MATHEMATICAL MODELING AND COMPUTING, Vol. 10, No. 2, pp. 583–592 (2023)

Model of money income diffusion in the European integration context

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(Received 10 October 2022; Revised 11 April 2023; Accepted 25 April 2023)

A model of money income diffusion is constructed taking into account the processes of social comparison and the development of the income formation spatial structure. To implement such model, the analytical theory of continued fractions is used and the corresponding differential equation solution is found in the form of a formal functional continued fraction. The values of the approach fractions of a continuous fraction describing the dynamics of changes in the level of money income give an approximation of the real values with almost predetermined, arbitrarily high accuracy. This allowed us to qualitatively describe the general dynamics of the money income diffusion process.

Keywords: *diffusion model; money income; functional continuous fraction.*
2010 MSC: 91B62, 34A25 **DOI:** 10.23939/mmc2023.02.583

1. Introduction

In the context of the socio-economic development of countries and individual regions and management practices, the problem of assessing inequality and forecasting population income distribution is considered. The problem of interregional differences in income distribution is a consequence of the historical development of territories, the strategy and tactics of production location, interregional interactions [1, 2].

The problem of interregional differences in income distribution is a consequence of the historical development of territories, the strategy, and tactics of production location, and interregional aspect in studies of the level, factors and trends of the income, we find in [3–7]. Money income is considered in the context of the relationship and interaction with various components that determine the level and quality of life of the population, including age aspects [8].

Now, the models and methods of the regression-correlation type have given way to more complex models, among which a particular group consists of models of the adaptive type, which allow for a more accurate display of trends by adaptation in the process of receiving new data [9]. Such modeling opportunities are especially relevant in the conditions of the formation of unfavorable trends of increasing inequality of household income distribution and simultaneous economic growth of the economy as a whole, which cannot be explained in the conditions of traditional forecasting methodology and organization. The latter is due to the insufficient orientation of indicators of socio-economic development in general and the level of income of the population in particular to solving the problems of its social protection.

The population income forecasting system solves the following problems: identification of the main trends in population income growth; determination of stable dynamic and structural characteristics of social development processes; determination of changes in the standard of living due to changes in the income of the population.

The research of L. Dmytryshyn was performed within the framework of the scholarship program of the fund "Excellence Initiative – Research University", University of Lodz, Poland (Agreement No.12/IDUB/SNU/2022).

Forecasting allows you to determine the possible effects of the external environment on the state of the research object in the future with the aim of developing a real and effective program of socio-economic development. The results of forecasts, taking into account strategic guidelines, are the basis for forming a system of comprehensive development programs goals. Forecasting the level of income has a number of features due to the socio-economic nature of the management object.

Forecasts of income level should, firstly, cover all its components, i.e., be comprehensive; secondly, to be related to the general economic forecast; thirdly, to take into account all mechanisms of income formation, which allow regulating the stochasticity of this process to a certain extent.

We pay attention on the development of empirical and theoretical studies on wealth and income distributions, as well as the connection and interplay between economics and econophysics [10, 11].

Our research is based on the diffusion models application adapted to the object of research (money income) and their connection with income inequality and social comparison criteria in the European integration context.

2. An income diffusion

The diffusion of income means the process of income distribution in a given socio-economic system, as well as from one system to another. Following [12], we consider the next types of the process diffusion of income.

Model of "center–periphery". This model can be considered as a classic case of state regulation of the redistribution of income through the collection of taxes on profits and personal income and the implementation of transfer payments to vulnerable sections of the population, as well as the regulation of marginal prices for basic necessities and determination of minimum wage rates, etc.

Model of reproduction of centers. In this model, as in the previous one, the main role is played by the center, but the management process is decentralized. Local centers are created on the ground, which independently spread diffusion, taking into account local specifics.

