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Air traffic modeling and optimization by solving two new models with a modified algorithm

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In this paper, we will try to manage arrivals in the approach area of Mohammed V airport by solving two proposed algorithms "Scheduling with speeds limitations" and "Scheduling with speeds corrections", with a modified Bat algorithm, this modification involved the speed equation as well as the frequency equation of the standard Bat algorithm to avoid the phenomenon of slow convergence to the best solution in the search space of this algorithm. These proposed algorithms will allow us to schedule traffic optimally and to increase the capacity of the airport and better manage the controlled space approach area of Mohammed V Airport.

Keywords: *air traffic management; algorithm; FCFS; scheduling; bat algorithm.*

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1. Introduction

The airspace is divided into geographical areas called control sectors, will soon reach its saturation level. To increase the amount of traffic that the system can absorb, it is necessary to reduce the workload of air traffic controllers by helping them in their task of aircraft separation. This is a challenging task with open issues between the world of research and the world of operational ATM (Air Traffic Management). The interface between these two domains is essential for the evolution of all ATS, systems, tools, and HMIs. The Air Traffic Management and Airport research program include technological issues (processing algorithms and operational systems, etc.), selection, training and training tools, human factors, and regulations. Its objective is to combine all the fields of expertise and technologies developed within the laboratories and transversal research programs to develop future ATM systems and more widely ATS in the operational world. Many algorithms have been developed to optimize and model air traffic like "An efficient genetic algorithm with uniform crossover for air traffic control" [1] and "VNS based algorithm for solving a $0-1$ nonlinear nonconvex model for the Collision Avoidance in Air Traffic Management" [2] and many others. To avoid the saturation of airspace we propose to solve two models which allow scheduling traffic using a meta-heuristic algorithm called modified Bat algorithm(BA), the proposed solution will allow us to improve the efficiency of airport operations and reduce flight delays as well as avoid congestion, both in airports and in the surrounding airspace.

2. Problematic

For several decades, air traffic management has been a mission that requires making decisions based on documents, standards, recommended practices, and procedures relating to the rules of the air which require controllers to manage traffic according to the order of entry to the control zone, this principle is called FCFS (First Come First Served) or (First In First Out) which designates that the first to cross the controlled space lands first and so on.

As an example, we assume that 2 aircrafts are arriving at the airport, A_1 and A_2, respectively, with speeds v_1 and v_2, we can then define the t_i (the time left for aircraft i for landing, ignoring all others), given that $t_1 = 16$ minutes, $t_2 = 18$ minutes respectively but aircraft A_2 will cross the area of approach earlier than A_1, so according to this principle A_2 will land after 18 min on the aerodrome, and it will only be able to receive A_1 just after 2 min (separation time threshold), which will create a minimum delay of 4 min for A_1, and a distance $d \geqslant 4v_i$ added to A_1's course. Following this example, we define the $test_i$ (the estimated impact time of aircraft i): $test_2 = 18\,\text{min}$ and $test_1 = 20\,\text{min}$.

Delay is the enemy of airlines, with daily expenses in the billions of dollars, so the objective of this paper is to facilitate decision-making for controllers and minimize delays. Therefore, optimizing the distance travelled, fuel consumption, and carbon dioxide emission rate. We will focus on the arrival flow, so the study we will do is limited to the controlled approach area.

3. Model 1: Scheduling with speeds limitations

Scheduling with speeds limitations is an algorithm that calculates the sequence of arrivals, the delay, and the distance added to the path of each aircraft while minimizing them with the Modified Bat Algorithm which is a meta-heuristic optimization method, this algorithm uses a BADA database (Base of Aircraft Data) which contains the speed profiles of each type of aircraft.

