt{\alpha^2+2\eta^2}(T-t)\right]-1\right)}.$$

Now, given the payoff of an equity warrant as such

$$\frac{1}{N+Mk}\max\left[kS(t)-NG,0\right], \tag{7}$$

where N is the number of common stocks, M is the number of outstanding warrants, G is the premium paid by the holder when k shares are received at maturity T. The partial differential equation (PDE) for the equity warrant value $V(t)$ driven by the processes (4), (5) and (6) is presented as follows

$$\begin{aligned}\frac{\partial V}{\partial t}&+\frac{1}{2}vS^2\frac{\partial^2 V}{\partial S^2}+\frac{1}{2}\sigma_0^2 v\frac{\partial^2 V}{\partial v^2}+\frac{1}{2}\eta^2 r\frac{\partial^2 V}{\partial r^2}+rS\frac{\partial V}{\partial S}+\kappa(\theta-v)\frac{\partial V}{\partial v}\\&+\left[\alpha\beta-\left(\alpha+B(t,T)\eta^2\right)r\right]\frac{\partial V}{\partial r}+\rho\sigma vS\frac{\partial^2 V}{\partial S\partial v}+\lambda\mathbb{E}[V(Y,t)-V(t)]-\lambda S\frac{\partial V}{\partial S}\mathbb{E}[Y-1]=0,\end{aligned} \tag{8}$$

with terminal condition shown in Equation (7). We now let

$$y=\frac{S}{P(r,t,T)},\quad \hat{E}(Y,y,t,L)=\frac{E(S,v,r,T,Y)}{P(r,t,T)},\quad L=v.$$

Then by solving the partial derivatives in Equation (8) with

$$\frac{\partial P}{\partial r}=-P(r,t,T)B(t,T),\quad \frac{\partial^2 P}{\partial r^2}=-P(r,t,T)B^2(t,T),\quad S^2=y^2P^2,$$

allows us to rewrite the PDE in Equation (8) as follows

$$\frac{\partial\hat{V}}{\partial t}+\frac{1}{2}\sigma^2 L\frac{\partial^2\hat{V}}{\partial L^2}+\left(\frac{1}{2}y^2L+\frac{1}{2}\eta^2 r(yB(t,T))^2\right)\frac{\partial^2\hat{V}}{\partial y^2}+\lambda\mathbb{E}[V(Y,t)-V(t)]-\lambda yP\frac{\partial\hat{V}}{\partial y}\mathbb{E}[Y-1]=0. \tag{9}$$

Let $x=\ln y$. Hence, the PDE in Equation (9) can be written as follows

$$\begin{aligned}\frac{\partial\hat{V}}{\partial t}&+\frac{1}{2}\sigma^2 L\frac{\partial^2\hat{V}}{\partial L^2}-\left(\frac{1}{2}L+\frac{1}{2}\eta^2 rB^2(t,T)\right)\frac{\partial\hat{V}}{\partial x}\\&+\left(\frac{1}{2}L+\frac{1}{2}\eta^2 rB^2(t,T)\right)\frac{\partial^2\hat{V}}{\partial x^2}+\lambda\mathbb{E}[V(Y,t)-V(t)]-\lambda P\frac{\partial\hat{V}}{\partial x}\mathbb{E}[Y-1]=0.\end{aligned} \tag{10}$$

Suppose that $\hat{E}(Y,y,t,L)=u(Y,\hat{\eta},\tau,\hat{\lambda})$ and $\hat{\eta}=x+\hat{\alpha}(t)$, where $\hat{\alpha}(T)=\omega(T)=0$, $\tau=\omega(t)$ and $\hat{\lambda}=L+h(t)$. We then solve the partial derivatives in Equation (10) and we rewrite the PDE as such:

$$\begin{aligned}&\left[\hat{\alpha}'(t)-\frac{1}{2}L-\frac{1}{2}\eta^2 rB^2(t,T)-\lambda Pe^{\mu+\frac{1}{2}\delta^2}\right]\frac{\partial u}{\partial\hat{\eta}}+\left[\frac{1}{2}L+\frac{1}{2}\eta^2 rB^2(t,T)\right]\frac{\partial^2 u}{\partial\hat{\eta}^2}\\&\qquad+\omega'(t)\frac{\partial u}{\partial\tau}+h'(t)\frac{\partial u}{\partial\hat{\lambda}}+\frac{1}{2}\sigma^2 L\frac{\partial^2 u}{\partial\hat{\lambda}^2}+\lambda\mathbb{E}[V(Y,t)-V(t)]=0,\end{aligned} \tag{11}$$

where the terms $\mathbb{E}[V(Y,t)-V(t)]$ involves the expectation operator and $\mathbb{E}[y_t-1]=e^{\mu+\frac{1}{2}\delta^2}-1\equiv k_j$, which is the mean of the relative asset price jump size.