The following five stages of diffusion implementation can be distinguished:

1) awareness (the economic agent knows about a new type of income, but there is not enough information);
2) interest (the economic agent is interested in a new type of income and is looking for additional information);
3) evaluation (the economic agent makes a decision, evaluating the benefits of obtaining this type of income at the present time and in the future);
4) testing (the economic agent accepts and uses a new type of income);
5) distribution (an economic agent increases his own total income due to the introduction of a new type of income).

The rate of diffusion is determined by following factors.

1) Relative advantages. In accordance with the theory of utility, an economic agent will prefer to receive income with greater utility, but less expended effort.
2) Compatibility with the environment, existing values and past experience. If a certain source of income had a negative past experience, then it is necessary to implement it in the future with special care.
3) Difficulty of development.
4) The possibility of testing before making a final decision. This factor is within the competence of state social policy management bodies, which are able to launch certain pilot projects and suspend them due to ineffectiveness, or generally the presence of negative implementation results.
5) Communication clarity. In essence, this is the extent to which the results of diffusion can be seen and appreciated by the population. For example, raising the level of the minimum wage, minimum pension, etc. with the simultaneous increase in the price of the minimum consumer basket for the majority of households/population does not clearly illustrate the state's social policy.

The effectiveness of diffusion is largely determined by social determination, i.e. how households/populations are ready to accept new types of income. Society as a collection of households/population may demand change, but may also tire of the confusion caused by the introduction of previous innovations in the social sphere. Therefore, the demand for this kind of innovation can be represented by diffusion phases determined in accordance with the life cycle of the households/population that represent such a society.

The life cycle reflects the possibility of the location of households/population in certain income groups during a certain evolutionary (historical) period of time of the change of generations (and the number of such groups is not taken into account here). First, households/populations form their own income level for the purpose of self-preservation. The need for self-assertion leads to the rapid growth of income, their differentiation is caused by different access of households/population to available sources of income. Reserves for the rapid growth of household/population income are drawn from the property of passion — that is, the characteristics of behavior and psyche, manifested in the effort of households/population to achieve a certain goal (in this case, increasing aggregate income) and using all possible (legal and illegal) ways to achieve it. Then the growth of income creates development inertia, which leads to a slow decline in income (for example, the distribution of the wealth of the super-rich in subsequent generations).

Therefore, the following phases of the diffusion process can be distinguished.

1) Phase of diffusion growth that occurs when the main part of households/population belongs to the group of low- and middle-income classes, which, due to a passionate impulse, are ready to change their share of income in society. Moreover, it is possible to distinguish hidden (latent) and explicit growth. The first is more characteristic of the low-income group of households/population and is manifested in the search for new, mainly non-money sources of income (for example, consumed products obtained from personal subsistence farming, self-production, benefits and non-cash subsidies for the payment of housing and communal services, electricity and fuel, goods and health care services, tourist services, tickets to recreation centers, transport and communication services, etc.). So, such growth is implicit, i.e., latent. The obvious growth lies in the search for new money sources of income. As a result the diffusion growth, a new social class can be formed, which is ready to move to the next phase through expansion.
2) Phase of stagnation that are characterized by the highest degree of passionate state of households/population. In this phase, passionate overheating is possible, when excess energy begins to be extinguished in internal conflicts, i.e., interest in finding new sources of income decreases due to a decrease in the usefulness of their distribution.
3) Phase of decline that characterized by a sharp decrease in the level of passion and may be accompanied by a split in the groups that represent this phase (middle-, high-income, super-rich). This phase can still be considered as an "age disease" of households/population, when there is a conflict of interests of generations.
4) Phase of inertia that characterized by the state regulation strengthening in the social sphere, including such regulation aimed at combining social policy and employment policy with the aim of raising the standard of living of households/population.
5) Phase of obscuration that are characterized by negative characteristics of life of households/population, which are manifested in the growth of corruption, crime and population decline. Such a socio-economic system can be "easy prey" for other, more passionately oriented systems. Therefore, the interest in the income level diffusion manifests itself in its illegal form.
6) Phase of regeneration are characterized by a short-term recovery of interest in the diffusion of new sources of income, however, the insignificant result achieved in this process leads to the transition to the last phase.
7) Phase of relict. The stay in this phase of households/population can be long enough in time, and therefore the characteristics inherent in it (the same as in the obscuration phase) can lead to revolutionary states.

