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Quasi-static problem of thermoelasticity for layered shallow cylindrical shells of irregular structure

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For rectangular layered shallow cylindrical shells of irregular structure, the quasi-static problem of unbound thermoelasticity is formulated. As a mathematical model, the equations of the shear theory of shallow shells of Timoshenko type are used. The closed solution for the formulated problem is found by the methods of integral transformations. The distribution of temperature, displacements, forces and moments in a two-layer cylindrical shell under local convective heating is analyzed numerically.

Keywords: *shallow cylindrical shell; layered; irregular structure; heat transfer; thermoelasticity.*

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1. Introduction

Shallow shells of layered structure are widely used in many branches of modern technology, in particular in the aerospace and construction industries, to increase the strength and rigidity of structures, as well as to protect them from low or high-temperature thermal effects. Therefore, the calculation of thermal stresses in such structures is an important engineering task.

Elements of layered structure have been studied [1–6] by many scientists. The improved models [2–5] that take into account the characteristic features of composite materials have been developed. The exact solutions of thermoelasticity problems for layered shells based on three-dimensional equations have been constructed [6, 7]. Analytical solutions [8–11] have been obtained using the equations of classical and various improved theories. Using the equation of interrelated thermoelasticity, the effect of the association coefficient on the dynamic behavior of composite shells is analyzed [12]. The paper [13] is focused on the thermoelectromechanical analysis of multilayer piezoelectric cylindrical shells of open profiles. A more detailed review of various models and methods is given in [1, 2]. In the paper [14], local heating by heat sources of a functionally gradient isotropic cylindrical shell is considered. In the paper [15], the stress-strain state of a layered cylindrical shell under local convective heating is examined.

This paper investigates the stress-strain state of layered isotropic shallow cylindrical shells under the unsteady temperature field.

2. Basic equations

Consider a rectangular shallow cylindrical shell of $a \times b$ dimensions with the middle surface radius R and constant thickness $2h$. The shell is made of isotropic material inhomogeneous in the transverse direction. The points of the shell space are placed in the orthogonal coordinate system x, y, z, with indices 1, 2, and 3 corresponding to these coordinates. Let the coefficients of the first quadratic form be equal to unity, the main curvatures k_1 and k_2 being constant, and the lines of these curvatures coinciding with the coordinate lines.

Let the shell be under the external force load and undergo heating by heat sources and the external environment by convective heat transfer through the side surfaces $z = \pm h$. To investigate the thermoelastic state of the shell, a two-dimensional mathematical shear model of the first order (a model of Timoshenko type) is employed. The model is based on the assumption of linear dependence of the tangential components of the displacement vector and temperature on the transverse coordinate [14]. For thermal stress problems, this model consists of two independent systems of equations, i.e. ones of thermoelasticity and thermal conduction.

System of thermoelasticity equations. The kinematic relations for the deformation components e_{ij} of an arbitrary point of the shell are of the following form:

$$e_{11} = \varepsilon_{11} + z\varkappa_{11}, \quad e_{22} = \varepsilon_{22} + z\varkappa_{22}, \quad e_{12} = \varepsilon_{12} + z\varkappa_{12}, \quad e_{13} = \varepsilon_{13}, \quad e_{23} = \varepsilon_{23}. \tag{1}$$

Here, the deformation components of the mean surface ε_{ij}, $\varkappa_{ij}$ are expressed through the generalized displacements u_i, w, γ_i of the surface points by the following formulas:

$$\begin{gathered}\varepsilon_{11} = \partial_1 u_1, \quad \varepsilon_{22} = \partial_2 u_2 + w/R, \quad \varepsilon_{12} = \partial_2 u_1 + \partial_1 u_2, \\ \varepsilon_{13} = \gamma_1 + \partial_1 w, \quad \varepsilon_{23} = \gamma_2 + \partial_2 w, \quad \varkappa_{11} = \partial_1\gamma_1, \\ \varkappa_{22} = \partial_2\gamma_2, \quad \varkappa_{12} = \partial_1\gamma_2 + \partial_2\gamma_1. \end{gathered} \tag{2}$$