Then, by substituting $\frac{\partial^2 u}{\partial \hat{\eta}^2} = \frac{\partial u}{\partial \tau}$ and $\frac{\partial^2 \hat{E}}{\partial \hat{\lambda}^2} = \frac{\partial u}{\partial \hat{\lambda}}$ into the PDE in Equation (11) yields

$$\begin{aligned}
\hat{\alpha}'(t) &= \frac{1}{2}L + \frac{1}{2}\eta^2 r B^2(t,T) - Pe^{\mu+\frac{1}{2}\delta^2},\\
\hat{\omega}'(t) &= -\frac{1}{2}L - \frac{1}{2}\eta^2 r B^2(t,T),\\
\hat{h}'(t) &= -\frac{1}{2}\sigma^2 L.
\end{aligned}$$

Subsequently, by integrating the above, results to

$$\begin{aligned}
\hat{\alpha}(t) &= \int_t^T \frac{1}{2}L + \frac{1}{2}\eta^2 r B^2(s,T) - Pe^{\mu+\frac{1}{2}\delta^2} ds,\\
\hat{\omega}(t) &= -\int_t^T \frac{1}{2}L + \frac{1}{2}\eta^2 r B^2(s,T)\, ds,\\
\hat{h}(t) &= -\int_t^T \frac{1}{2}\sigma^2 L\, ds,
\end{aligned}$$

which by the explicit solution of an one-dimensional heat equation, we have the following presentation of the payoff

$$\begin{aligned}
\hat{E} = u(Y,\hat{\eta},\tau,\hat{\lambda}) &= \sum_{i=0}^{\infty} \frac{e^{-\bar{\lambda}\tau}(\bar{\lambda}\tau)^i}{i!}\left[\frac{1}{2\sqrt{\pi\tau}}\int_{\ln\frac{NG}{k}}^{+\infty} \frac{1}{N+Mk}(ke^{\hat{\eta}} - NG)e^{-\frac{(\hat{\eta}-\xi)^2}{4\tau}} d\xi\right],\\
&= \sum_{i=0}^{\infty} \frac{e^{-\bar{\lambda}\tau}(\bar{\lambda}\tau)^i}{i!}\left[\frac{k}{2\sqrt{\pi\tau}}\int_{\ln\frac{NG}{k}}^{+\infty} \frac{e^{\xi}e^{-\frac{(\hat{\eta}-\xi)^2}{4\tau}}}{N+Mk} d\xi - \frac{NG}{2\sqrt{\pi\tau}}\int_{\ln\frac{NG}{k}}^{+\infty} \frac{e^{-\frac{(\hat{\eta}-\xi)^2}{4\tau}}}{N+Mk} d\xi\right], \qquad (12)
\end{aligned}$$

where by solving the integration terms in Equation (12) returns the pricing formula for equity warrants with jumps, stochastic volatility and stochastic interest rates as given in the following theorem.

Theorem 1. *Given an underlying asset price S where its dynamics incorporates jumps, stochastic volatility, and stochastic interest rates, N number of common stocks, M number of outstanding warrants, G premium paid by the holder when k shares are received at maturity T. Then the price of an equity warrant, V, discounted at a risk-free interest rate, r, of its payoff given as such*

$$\frac{e^{-r(T-t)}}{N+Mk}\left[\max\left(kS_T - NG, 0\right)\right],$$

is obtained as follows

$$V = e^{-r(T-t)} \sum_{n=0}^{\infty} \frac{(\lambda\tau)^i e^{-\lambda\tau})}{i} \left(\frac{1}{N+Mk}\left[kS(T)\phi(d_1)e^{r_i(T-t)} - NG\phi(d_2)\right]\right), \qquad (13)$$