Thus, the phases of the diffusion process can be considered as a learning process of a given socio-economic system, which is represented by a set of households/population. The learning process includes innovative perception, evaluation and decision-making, i.e., a whole range of cognitive factors.

Consideration of the diffusion process aspects in this study is based on the diffusion influence analysis on forms of consumption and on the lifestyle change, in general. We consider the function of social comparison, which allows us to describe the relationships between reference groups of the population by income level and the possibility of transition from one group to another (income mobility), as well as to characterize the relationship between inequality and income mobility with economic growth.

3. Diffusion model

The diffusion model of the population money income can be represented in the form of a differential equation:

$$u'(t) = \frac{1}{t\gamma(t)} - Gu(t) - \gamma(t)\, u^2(t), \tag{1}$$

where $u(t)$ is an unknown function of variables t; G is the Gini coefficient; $\gamma(t) = e^{\alpha t}$ is an infinitely differentiate function with the parameter α. The essence of $\gamma(t)$ is that the income of the reference groups of the population of different regions is compared with the income of their closest "neighbors" in hierarchical income levels.

We use the analytical theory of continuous fractions to find a solution of the money income diffusion model [13].

The substitution

$$y(t) = u(t)\gamma(t)$$

leads to a Riccati equation

$$y'(t) = \frac{1}{t} + \left(\frac{\gamma'(t)}{\gamma(t)} - G\right) y(t) - y^2(t)$$

or

$$y'(t) = \frac{1}{t} + (\alpha - G)\, y(t) - y^2(t). \tag{2}$$

The way to obtain a second-order linear equation from a Riccati equation (2) is to replace (see [13, Chapter XII])

$$y(t) \quad \text{by} \quad \frac{w'(t)}{w(t)}.$$

We then obtain

$$w(t) = P_0(t)\, w'(t) + Q_0(t)\, w''(t),$$

where

$$P_0(t) = (G - \alpha)t, \quad Q_0(t) = t.$$

We differentiate and rearrange

$$w'(t) = P_1(t)\, w''(t) + Q_1(t)\, w'''(t),$$

where

$$P_1(t) = \frac{P_0(t) + Q_0'(t)}{1 - P_0'(t)}, \quad Q_1(t) = \frac{Q_0(t)}{1 - P_0'(t)}.$$

We proceed and find generally

$$w^{(n)}(t) = P_n(t)\, w^{(n+1)}(t) + Q_n(t)\, w^{(n+2)}(t),$$

where

$$P_n(t) = \frac{P_{n-1}(t) + Q_{n-1}'(t)}{1 - P_{n-1}'(t)}, \quad Q_n(t) = \frac{Q_{n-1}(t)}{1 - P_{n-1}'(t)}.$$

Since (if no denominator is 0)

$$\begin{aligned}
\frac{w(t)}{w'(t)} &= P_0(t) + \frac{Q_0(t)}{w'(t)/w''(t)},\\
\frac{w'(t)}{w''(t)} &= P_1(t) + \frac{Q_1(t)}{w''(t)/w'''(t)},\\
&\cdots\cdots\cdots\cdots,\\
\frac{w^{(n)}(t)}{w^{(n+1)}(t)} &= P_n(t) + \frac{Q_n(t)}{w^{(n+1)}(t)/w^{(n+2)}(t)},
\end{aligned}$$

we obtain the formal identity

$$\frac{w(t)}{w'(t)} = P_0(t) + \cfrac{Q_0(t)}{P_1(t) + \cfrac{Q_1(t)}{P_2(t) + \ddots + \cfrac{Q_n(t)}{w^{n+1}(t)/w^{n+2}(t)}}}.$$