The physical equations for stresses and strains are written as follows:

$$\begin{gathered}\sigma_{11} = \frac{E(z)}{1-\nu^2}\big[e_{11} + \nu\, e_{22} - (1+\nu)\alpha_t(z)t\big], \quad \sigma_{22} = \frac{E(z)}{1-\nu^2}\big[e_{22} + \nu\, e_{11} - (1+\nu)\alpha_t(z)t\big], \\ \sigma_{12} = \frac{E(z)}{2(1+\nu)}e_{12}, \quad \sigma_{13} = \frac{E(z)}{2(1+\nu)}e_{13}, \quad \sigma_{23} = \frac{E(z)}{2(1+\nu)}e_{23}. \end{gathered} \tag{3}$$

Here, ν is Poisson's ratio that is considered constant; $E(z)$ and $\alpha_t(z)$ are the elastic modulus and the coefficient of thermal linear expansion that depend on the coordinate z; $t(x, y, z, \tau)$ is the temperature field.

The physical equations for internal forces N_i, N_{12}, Q_i and moments M_i, M_{12} in the mean surface of the shell are obtained from relations (3) by integrating them over the shell thickness [14],

$$\begin{gathered}\begin{pmatrix} N_1 \\ N_2 \\ M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} A & A_1 & B & B_1 \\ A_1 & A & B_1 & B \\ B & B_1 & D & D_1 \\ B_1 & B & D_1 & D \end{pmatrix} \begin{pmatrix} \partial_1 u_1 \\ \partial_2 u_2 + w/R \\ \partial_1\gamma_1 \\ \partial_2\gamma_2 \end{pmatrix} - \begin{pmatrix} A^t \\ A^t \\ B^t \\ B^t \end{pmatrix} T_1 - \begin{pmatrix} B^t \\ B^t \\ D^t \\ D^t \end{pmatrix} \frac{T_2}{h}, \\ \begin{pmatrix} N_{12} \\ M_{12} \end{pmatrix} = \begin{pmatrix} A_6 & B_6 \\ B_6 & D_6 \end{pmatrix} \begin{pmatrix} \partial_1 u_2 + \partial_2 u_1 \\ \partial_1\gamma_2 + \partial_2\gamma_1 \end{pmatrix}, \\ Q_1 = k' A_6\left(\gamma_1 + \partial_1 w\right), \\ Q_2 = k' A_6\left(\gamma_2 + \partial_2 w\right). \end{gathered} \tag{4}$$

Here,

$$\begin{gathered}\{A, B, D\} = \frac{1}{1-\nu^2}\{E_1, E_2, E_3\}, \quad \{A_1, B_1, D_1\} = \frac{\nu}{1-\nu^2}\{E_1, E_2, E_3\}, \\ \{A_6, B_6, D_6\} = \frac{1}{2(1+\nu)}\{E_1, E_2, E_3\}, \quad \{A^t, B^t, D^t\} = \frac{1}{1-\nu}\{\beta_1, \beta_2, \beta_3\}, \\ E_i = \int_{-h}^{h} E(z)\, z^{i-1} dz, \quad \beta_i = \int_{-h}^{h} E(z)\, \alpha_t(z) z^{i-1} dz \quad (i = 1, 2, 3), \end{gathered} \tag{5}$$

$T_i = \frac{2i-1}{2h^i}\int_{-h}^{h} t\, z^{i-1}\, dz$ $(i = 1, 2)$, are the temperature characteristics integral over the thickness h of the shell; $\partial_1 = \frac{\partial}{\partial x}$, $\partial_2 = \frac{\partial}{\partial y}$, and k' is the rate of shear [14].

The equilibrium equations have the following form:

$$\begin{gathered}\partial_1 N_1 + \partial_2 N_{12} = -q_1, \quad \partial_1 N_{12} + \partial_2 N_2 = -q_2, \\ \partial_1 Q_1 + \partial_2 Q_2 - N_2/R = -q_3, \quad \partial_1 M_1 + \partial_2 M_{12} - Q_1 = -m_1, \\ \partial_1 M_{12} + \partial_2 M_2 - Q_2 = -m_2, \end{gathered} \tag{6}$$

where q_i, m_i are the surface load components [1, 14].