where $\phi(\cdot)$ is a cumulative distribution function, $\tau = T - t$, and

$$\begin{aligned}
d_1 &= \frac{\ln\frac{kS}{NG} - \ln P(r,t,T) + \frac{1}{2}L(T-t) + \frac{1}{2}Q}{\sqrt{L(T-t)+Q}},\\
d_2 &= \frac{\ln\frac{kS}{NG} - \ln P(r,t,T) - \frac{1}{2}L(T-t) - \frac{1}{2}Q}{\sqrt{L(T-t)+Q}},\\
Q &= \eta^2 r_i \int_t^T \left(\frac{2(e^{2R} - 2e^R + 1)}{2((\alpha)^2 + 2\eta^2) + (e^R - 1)(C)}\right),\\
R &= (T-s)\sqrt{(\alpha)^2 + 2\eta^2},\\
C &= \alpha\sqrt{(\alpha)^2 + 2\eta^2} + (\alpha)^2 + 3\eta^2 + (e^R)\left((\alpha)^2 + \alpha\sqrt{(\alpha)^2 + 2\eta^2} + \eta^2\right),
\end{aligned}$$

$$k_j = e^{\mu+\frac{1}{2}\delta^2} - 1,$$
$$\sigma_i^2 = \sigma_s^2 + \frac{i\delta^2}{\tau},$$
$$r_i = r - \lambda k_j + \frac{i \ln(1+k_j)}{\tau}.$$

This completes the derivation.

3. Monte Carlo simulation technique

The main concern in financial matters is to estimate the certain amount of derivative securities and portfolio risks. In discussion of probability theory, the mathematical expectation of a random variable estimated this certain value. A well-known method for calculating mathematical expectation is Monte Carlo simulation. The Monte Carlo method has advantages over other methods. For instance, it is straightforward to apply when pricing derivatives since it estimates after simulating sample paths of the underlying. In this section, we provide the setup to price equity warrants under the processes in Section 2 using Monte Carlo simulation via Euler discretization.

By letting $t = t_0 < t_1 < t < M = T$ be a partition of the time interval $[t, T]$ of M equal segments of length Δt_i where $\Delta t_i = t_i - t_{i-1}$ and $t_i = \frac{iT}{M}$ for $i = 1, \ldots, M$, Equations (4), (5) and (6) are discretized as follows

$$S_i = S_{i-1} + (r_{i-1} - \lambda k_j) S_{i-1} \Delta t_i + \sqrt{V_{i-1}\Delta t_i} Z_1 + S_{i-1} \sum_{i}^{N_t} Y_i, \tag{14}$$

$$V_i = V_{i-1} + \kappa(\theta - V_{i-1})\Delta t_i + \sigma_v \sqrt{V_{i-1}\Delta t_i} Z_2, \tag{15}$$

$$r_i = r_{i-1} + \alpha(\beta - r_{i-1})\Delta t_i + \eta \sqrt{r_{i-1}\Delta t_i} Z_3, \tag{16}$$

where $V_i^+ = \max(V_i, 0)$ and $Z_1 = \rho Z_2 + \sqrt{1-\rho^2} Z$, to which $Z \sim \mathcal{N}(0,1)$ is independent of $Z_2 \sim \mathcal{N}(0,1)$. Given the mean price $\bar{V}$, the confidence interval at 95% is also computed as such

$$\left[\bar{V} - \frac{1.96\sigma}{\sqrt{M}}, \bar{V} + \frac{1.96\sigma}{\sqrt{M}}\right].$$

Hence, the setup for pricing equity warrants using Monte Carlo simulation technique is now complete.

4. Numerical experiment

This section documents the numerical results by pricing the equity warrants using the pricing formula obtained in Section 2 and the Monte Carlo simulation technique described in Section 3.

For numerical experiment, we selected 14 equity warrants provided in [9] as shown in Table 1.

Meanwhile, Table 2 lists the other parameters that are used to compute the prices of equity warrants using both Equation (13) and (14), where θ, σ_v, β, η, $r(0)$, $v(0)$, ρ, κ and α follow [9].