3.1. Delays calculation

If we have two aircrafts A_i and A_{i+1}, and A_i is received the first, then after 2 min (separation threshold) the aerodrome can receive A_{i+1}, this proposal has decreased the delay to zero. Aircraft A_i is more efficient than A_{i+1}, i.e. $t_i < t_{i+1}$, so if we order the aircraft in an order where the most efficient will be received, etc., then aircraft A_i can be landed from time $test_{i-1} + 2\,\text{min}$, so the challenge is to minimize the estimated time of impact of aircraft ith $test_i$ under the condition: $test_i \geqslant test_{i-1} + 2\,\text{min}$, minimizing $test_i$ implies minimizing the distances travelled. The problem imposed by this principle is to find the optimal scheduling order that minimizes the delays, and then by following this ordering how to continuously land aircrafts while minimizing the distances and separations between aircrafts on the airfield?

3.2. Distances calculation

Inspired by the thesis (Aircraft Route Network Optimization in Terminal Maneuvering Area presented by: MANLIANG), the response to the previous section's question is: the inclusion of an additional arc S2, two arcs S_1 and S_2 will enable us to switch the ordering while staying at equal distance from SLK (Siera Lema Kilo) point.

The idea behind this suggestion is that once we have identified the permutation p, we should permute the planes with the order of landing: $\{A_{p(1)}, A_{p(2)}, \ldots, A_{p(n)}\}$, with $A_{p(i)}$ for $i = 2, \ldots, n$ crosses R_i in both arcs.

3.3. Schedule with speeds limitations algorithm (s.s.l)

After the detection of the type of n aircraft by the radar, the radar calculates the estimated time of landing for each aircraft A_i according to the BADA (Base of Aircraft Data) [3], then after that time, order the aircraft by decreasing order of performance to obtain the optimized permutation p: $p\colon j \longrightarrow p(j)$, with $j \in \{1, 2, 3, \ldots, n\}$ is the landing order of the aircraft $A_{p(j)}$, so the ordering is $\{A_{p(1)}, A_{p(2)}, \ldots, A_{p(n)}\}$, so $A_{p(1)}$ lands at $test_1 = t_{p(1)}$, such as for $A_{p(2)}$ may land from $test_2 = \max(test_1 + 2\,\text{min}, t_{p(2)})$, as we cannot land an aircraft before its estimated time (speeds limitations) and the aerodrome is ready to receive $A_{p(2)}$ after $test_1 + 2\,\text{min}$, this separation takes into consideration the release of the aerodrome and the final separation. We define $R_i = test_i - t(p(i))$ the aircraft delay $A_{p(i)}$ with i being the landing order, it is clear that $R_1 = 0$, the aircraft delay $A_{p(1)}$ is reduced to zero. We then calculate the distance travelled in arc S_1: $d_{i,1}$ and the distance travelled in arc S_2: $d_{i,2}$ for each aircraft $A_{p(i)}$.

4. Model 2: Scheduling with speeds corrections

Scheduling with speeds corrections is an algorithm that calculates the sequence of arrivals, the delay and the distance added to the route of each aircraft like scheduling with speed limitations except at the level of recorded speed profiles in BADA, we will exploit them while adjusting them within an interval of $[-10\%; 10\%]$, so, in addition, this algorithm calculates the optimal speeds to minimize the delay and the distances traveled.

The gain of this procedure is enormous compared to the FCFS model and the model with speed limitations, for A_i will land before its estimated time calculated by the radar, which will allow us to reduce the delay, so we speak of gain, because we will land the planes before their t_i (estimated time calculated by the radar with speed limitations).

4.1. Delays calculation

In this section, optimization comes into play in the calculation of delays, the idea is the same as in the preceding section, depending on the estimated landing of each aircraft, we will order the aircraft by descending order of performance to find the optimal permutation p. $A_{p(1)}$ will be requested to accelerate as much as possible, thus adding 10% of the speeds recorded in BADA, then updating the estimated touchdown time with speed corrections $testv_1 < t_{p(1)}$, then optimizing to compute a, b and c speed percentages in the interval $[-10\%; 10\%]$ for the $A_{p(2)}$ aircraft to land at $testv_2 \geqslant testv_1 + 2\,\text{min}$, repeat the same procedure for the rest of the $A_{p(i)}$ aircraft, with $testv_i \geqslant testv_{i-1} + 2\,\text{min}$, then calculate $Rv_i = testv_i - t_{p(i)}$, R_i is the delay of the $A_{p(i)}$ aircraft after speed corrections, in case $Rv_i > 0$, otherwise, Rv_i is the gain of the $A_{p(i)}$ aircraft.