In this way, we obtain the continued fraction

$$P_0(t) + \cfrac{Q_0(t)}{P_1(t) + \cfrac{Q_1(t)}{P_2(t) + \ddots + \cfrac{Q_n(t)}{P_{n+1}(t) + \ddots}}},$$

where

$$P_n(t) = \frac{n + (G-\alpha)t}{1 - n(G-\alpha)}, \quad Q_n(t) = \frac{t}{1 - n(G-\alpha)}, \quad n \geqslant 0.$$

Next, we will investigate the convergence of the continued fraction

$$\cfrac{1}{P_0(t) + \cfrac{Q_0(t)}{P_1(t) + \cfrac{Q_1(t)}{P_2(t) + \ddots + \cfrac{Q_n(t)}{P_{n+1}(t) + \ddots}}}}. \tag{3}$$

By equivalent transformations (see [14, §2.3] to learn more) reduce it to the form

$$\cfrac{1}{d_0 t + \cfrac{t}{e_1 + d_1 t + \cfrac{t}{e_2 + d_2 t + \ddots + \cfrac{t}{e_{n+1} + d_{n+1} t + \ddots}}}}, \tag{4}$$

where

$$d_0 = G - \alpha, \quad e_{n+1} = \prod_{k=1}^{n+1} \frac{n+1}{(1 - k(G-\alpha))^{(-1)^{n+k-1}}}, \quad d_{n+1} = \prod_{k=1}^{n+1} \frac{G-\alpha}{(1 - k(G-\alpha))^{(-1)^{n+k-1}}}, \quad n \geqslant 0.$$

It is clear that for $n \geqslant 0$,

$$|e_{n+1}| \geqslant k_1 = \begin{cases} 2, & \text{if} \quad |1 + \alpha - G| \leqslant 1/2, \\ 1/|1 + \alpha - G|, & \text{if} \quad |1 + \alpha - G| > 1/2, \end{cases}$$

and

$$|d_{n+1}| \leqslant k_2 = \begin{cases} |G - \alpha|, & \text{if} \quad |1 + \alpha - G| \geqslant 1, \\ |G - \alpha|/|1 + \alpha - G|, & \text{if} \quad |1 + \alpha - G| < 1, \end{cases}.$$

It follows from [14, Theorem 4.62] that for all t in the closed circular disk

$$G = \left\{ t\colon |t| \leqslant \left((\sqrt{1 + k_1 k_2} - 1)/k_2 \right)^2 \right\}$$

the continued fraction

$$d_0 t + \cfrac{t}{e_1 + d_1 t + \cfrac{t}{e_2 + d_2 t + \ddots + \cfrac{t}{e_{n+1} + d_{n+1} t + \ddots}}} \tag{5}$$

converges to a function $g(t)$. Moreover, the convergence is uniform on every compact subset of $\mathrm{Int}(G)$, and $g(t)$ is holomorphic on $\mathrm{Int}(G)$. Then the continued fraction (4), and therefore the continued fraction (3), as its equivalent, converges to a function $f(t)$ meromorphic in G, and the convergence is uniform on every compact subset of $\mathrm{Int}(G)$ which contains no poles of $f(t)$.

Now, it follows from [14, Theorem 7.17] that the continued fraction (5) corresponds at $t = 0$ to a uniquely determined formal power series

$$L = \sum_{k=1}^{\infty} c_k t^k$$

(we refer the reader to the book [14, §5.1] to learn more about correspondence). The order of correspondence of the nth approximant $f_n(t)$ is $\nu_n = n + 1$, and the Taylor series at $t = 0$ of $f_n(t)$ has the form

$$f_n(t) = \sum_{k=1}^{n} c_k t^k + \sum_{k=n+1}^{\infty} \gamma_k^{(n)} t^k. \tag{6}$$