Using the above relations, the equilibrium equations (6) are written in the form of generalized displacements

$$\sum_{k=1}^{5} L_{rk} Y_k = b_r \quad (r,k = 1,2,\ldots,5)\,. \tag{7}$$

Here, $Y_i = u_i$; $Y_3 = w$; $Y_{3+i} = \gamma_i$ $(i = 1,2)$. The differential operators L_{rk} $(L_{rk} = L_{kr})$ and free terms b_r have the following form

$$L_{11} = A\partial_{11}^2 + A_6\partial_{22}^2,\ L_{12} = (A_1 + A_6)\partial_{12}^2,\ L_{13} = A_1/R\partial_1,\ L_{14} = B\partial_{11}^2 + B_6\partial_{22}^2,\ L_{15} = (B_1 + B_6)\partial_{12}^2,$$
$$L_{22} = A_6\partial_{11}^2 + A\partial_{22}^2, \quad L_{23} = A/R\partial_2, \quad L_{24} = (B_1 + B_6)\partial_{12}^2, \quad L_{25} = B_6\partial_{11}^2 + B\partial_{22}^2,$$
$$L_{34} = \left(B_1/R - k'A_6\right)\partial_1, \quad L_{33} = k'A_6(\partial_{11}^2 + \partial_{22}^2) - A/R^2, \quad L_{35} = \left(B/R - k'A_6\right)\partial_2,$$
$$L_{44} = D\partial_{11}^2 + D_6\partial_{22}^2 - k'A_6, \quad L_{45} = (D_1 + D_6)\partial_{12}^2, \quad L_{55} = D_6\partial_{11}^2 + D\partial_{22}^2 - k'A_6,$$
$$b_1 = -q_1 + A^t\partial_1 T_1 + B^t\partial_1 T_2/h, \quad b_2 = -q_2 + A^t\partial_2 T_1 + B^t\partial_2 T_2/h,$$
$$b_3 = q_3 + \left(A^t T_1 + B^t T_2/h\right)/R, \quad b_4 = -m_1 + B^t\partial_1 T_1 + D^t\partial_1 T_2/h,$$
$$b_5 = -m_2 + B^t\partial_2 T_1 + D^t\partial_2 T_2/h.$$

To achieve the uniqueness of the solution of system (7), it is necessary to set the appropriate boundary conditions:

1) one value from each pair $\{N_1, u_1\}$, $\{N_{12}, u_2\}$, $\{Q_1, w\}$, $\{M_1, \gamma_1\}$, $\{M_{12}, \gamma_2\}$ with $x = 0$, $x = a$ on the edges;
2) one value from each pair (N_2, u_2), (N_{12}, u_1), (Q_2, w), (M_2, γ_2), (M_{12}, γ_1) with $y = 0$, $y = b$ on the edges.

These conditions along with the system of equations (7) constitute the boundary value problem of the theory of thermal stresses for inhomogeneous isotropic shallow cylindrical panels in displacements. Given the known displacements, the mean surface deformations and moments of a force are determined using the relations (2) and (4) respectively, and the thermal stresses and deformations at any point of the shell are calculated by formulas (1) and (3).

System of heat conduction equations. The integral characteristics of temperature T_1 and T_2, which are part of the free terms of the system (6), are to be determined from the corresponding equations of heat conduction under boundary conditions set on the surfaces $z = \pm h$ and at the ends of the shell. For convective heat transfer on the surfaces $z = \pm h$, the system of heat conduction equations with linear temperature dependence on the transverse coordinate has the form

$$\Delta_1 T_1 - \varepsilon_1^t T_1 + \Delta_2 T_2 + \left(\frac{\Lambda_1}{hR} - \varepsilon_2^t\right) T_2 - C_1\partial_\tau T_1 = -f_1^z,$$
$$\Delta_2 T_1 - \varepsilon_2^t T_1 + \Delta_3 T_2 + \left(\frac{\Lambda_2}{hR} - \frac{\Lambda_1}{h^2} - \varepsilon_1^t\right) T_2 - C_3\partial_\tau T_2 = -f_2^z. \tag{8}$$

Here,

$$\{\Lambda_i, C_i\} = \int_{-h}^{h} \{\lambda(z), c_e(z)\} \left(\frac{z}{h}\right)^{i-1} dz \quad (i = 1,2,3),$$
$$\Delta_i = \Lambda_i\left(\partial_{11}^2 + \partial_{22}^2\right), \quad f_j^z = t_1^z\varepsilon_j^t + t_2^z\varepsilon_{3-j}^t + W_j^t = Q_j^z(x,y)F_j^z(\tau),$$
$$\varepsilon_j^t = \left(\alpha^+ - (-1)^j\alpha^-\right), \quad t_j^z = \frac{1}{2}\left(t_z^+ - (-1)^j t_z^-\right), \quad W_j^t = \int_{-h}^{h} w_t \left(\frac{z}{h}\right)^{j-1} dz\ (j = 1,2), \quad \partial_\tau = \frac{\partial}{\partial\tau},$$

$\lambda(z)$ is the heat conduction coefficient; t_z^+, t_z^- are ambient temperatures on surfaces $z = h$ and $z = -h$ respectively; α^+, α^- are heat transfer coefficients from these surfaces; $c_e(z)$ is the specific heat capacity; τ is a time variable; w_t is the density of heat sources.