In Table 3, we document the actual and calculated prices of the selected equity warrants using JSVSR model given by Equation (13), and the model in [9].

It can be seen in Figure 1 that the actual and calculated prices using JSVSR and [9] models are very close to each other.

In order to measure the accuracy of Equation (13), we calculate the relative error, ε as follows, taken relative to Equation (13).

$$\varepsilon = \left| \frac{\text{Actual Price} - \text{Model Price}}{\text{Model Price}} \right| \times 100\%.$$

Table 1. List of selected equity warrants.

Names	Warrants O/S (‘000,000) M	Exercise Price G	Share per Warrant k	Shares O/S (‘000,000) N	Maturity T	Stock Price S
APPASIA-WA	135690400	0.13	1	341748000	10	0.0847
AZRB-WA	116201952	0.63	1	596435000	10	0.3052
BIMB-WA	426715078	4.72	1	1764283000	10	4.1956
BTM-WB	26295146	0.20	1	141344000	10	0.1250
DIGISTA-WB	74024334	0.26	1	650966000	10	0.0371
DNONCE-WA	51920700	0.25	1	261296000	5	0.2533
DOMINAN-WA	45643879	1.30	1	165240000	5	1.2082
DPS-WB	194261746	0.10	1	587770000	10	0.0571
ECOWLD-WA	525392340	2.08	1	2944368000	7	0.6694
EG-WC	68963282	0.42	1	257423000	5	0.3351
GPA-WA	490243800	0.10	1	980488000	10	0.0704
JIANKUN-WA	75586889	0.32	1	166845000	7	0.2749
LBS-WB	99949262	0.56	1	1592579000	5	0.4471
PENSONI-WB	64834000	0.60	1	129668000	10	0.2932
**O/S: Outstanding*						

Table 2. Chosen parameter values.

θ	σ_v	β	η	$r(0)$	$v(0)$	ρ	κ	α	μ	δ	λ	σ_s
0.5	0.1	1	0.7477	1	1	0.5	4	0.2403	0.0002	0.2570	0.01	0.0390

Table 3. Warrant Prices: JSVSR Model and [9] Model.

Names	Actual Price	JSVSR Model	[9] Model
APPASIA-WA	0.088	0.088000000000006	0.088000000148371
AZRB-WA	0.185	0.185000000000113	0.185000000252882
BIMB-WA	0.228	0.228000000000078	0.228000000898383
BTM-WB	0.118	0.118000000000057	0.118000000811604
DIGISTA-WB	0.018	0.018000000000001	0.018000000015144
DNONCE-WA	0.143	0.143000000000579	0.143000000000000
DOMINAN-WA	0.035	0.035000000051150	0.035000000000000
DPS-WB	0.038	0.038000000000001	0.038000000046692
ECOWLD-WA	0.238	0.238000000000150	0.238000000000001
EG-WC	0.073	0.073000000001358	0.073000000000000
GPA-WA	0.038	0.038000000000000	0.038000000026699
JIANKUN-WA	0.068	0.068000000000601	0.068000000001654
LBS-WB	0.053	0.053000000000135	0.053000000000000
PENSONI-WB	0.060	0.060000000003813	0.060000001570295

Moreover, for M sample size, we calculate the mean absolute error (MAE), the mean absolute percentage error (MAPE), and the root mean squared error (RMSE), as follows, respectively

$$\mathrm{MAE} = \frac{\sum_{i=0}^{M} |\text{Actual Price} - \text{Model Price}|}{\mathrm{M}},$$

$$\mathrm{MAPE} = \frac{\sum_{i=0}^{M} \left|\frac{\text{Actual Price}-\text{Model Price}}{\text{Actual Price}}\right|}{\mathrm{M}},$$

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=0}^{M} |\text{Actual Price} - \text{Model Price}|^2}{\mathrm{M}}}.$$