4.2. Distances calculation

The idea is the same as in the preceding section: to cover the delay on two arcs, except that we will take the corrected speeds calculated in the delay section into consideration.

4.3. Schedule with speeds corrections algorithm (s.s.c)

Order the aircrafts in descending order of performance to determine the permutation p. Then request $A_{p(1)}$ to accelerate as much as possible, thus adding 10% of the speeds recorded in BADA, then updating the estimated time reached with speed corrections $testv_1 < t_{p(1)}$, after calculating a, b and c speed percentages in the interval $[-10\%; 10\%]$ for the $A_{p(2)}$ aircraft to land at $testv_2 \geqslant testv_1 + 2\,\text{min}$, repeat the process for the rest of the $A_{p(i)}$ aircraft, with $testv_i \geqslant testv_{i-1} + 2\,\text{min}$, after calculating $Rv_i = testv_i - t_{p(i)}$, Rv_i is the delay of the $A_{p(i)}$ aircraft after speed correction, in case $Rv_i > 0$, we calculate the distance travelled in the arc S_1: $d_{i,1}$ and the distance travelled in arc S_2: $d_{i,2}$ for each aircraft $A_{p(i)}$.

5. Standard bat algorithm

The Bat Algorithm (BA) was introduced by Yang in 2010 [4], it simulates the echolocation behavior of microbats because it can generate high echolocation. The bat produces a high-pitched sound to detect its prey which resonates with a specific frequency. Echolocation is the process of detecting an object by reflected sound. It is used to know how far the prey is from the background object. By observing the bounce frequency of sound, bats can distinguish between prey and obstacle and can sense the distance between them in their close surroundings. They fly randomly with a certain speed, frequency, and volume to search for food. The solution of the objective function is to find prey at a minimum distance. The frequency and zoom settings maintain the balance between the exploration and exploitation processes. The algorithm continues until the convergence criteria are satisfied [3, 5].

5.1. Bat algorithm initialization

The initial population is randomly generated for n numbers of bats. Each individual in the population is described by a real-valued vector of dimension d.

The following equation is used to generate the initial population:

$$X_{ij} = X_{\min j} + \text{rand}(0,1)(X_{\max j} - X_{\min j}).$$

Where $i = 1, 2, \dots, n$, $j = 1, 2, \dots, d$, $X_{\max j}$ and $X_{\min j}$ are the upper and lower bounds for the dimension j.

5.2. Update positions, speeds, and frequencies

It is necessary to define the rules for updating their positions x_i and velocities v_i at each iteration t. Among all the solutions, there is a current best solution x^*. To obtain the new solutions x_i^t and velocities v_i^t at step t, the following equations are applied:

$$\begin{aligned} f_i &= f_{\min} + (f_{\max} - f_{\min})\beta, \\ v_i^t &= v_i^{t-1} + (x_i^t - x^*) f_i, \\ x_i^t &= x_i^{t-1} + v_i^t. \end{aligned}$$

Where β drawn randomly in $[0,1]$.

When a solution is selected from the best common solutions, a new solution for each bat is generated locally using a transformation incorporating a random factor.

$$X_{new} = X_{old} + \varepsilon A^t.$$

Where $\varepsilon \in [-1, 1]$ is a random number. A^t is the average volume of all bats at this processing step.

5.3. Update volume and pulse rate

The volume A^i and the pulsation rate r^i must be updated at each iteration using the following equations:

$$\begin{aligned} A_i^{t+1} &= \alpha A_i^t, \\ r^{t+1} &= r_i^0 \big[1 - \exp(-\gamma t)\big]. \end{aligned}$$

Where α and γ are constants.