To achieve the uniqueness of the solution of system (6) on each edge, $x = 0$, $x = a$ and $y = 0$, $y = b$, it is necessary to specify two combinations of values of the type

$$a_1 T_1 + a_2 T_2 + a_3\frac{\partial T_1}{\partial x} + a_4\frac{\partial T_2}{\partial x} + a_5\frac{\partial T_1}{\partial y} + a_6\frac{\partial T_2}{\partial y},$$

where $a_i = \text{const}$; T_1 and T_2 are the values of temperature characteristics at the initial time.

3. Problem solving method

Let the shell consist of a package of rigidly connected N homogeneous isotropic layers of different thicknesses h_k. It is assumed that the hypothesis about the temperature pattern along the thickness of the shell applies to the entire package of layers. Then, according to the procedure described in [3], the thermophysical characteristics of the layered shell as a whole are represented by asymmetric unit functions $S_{\pm}(z)$ of the form

$$q(z) = q_1 + \sum_{k=1}^{N-1} (q_{k+1} - q_k)\, S_+(z - z_k).$$

Here, $q(z) = \{E(z), \alpha_t(z), \lambda(z), c_e(z)\}$ $q_k = \left\{E^{(k)}, \alpha_t^{(k)}, \lambda^{(k)}, c_e^{(k)}\right\}$ are the physical and mechanical characteristics of the k-th layer; z_k is the distribution limit coordinate of the k-th and k-th layers, with $z_k = -h + \sum_{m=1}^{k} h_m$;

$$S_+(x) = \left\{ \begin{array}{ll} 1, & x > 0, \\ 0, & x \leqslant 0, \end{array} \right. \qquad S_-(x) = \left\{ \begin{array}{ll} 1, & x \geqslant 0, \\ 0, & x < 0. \end{array} \right.$$

Substituting the relation (5) with (4), the expressions of the integral characteristics E_i, β_i, $\Lambda^{(n)}$, $C^{(n)}$ through the physical properties of the layers $E^{(k)}$, $\alpha_t^{(k)}$, $\lambda^{(k)}$, $c_e^{(k)}$ are obtained. For E_i, they take the following form:

$$E_1 = 2hE^{(1)} + \sum_{k=1}^{N-1} \left(E^{(k+1)} - E^{(k)}\right)(h - z_k), \quad E_2 = \frac{1}{2h} \sum_{k=1}^{N-1} \left(E^{(k+1)} - E^{(k)}\right)(h^2 - z_k^2),$$

$$E_3 = \frac{2h}{3} E^{(1)} + \frac{1}{3h^2} \sum_{k=1}^{N-1} \left(E^{(k+1)} - E^{(k)}\right)(h^3 - z_k^3), \tag{9}$$

which is analogical for other integral characteristics.

Depending on the structure of layered constructions, the relations (9) may vary.

Let the edges of the shell be hinged and maintained at zero temperature. Then the boundary conditions will be as follows:
when $x = 0$ and $x = a$:

$$w = u_2 = \gamma_2 = 0, \quad N_1 = M_1 = 0, \tag{10}$$

$$T_1 = T_2 = 0; \tag{11}$$

when $y = 0$ and $y = b$:

$$w = u_1 = \gamma_1 = 0, \quad N_2 = M_2 = 0, \tag{12}$$

$$T_1 = T_2 = 0. \tag{13}$$

At the initial moment of time $\tau = 0$, the temperature characteristics are set

$$T_1(x, y, 0) = T_1^0(x, y), \quad T_2(x, y, 0) = T_2^0(x, y). \tag{14}$$

Solution for the heat conduction problem. Equation (8) after applying the double finite Fourier transform by the coordinates x, y according to the boundary conditions (11) and (13) takes the form