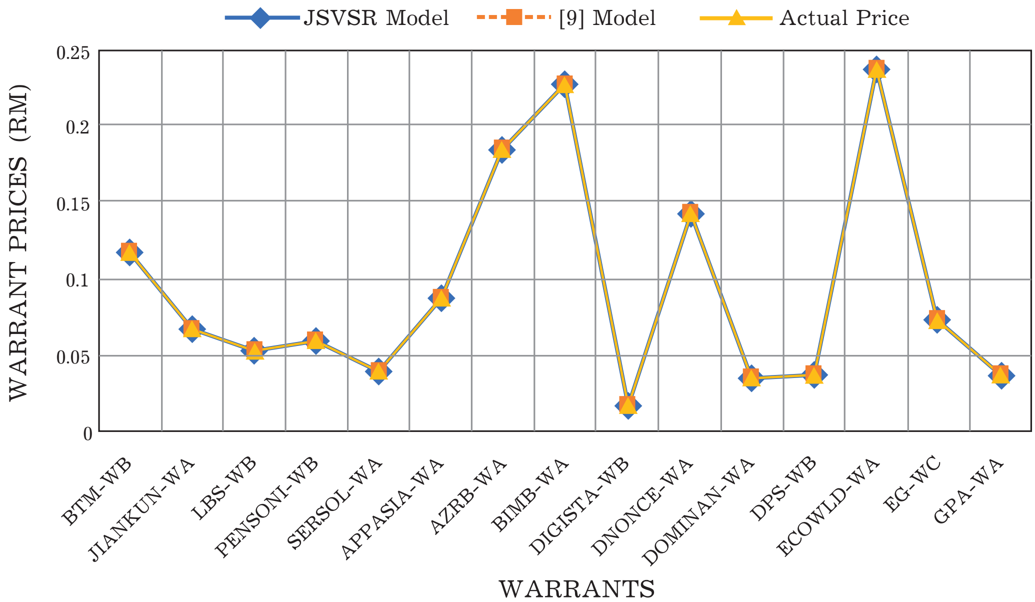

Fig. 1. Warrant Prices: Actual, JSVSR Model and [9] Model.

Table 4 tabulates the relative errors for the JSVSR model and [9] model. The other measurement errors are shown in Table 5.

Table 4. Relative Errors, ε: JSVSR Model and [9] Model.

Names	ε (JSVSR)	ε (Ref. [9])
APPASIA-WA	6.8285E-12	1.6860E-07
AZRB-WA	6.1077E-11	1.3669E-07
BIMB-WA	3.4208E-11	3.9403E-07
BTM-WB	4.8314E-11	6.8780E-07
DIGISTA-WB	5.5704E-12	8.4133E-08
DNONCE-WA	4.0490E-10	0
DOMINAN-WA	1.4614E-07	0
DPS-WB	2.6295E-12	1.2287E-07
ECOWLD-WA	6.3033E-11	4.1983E-13
EG-WC	1.8603E-09	0
GPA-WA	0	7.0261E-08
JIANKUN-WA	8.8381E-10	2.4323E-09
LBS-WB	2.5472E-10	0
PENSONI-WB	6.3550E-09	2.6172E-06

Table 5. Error Measurements: MAE, MAPE and RMSE.

Errors	JSVSR Model	[9] Model
MAE	3.870536E-12	2.516995E-10
MAPE	1.041088E-10	2.862267E-09
RMSE	1.325007E-11	5.177336E-10

To compute the price of the equity warrants using Monte Carlo simulation (14), we used 10 000 simulations. The actual and computed prices are shown in Table 6, with its 95% confidence interval (CI).

The prices in Table 6 are plotted in Figure 2.

The relative errors for computation of the prices using Monte Carlo simulation are given in Table 7.

Table 6. Warrant Prices via Monte Carlo Simulation: JSVSR Model and [9] Model.