Let $A_n(t)$ and $B_n(t)$ denote the nth numerator and denominator, respectively, of (3) and (4). Then from the difference equations

$$\begin{gathered} A_{-1}(t) = 1, \quad A_0(t) = d_0 t, \quad B_{-1}(t) = 0, \quad B_0(t) = 1, \\ A_n(t) = (e_n + d_n t) A_{n-1}(t) + t A_{n-2}(t), \quad n \geqslant 1, \\ B_n(t) = (e_n + d_n t) B_{n-1}(t) + t B_{n-2}(t), \quad n \geqslant 1, \end{gathered}$$

we have

$$\begin{aligned} A_n(t) &= a_{n,1} t + \ldots + a_{n,n+1} t^{n+1}, \quad n \geqslant 1, \\ B_n(t) &= b_{n,0} + b_{n,1} t + \ldots + b_{n,n} t^n, \quad n \geqslant 1, \end{aligned}$$

where, in particular,

$$a_{n,n} = (1 + e_1 d_0) d_2 d_3 \ldots d_n, \quad a_{n,n+1} = d_0 d_1 \ldots d_n, \quad b_{n,0} = e_1 e_2 \ldots e_n, \quad b_{n,n} = d_1 d_2 \ldots d_n. \tag{7}$$

It follows from (6) and (7) that $\Lambda(B_n) - A_n$, where $\Lambda(B_n)$ is Taylor expansion in a neighbourhood of the origin, is a formal power series of the form

$$\Lambda(B_n) - A_n = e_1 e_2 \ldots e_n \big(c_{n+1} - \gamma_{n+1}^{(n)}\big) t^{n+1} + \lambda_{n+2} t^{n+2} + \lambda_{n+3} t^{n+3} + \ldots.$$

Equating coefficients of similar powers of t on both sides and using (7), we obtain the equations

$$\begin{gathered} c_1 b_{n,o} = a_{n,1}, \\ c_2 b_{n,o} + c_1 b_{n,1} = a_{n,2}, \\ \ldots\ldots\ldots\ldots \\ c_n b_{n,o} + c_{n-1} b_{n,1} + \ldots + c_1 b_{n,n-1} = a_{n,n}. \end{gathered}$$

Hence, the coefficients c_k, $1 \leqslant k \leqslant n$, are uniquely determined by $a_{i,j+1}$ and $b_{i,j}$, $1 \leqslant i \leqslant n$, $0 \leqslant j \leqslant n$, and therefore also by the coefficients of the continued fraction (5).

Let now $c_1 \neq 0$. Then we can compute the coefficients of the inverse series

$$\sum_{k=0}^{\infty} \alpha_k t^k \quad \text{to} \quad \sum_{k=0}^{\infty} \frac{c_{k+1}}{c_1} t^k$$

via the explicit recursive formula

$$\alpha_0 = 1, \quad \alpha_k = -\sum_{n=1}^{k} c_n \alpha_{k-n}, \quad k \geqslant 1.$$

It can be shown (see [15]) that series

$$\frac{1}{c_1 t} \sum_{k=0}^{\infty} \alpha_k t^k$$

satisfies equation (2). Then $\tilde{y}(t) = f(t)$, i.e. continued fraction (3), is a partial solution of equation (2). Thus

$$\tilde{u}(t) = \frac{\tilde{y}(t)}{\gamma(t)} = f(t) e^{-\alpha t}$$

is a partial solution of equation (1).

Finally, we have the following approximants

$$\tilde{u}_0(t) = \frac{e^{-\alpha t}}{(G-\alpha)t},$$

$$\tilde{u}_1(t) = \frac{e^{-\alpha t}}{(G-\alpha)t + \dfrac{t}{(1+(G-\alpha)t)/(1-(G-\alpha))}},$$

$$\tilde{u}_2(t) = \frac{e^{-\alpha t}}{(G-\alpha)t + \dfrac{t}{(1+(G-\alpha)t)/(1-(G-\alpha)) + \dfrac{t/(1-(G-\alpha))}{(2+(G-\alpha)t)/(1-2(G-\alpha))}}},$$

$$\ldots\ldots\ldots\ldots$$

for the partial solution $\tilde{u}(t)$ of differential equation (1).