$$\frac{dT_{1mn}}{d\tau_1} + G_1 T_{1mn} + G_2 T_{2mn} = f_{1mn}^z, \tag{15}$$

$$\frac{dT_{2mn}}{d\tau_1} + G_3 T_{1mn} + G_4 T_{2mn} = f_{2mn}^z. \tag{16}$$

Here, $G_1 = \tilde{\Lambda}_1 \left(\mu_m^2 + \mu_n^2\right) + \mathrm{Bi}_1$, $G_2 = \tilde{\Lambda}_2 \left(\mu_m^2 + \mu_n^2\right) - \delta\tilde{\Lambda}_1 + \mathrm{Bi}_2$, $G_3 = \left[\tilde{\Lambda}_2 \left(\mu_m^2 + \mu_n^2\right) + \mathrm{Bi}_2\right]\tilde{C}$, $G_3 = \left[\tilde{\Lambda}_3 \left(\mu_m^2 + \mu_n^2\right) + \mathrm{Bi}_1 + \tilde{\Lambda}_1 - \delta\tilde{\Lambda}_2\right]\tilde{C}$, $\mu_m = \frac{\pi m h}{a}$, $\mu_n = \frac{\pi n h}{b}$, $\delta = \frac{h}{R}$, $\tau_1 = \frac{2\lambda_0}{hC_1}\tau$, $\tilde{C} = \frac{C_1}{C_3}$, $\tilde{\Lambda}_i = \frac{\Lambda_i}{2h\lambda_0}$, $\mathrm{Bi}_i = \frac{\varepsilon_i^t h}{2\lambda_0}$, λ_0 is a characteristic heat conductivity coefficient;

$$f_{1mn}^z = \mathrm{Bi}_1 t_{1mn}^z + \mathrm{Bi}_2 t_{2mn}^z + W_{1mn}^t \frac{h}{2\lambda_0} = Q_{1mn}^z(x, y) F_1^z(\tau);$$

$$f_{2mn}^z = \left(\mathrm{Bi}_2 t_{1mn}^z + \mathrm{Bi}_1 t_{2mn}^z + W_{2mn}^t \frac{h}{2\lambda_0}\right) \tilde{C} = Q_{2mn}^z(x, y) F_2^z(\tau).$$

The solution of the system of equations (16) is obtained by the integral Laplace transform using the initial conditions (14) in the form

$$T_{1mn} = \sum_{\substack{j=1 \\ k\neq j}}^{2} \frac{(p_j - G_4)Q^z_{1nm}Z_{1j}(\tau) + G_2Q^z_{2nm}Z_{2j}(\tau) + \left[(p_j - G_4)T^0_{1nm} + G_2T^0_{2nm}\right]\exp(-p_j\tau_1)}{p_j - p_k},$$

$$T_{2mn} = \sum_{\substack{j=1 \\ k\neq j}}^{2} \frac{(p_j - G_1)Q^z_{2nm}Z_{2j}(\tau) + G_3Q^z_{1nm}Z_{1j}(\tau) + \left[(p_j - G_1)T^0_{2nm} + G_3T^0_{1nm}\right]\exp(-p_j\tau_1)}{p_j - p_k}.$$

Here,

$$p_i = \frac{G_1 + G_4}{2} + (-1)^i\sqrt{\frac{(G_1 - G_4)^2}{4} + G_2G_3},$$

$$\left\{Q^z_{jnm}, T^0_{jnm}\right\} = \frac{4}{ab}\int_0^a\int_0^b \left\{Q^z_j, T^0_j\right\}(x, y)\sin\frac{\pi m}{a}x\sin\frac{\pi n}{b}y\,dx\,dy \quad (j = 1, 2),$$

$$Z_{ij} = \int_0^{\tau_1} F_i(u)\exp\left(-p_j(\tau_1 - u)\right)du \quad (i, j = 1,\ 2).$$

The temperature characteristics T_s $(s = 1, 2)$ using Fourier coefficients T_{smn} $(s = 1, 2)$ are expressed by the formulas

$$\{T_1, T_2\} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\{T_{1mn}, T_{2mn}\}\sin\frac{\pi m}{a}x\sin\frac{\pi n}{b}y. \tag{17}$$