Names	Actual Price	JSVSR Model 95%CI	[9] Model 95%CI
APPASIA-WA	0.0880	0.0811 $(-0.0823, 0.2601)$	0.0782 $(-0.0583, 0.2146)$
AZRB-WA	0.1850	0.1881 $(-0.1121, 0.3458)$	0.1727 $(-0.1629, 0.5083)$
BIMB-WA	0.2280	0.2653 $(0.0188, 0.5117)$	0.1867 $(-0.1379, 0.5114)$
BTM-WB	0.1180	0.1179 $(-0.1070, 0.3428)$	0.1684 $(-0.1602, 0.4970)$
DIGISTA-WA	0.0180	0.01916 $(-0.0179, 0.0563)$	0.0119 $(-0.0114, 0.0351)$
DOMINAN-WA	0.0350	0.0329 $(-0.01299, 0.0788)$	0.0255 $(-0.0186, 0.0696)$
DNONCE-WB	0.1430	0.1676 $(-0.0645, 0.3997)$	0.1035 $(-0.0953, 0.1564)$
DPS-WB	0.0380	0.0343 $(-0.0329, 0.1016)$	0.0431 $(-0.0249, 0.1511)$
ECOWLD-WA	0.2380	0.2653 $(-0.0465, 0.3737)$	0.1867 $(-0.2032, 0.6268)$
EG-WC	0.0730	0.0748 $(-0.0327, 0.1185)$	0.0771 $(-0.1510, 0.5052)$
GPA-WA	0.0380	0.0356 $(-0.0189, 0.0767)$	0.0408 $(-0.0270, 0.0834)$
JIANKUN-WA	0.0680	0.0608 $(-0.0466, 0.1682)$	0.0316 $(-0.0250, 0.0882)$
LBS-WB	0.0530	0.0521 $(-0.0206, 0.0969)$	0.0433 $(-0.0499, 0.1541)$
PENSONI-WA	0.0600	0.0547 $(-0.0765, 0.2694)$	0.0523 $(-0.0484, 0.2369)$

Table 7. Relative Errors, ε: JSVSR Model and [9] Model via Monte Carlo Simulation.

Names	ε (JSVSR)	ε ([9])
APPASIA-WA	7.8676	11.1424
AZRB-WA	1.6885	6.6264
BIMB-WA	16.3436	18.0852
BTM-WB	0.0913	42.6951
DIGISTA-WA	6.4746	33.9953
DNONCE-WA	17.2241	27.6520
DOMINAN-WB	5.9689	27.1012
DPS-WB	9.6468	13.3609
ECOWLD-WA	6.7637	11.0239
EG-WC	2.5018	5.6259
GPA-WA	6.4345	7.2767
JIANKUN-WA	10.5679	53.5206
LBS-WB	1.7699	18.3560
PENSONI-WB	8.7789	12.7639

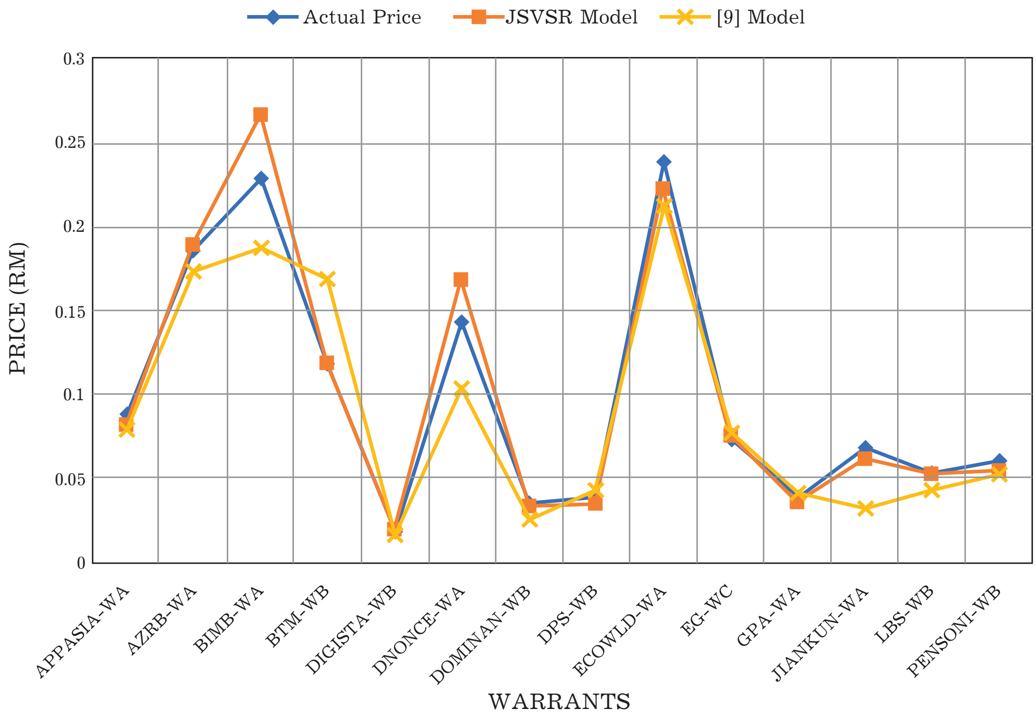

Fig. 2. Warrant Prices via Monte Carlo Simulation: Actual, JSVSR Model and [9] Model.