4. Application

The practical approbation of the diffusion model (1) was carried out on the basis of statistical information of money income of the population [16, 17] for Ukraine and the countries of the European Union that directly border with it (Table 1). This is primarily due to the need for a detailed analysis of various aspects of cross-border cooperation, including those related to income distribution and related migration processes. It is clear that this also fits into the national concept of deepening the Euro integration processes, the need to ensure important criteria for the possibility of Ukraine becoming a member of the EU.

As can be seen from Table 2, already the 6th approximant of continued fraction gives an approximation of real values with a relative error of less than 1%, where, in particular, to approximate the value of 657.42 (Czechia, 2012), we set $G = 24.9$ and $\alpha = 11.2457$. The following approximants give practically predetermined, arbitrarily high accuracy.

Table 1. Mean and median income by broad group of citizens (population aged 18 and over), EUR/month.

	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Czechia	630.58	657.42	646.08	640.00	622.50	658.50	691.42	755.92	829.83	889.83
Hungary	382.92	396.33	377.42	386.08	388.00	407.75	427.17	460.83	497.25	550.17
Poland	424.67	430.58	439.92	452.50	469.75	494.58	495.92	546.42	591.83	659.83
Romania	182.75	178.58	176.42	188.33	200.83	211.08	237.08	283.67	330.42	366.17
Slovakia	535.83	587.83	573.58	580.00	593.33	597.25	612.92	649.92	698.42	746.00
Ukraine	198.33	249.50	255.92	173.25	133.42	133.25	160.17	184.83	242.75	246.58

Table 2. Relative error of 6th approximant calculated on the model (1), %.

	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Czechia	0.2102	0.6501	0.7951	0.5649	0.4837	0.3152	0.4709	0.2834	0.1938	0.2695
Hungary	0.3105	0.1617	0.4636	0.3611	0.1684	0.4569	0.5174	0.1565	0.1206	0.4121
Poland	0.3021	0.9734	0.1405	0.7426	0.8894	0.2555	0.4120	0.5964	0.3160	0.8651
Romania	0.3850	0.6410	0.6188	0.2237	0.8073	0.1783	0.2111	0.6132	0.7652	0.9290
Slovakia	0.3171	0.2226	0.3102	0.4301	0.0193	0.1230	0.5223	0.7105	0.4125	0.1755
Ukraine	0.0930	0.2567	0.4203	0.5679	0.7882	0.3938	0.3649	0.1039	0.2443	0.5011

This allows us, in particular, to obtain sufficiently reliable forecasted values of money income on the basis of model (1) and to determine the dynamics of the further diffusion process (Fig. 1).

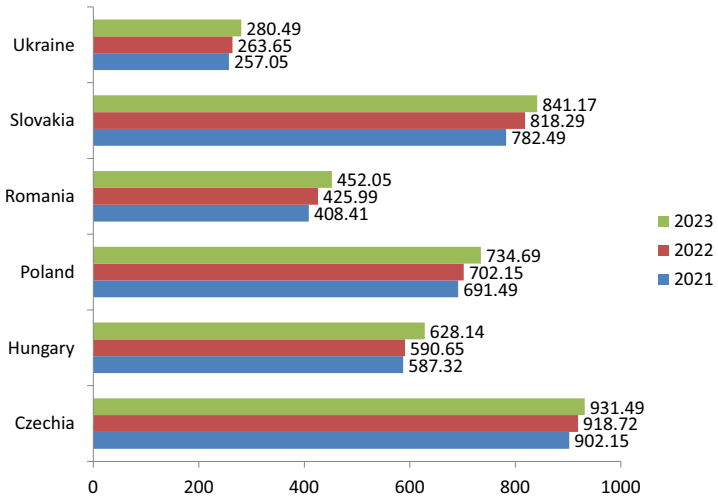

Fig. 1. Forecast values of mean and median income by broad group of citizens (population aged 18 and over), EUR/month.

As can be seen from Fig. 1, the Czech Republic and Slovakia occupy the leading positions in terms of forecast values of money income. As expected, Ukraine is a pronounced outsider. As for trends, as in the retrospective period, monotonous growth of average per capita money income is maintained in all studied countries, and growth rates do not differ significantly.