Solution for the thermoelasticity problem. The solution of the system of equilibrium equations (7), which meets the boundary conditions (10), (12) with a known temperature field (17), is reached by the method of finite double Fourier transform by coordinates x, y. As a result, a system of algebraic equations for determining the Fourier coefficients U_{1mn}, U_{2mn}, W_{mn}, Γ_{1mn}, Γ_{2mn} of the unknown generalized displacements is obtained and written in the matrix form

$$\mathbf{M}\,\mathbf{U} = \mathbf{V}\,T_{1mn} + \mathbf{S}\,T_{2mn}. \tag{18}$$

Here, $\mathbf{U} = \{U_{1mn}, U_{2mn}, W_{mn}, \Gamma_{1mn}, \Gamma_{2mn}\}^T$, $\mathbf{M} = (m_{ij})_{5\times5}$, $\mathbf{V} = (v_i)_{5\times1}$, $\mathbf{S} = (s_i)_{5\times1}$. The matrix coefficients m_{ij}, v_i and s_i are calculated on the basis of the expressions of differential operators of the system (7).

The solution of system (18) is obtained in the form

$$\mathbf{U} = \frac{1}{|\mathbf{M}|}\mathbf{M}^*\left(\mathbf{V}\,T_{1mn} + \mathbf{S}\,T_{2mn}\right), \tag{19}$$

where $|\mathbf{M}|$ is the determinant of the matrix $\mathbf{M}$, and $\mathbf{M}^*$ is the adjoined matrix.

The generalized displacements through their Fourier coefficients are expressed by the formulas

$$\{u_1, \gamma_1\} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\{U_{1mn}, \Gamma_{1mn}\}\cos\frac{\pi m}{a}x\,\sin\frac{\pi n}{b}y,$$

$$\{u_2, \gamma_2\} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\{U_{2mn}, \Gamma_{2mn}\}\sin\frac{\pi m}{a}x\,\cos\frac{\pi n}{b}y,$$

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\sin\frac{\pi m}{a}x\,\sin\frac{\pi n}{b}y. \tag{20}$$

According to the known generalized displacements (20) and temperature characteristics (17) of the stress and force, the moments in the shell are calculated by formulas (3) and (4) respectively.

4. Analysis of numerical results

Numerical studies were performed for a two-layer cylindrical panel heated by the environment through convective heat transfer. The temperature of the medium on the outer surface $z = h$ is represented by the function

$$t_z^+(x,y,\tau) = t^*\left[S_-(x-x_0+d_1) - S_+(x-x_0-d_1)\right] \times \left[S_-(y-y_0+d_2) - S_+(y-y_0-d_2)\right]\left(1-\exp(-\beta^*\tau)\right),$$

where t^*, $\beta^* = \text{const}$; x_0, y_0 are the coordinates of the heating area center; $2d_1 \times 2d_2$ is the size of this domain, which equals zero $t_z^- = 0$ on the inner surface $z = -h$.

It is assumed that the coefficients of heat transfer from the surfaces of the shell are the same $\alpha^+ = \alpha^- = \alpha_z$, with no surface forces and heat sources and the temperature of the shell at the initial moment equaling zero $T_1^0 = T_2^0 = 0$.

The material of the shell layers is titanium alloy and aluminum alloy [2]. The bottom layer is made of titanium alloy with the following physical and mechanical properties:

$$E^{(1)} = 110\ \text{GPa}, \quad \nu^{(1)} = 0.32, \quad \alpha_t^{(1)} = 8.6\cdot10^{-6}\ 1/\text{K}, \quad \lambda^{(1)} = 21.9\ \text{W/mK}, \quad c_e^{(1)} = 560\ \text{J/(kg K)}.$$

The top layer of the shell is made of aluminum alloy, for which:

$$E^{(2)} = 73\ \text{GPa}, \quad \nu^{(2)} = 0.3, \quad \alpha_t^{(2)} = 25\cdot10^{-6} 1/\text{K}, \quad \lambda^{(2)} = 130\ \text{W/mK}, \quad c_e^{(2)} = 897\ \text{J/(kg K)}.$$

The other parameters are as follows:

$$a/b = 1, \quad h/R = 0.05, \quad h_1/h_2 = 1, \quad x_0 = a/2, \quad y_0 = b/2,$$
$$d_1/a = 0.25, \quad d_2/b = 0.25, \quad \beta^* = 1, \quad k' = 5/6, \quad \text{Bi} = 1.$$