In addition, we calculated the warrant prices using Monte Carlo simulation with different number of simulations to test for convergence of this method, which is given in Table 8 with the relative errors computed against the actual price of the warrants.

Table 8. Warrant Prices via Monte Carlo Simulation: Different Number of Simulations, M.

M	APPASIA-WA	AZRB-WA	BIMB-WA	BTM-WB	DIGISTA-WB	DNONCE-WA	ε DOMINAN-WB	DPS-WB	ECOWLD-WA	EG-WC	GPA-WA	JIANKUN-WA	LBS-WB	PENSONI-WB
100	24.8879	36.7980	179.4103	93.7257	6.4746	60.7649	64.1873	61.1943	31.2562	56.2895	24.0098	79.6796	77.8535	53.9041
1 000	10.6304	14.4799	47.3913	12.9974	2.7654	26.2603	38.1114	9.5145	10.0252	12.0473	16.5880	16.6739	29.3674	23.4805
10 000	7.8676	1.6885	16.3436	0.0913	2.1333	17.2241	5.9689	9.6468	6.7637	2.5018	6.4345	10.5679	1.7699	8.7789

In Table 8, it can be seen that as the number of simulations increases, the relative error between the simulated prices and the actual prices the prices decreases. This shows that as we increase the number of simulations, the simulated prices converge to the actual prices of a given warrant.[1]

5. Conclusion

In this paper, we study a more general and realistic pricing model for equity warrants that encaptures stochastic volatility, jumps, and stochastic interest rates, which we name the JSVSR model. We obtain the semi-analytical form of the pricing formula under the extended asset price process, and we also model the processes via Euler discretization in order to price the warrants using Monte Carlo simulation. Numerical results show that the JSVSR model produces a slightly more accurate price than the model without jumps when computing using the pricing formula and the Monte Carlo simulation technique. This can be seen by the error measurements that we computed. This shows that by incorporating more features or properties of a given underlying asset, the value of a derivative may be priced more accurately than the Black–Scholes [1] model.

Future studies may include the incorporation of other factors, such as transaction cost, to model the equity warrant prices in the market. Other Monte Carlo simulation techniques, such as variance reduction, may also be used to test if this is more accurate than the regular Euler discretization.

[1]The computation was implemented using a device with an AMD Ryzen 5 with Radeon Vega Mobile Gfx 2.10 GHz.

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Цінові варанти на акції зі стрибками, стохастичною волатильністю та стохастичними процентними ставками

Савал А. С.[1], Ібрагім С. Н. І.[1,2], Рослан Т. Р. Н.[3]

[1]*Інститут математичних досліджень, Університет Путра Малайзії, 43400 UPM Серданг, Селангор, Малайзія*

[2]*Кафедра математики та статистики, Факультет природничих наук, Університет Путра Малайзії, 43400 UPM Серданг, Селангор, Малайзія*

[3]*Вища школа бізнесу Османа Йопа Абдулли, Університет Утара Малайзії, 50300 Куала-Лумпур, Малайзія*

Варант — це дериватив, який надає право, але не зобов'язання, купувати або продавати цінні папери за певною ціною до закінчення терміну дії. Метод оцінки вартості варантів був натхненний оцінкою опціонів через певну схожість цих двох деривативів. Формула варантної ціни за Блеком–Шоулзом доступна в літературі. Однак відомо, що формула Блека–Шоулза має низку недоліків; тому це дослідження має на меті розробити формулу ціноутворення для варантів шляхом включення стрибків, стохастичної волатильності та стохастичних процентних ставок до моделі Блека–Шоулза. У цьому дослідженні представлена формула ціноутворення в закритій формі, для виведення якої використовуються стохастичні диференціальні рівняння (СДР), які включають задачу Коші та рівняння теплопровідності.

Ключові слова: *варант акції, стрибок, стохастична волатильність, стохастична процентна ставка.*