It is also important to analyze the influence of the diffusion model parameters on the money income change volume (u), in particular, the parameters G and α. For comparison, let us consider the spatial interpretation of money income of Ukraine and Czech Republic (Figs. 2, 3). An money income increase occurs with a simultaneous change in the parameter of social comparison and the Gini coefficient. The rate of diffusion for Czech Republic (299.58) is higher than for Ukraine (82.34) when the social comparison

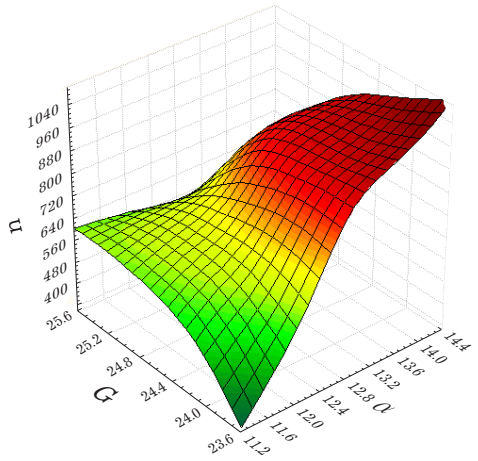

Fig. 2. Spatial interpretation of money income of Czechia.

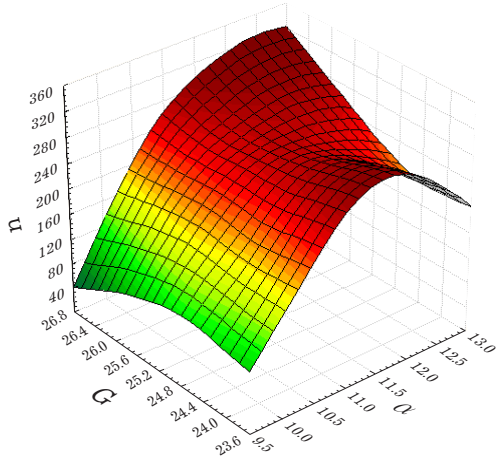

Fig. 3. Spatial interpretation of money income of Ukraine.

coefficient changes by 2.3. Moreover, the values of the Gini coefficient vary within the range from 24 to 26.5%, which characterizes the corresponding level of differentiation of money income. This level of differentiation of money income is an incentive for their increase, which also follows from the positions of upward comparison.

5. Conclusion

Thus, a predictive diffusion model of money income was developed, taking into account the processes of social comparison and the development of the spatial structure of the formation of money income. On the basis of the developed model, a forecast of the volume of money income of Ukraine and the countries of the European Union, which directly border it, was made. An analysis of the influence of the parameters of the diffusion model on changes in money income was carried out. An essential feature of our research is the use of the analytical theory of continuous fractions to find a solution to the model of diffusion of money income. In comparison with power series, continued fractions have a wider convergence domain and are endowed with the property of numerical stability. Their multidimensional generalizations – branched continued fractions – also have similar properties (see, for example, [18–21]. This creates prospects for their use to build other models of money income. Moreover, this approach can be applied to the description of various processes that can be analytically presented in the form of differential equations or their systems.

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Модель дифузії грошових доходів у контексті євроінтеграції

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Побудовано модель дифузії грошових доходів населення з урахуванням процесів соціального порівняння і розвитку просторової структури формування доходів. Для реалізації такої моделі використано аналітичну теорію неперервних дробів і знайдено розв'язок відповідного диференціального рівняння у формі формального функціонального неперервного дробу. Значення підхідних дробів неперервного дробу, який описує зміну рівня грошових доходів, дають наближення реальних значень з наперед заданою, як завгодно великою точністю. Це дозволило якісно описати загальну динаміку процесу дифузії грошових доходів.

Ключові слова: *модель дифузії; грошові доходи; функціональний неперервний дріб.*