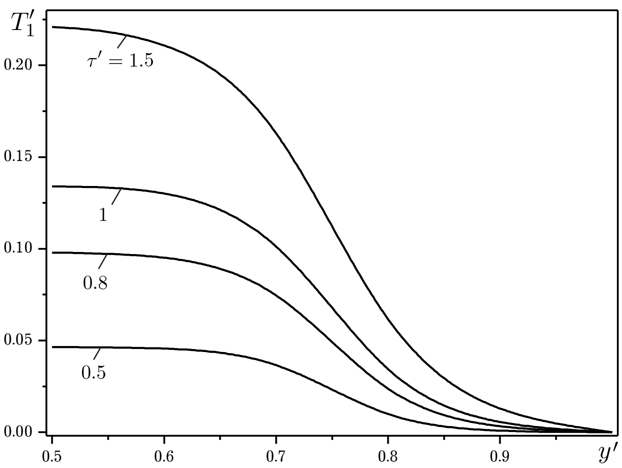

Fig. 1.

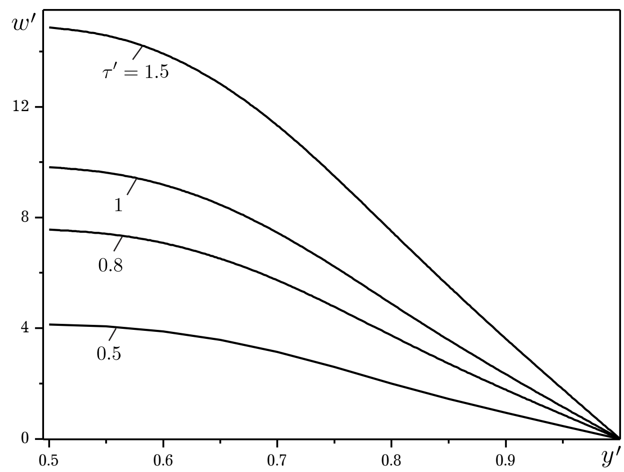

Fig. 2.

For the parameters given, the change in the dimensionless average temperature $T_1' = \frac{T_1}{t^*}$, deflection $w' = \frac{w}{h\alpha_t^{(1)}t^*}$, normal forces $N_i' = \frac{N_i}{E^{(1)}h\alpha_t^{(1)}t^*}$, and bending moments $M_i' = \frac{M_i}{E^{(1)}h^2\alpha_t^{(1)}t^*}$ $(i = 1,2)$ with the dimensionless coordinate $y' = y/b$ $(0.5 \leqslant y' \leqslant 1)$ at the point $x' = x/a = 0.5$ for the dimensionless time $\tau' = \frac{\lambda^{(1)}\tau}{c_e^{(1)}h^2}$ equaling 0.5, 0.8, 1, and 1.5 is calculated and illustrated in Figs. 1–6.

The maximum values of the average temperature T_1' and deflection w' are recorded in the center of the heating region, and when approaching the edges monotonically decrease to zero. The normal forces N_i' along the coordinate y' are naturally oscillatory. First, they are tensile in the center of the heating region and then (for N_1' if $\tau' \geqslant 0.7$, and for N_2' if $\tau' \geqslant 1.2$) compressive, with being always tensile outside the heating region. The change of the bending moment M_1' is oscillating: from the maximum negative values in the heating area center to positive values in the unheated area. Accordingly, the

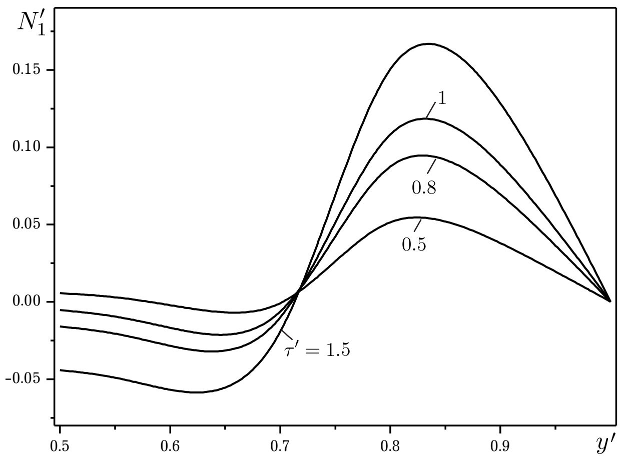

Fig. 3.

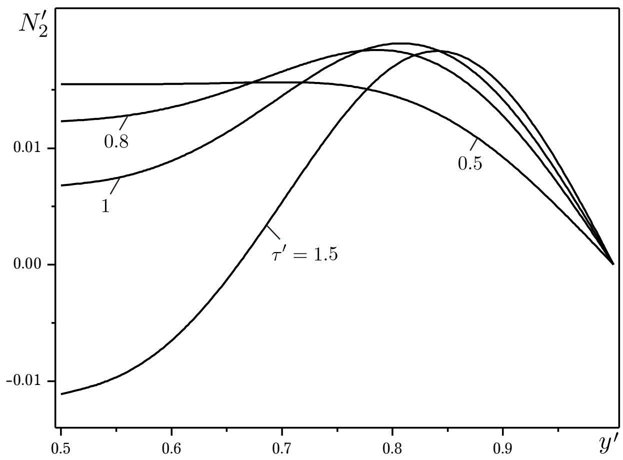

Fig. 4.

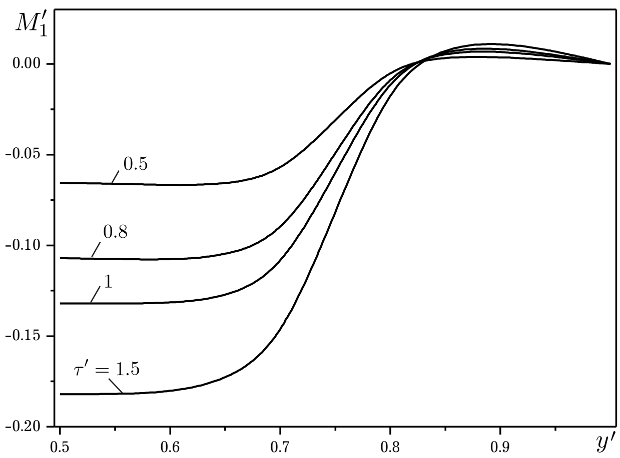

Fig. 5.

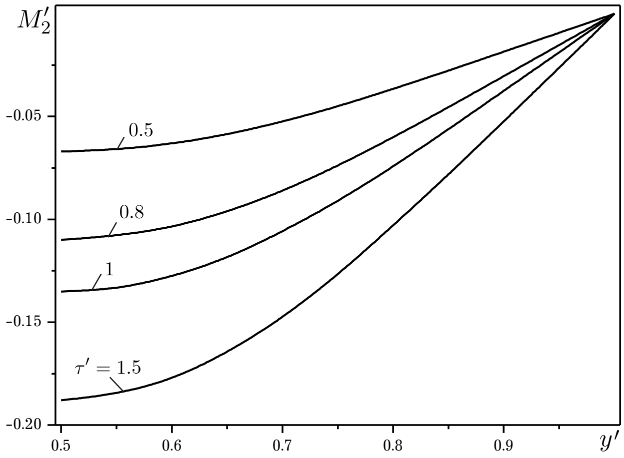

Fig. 6.

moment M_2' monotonically decreases from the maximum negative values in the heating area center to zero on the edge of the shell. Over time, the parameters of the stress-strain state напружено-д стан increase and enter steady-state operating conditions when $\tau' > 10$.

5. Conclusions

The stress-strain state of a two-layered isotropic rectangular shallow cylindrical panel, which is heated by the environment by convective heat exchange, is examined using the equations of the linear shear theory of the first order. The closed solution is obtained by the methods of Fourier and Laplace transforms. The figures illustrate the dependence of the temperature field, deflection and internal forces and moments on geometric parameters and time. The results obtained can be used for analyzing the stress-strain state of shallow-coated cylindrical shells.

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Квазістатична задача термопружності для шаруватих пологих циліндричних оболонок нерегулярної структури

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Для прямокутних в плані шаруватих пологих циліндричних оболонок нерегулярної структури сформульована квазістатична задача незв'язаної термопружності. За математичну модель використано рівняння зсувної теорії пологих оболонок типу Тимошенка. Замкнутий розв'язок сформульованої задачі знайдено методами інтегральних перетворень. Чисельно проаналізовано розподіл температури, переміщень, зусиль і моментів у двошаровій циліндричній оболонці за локального конвективного нагрівання.

Ключові слова: *полога циліндрична оболонка; шарувата; нерегулярної структури; теплообмін; термопружність.*