6. Modified bat algorithm

In this section, we develop our proposed modification of the standard bat algorithm.

Indeed, the phenomenon of slow convergence to the best solution in the search space of the standard bat algorithm still exists, so to overcome this limitation, we propose to replace its velocity equation with the following equation:

$$V_{k+1} = \omega V_k + (b_1(P_i - X_k) + b_2(P_g - X_k)) f_k.$$

Such as: P_i is the best solution, P_g is the best solution in neighborhood, X_k is current position, ω is inertia, b_1, b_2 are randomly fired in $[0,1]$.

We also propose to replace its frequency update equation with the following equation:

$$f_i = (\min(f_i + \delta, f_{\max}) - \max(f_{min}, f_i - \delta))\gamma + \max(f_{\min}, f_i - \delta).$$

Such as: δ is change step in frequency, γ is random variable uniformly distributed on $[0,1]$.

Algorithm 1 Pseudo code of the modified bat algorithm

Require: $x_1, \ldots, x_d$;
Ensure: $f(x)$, $x = (x_1, \ldots, x_d)$;
1: Initialize the bat population x_i and the speed v_i, $i = 1, 2, \ldots, n$;
2: Define the pulse frequency f_i of each position x_i
3: Initialize the heartbeat rate r_i and the volume A_i
4: **while** $t <$ maximum number of iterations
5: Generate new solutions by adjusting frequency and updating speeds and positions/solutions with the following equations:
$v_{i+1} = \omega v_i + (b_1(P_i - x_i) + b_2(P_g - x_i))f_i$,
$f_i = (\min(f_i + \delta, f_{\max}) - \max(f_{\min}, f_i - \delta))\gamma + \max(f_{\min}, f_i - \delta)$
6: **if** $rand > r_i$ **then**
7: Select a solution among the best solutions
8: Generate a local solution around the best selected solution x^*
9: **if** $rand < A_i$ and $f(x_i) < f(x^*)$ **then**
10: Accept new solutions
11: Increase r_i and decrease A_i
12: Find the best solution x^*
13: Display the results given by the best solution x^*

7. Numerical simulation

In the following section, we present a simulation of the suggested algorithm, using the Terminal Area of Mohammed V Airport as a study case.

7.1. Parameters and results

We are going to simulate this algorithm by a generation of the positions $A(i)$ for 4 aircrafts arriving from the 3 areas (SIERA1, SIERA2, SIERA3). Concerning the speeds, we generated 4 speed profiles randomly from the database. Then we will apply the scheduling algorithms.

The 4 speeds profiles are:

$$V = \begin{bmatrix} 320 & 260 & 210 & 160 & 120 \\ 310 & 250 & 200 & 150 & 100 \\ 290 & 230 & 180 & 130 & 90 \\ 230 & 170 & 120 & 100 & 70 \end{bmatrix}.$$

As a reference order, we have taken the descending order of speed where A_1 has the highest speed profile and A_4 has the lowest.

The position $A(i)$ of each aircraft A_i in the circle zone of $R3 = 30\,\mathrm{Nm}$ are:

$$A = \begin{bmatrix} 10 & 8 & 18 & 2 \end{bmatrix}.$$

We need to know $O(i)$ the origin of each aircraft A_i, a random number u_i has been drawn at random for each A_i in the interval $[0,1]$ (uniform distribution):

– If $u_i < \frac{1}{3}$, then A_i comes from zone SIERA1, we set $O(i) = 0$;
– If $\frac{1}{3} \leqslant u_i < \frac{2}{3}$, then A_i comes from zone SIERA2, we set $O(i) = 1$;
– If $\frac{2}{3} \leqslant u_i \leqslant 1$, then A_i comes from the SIERA3 zone, we set $O(i) = 2$;

$$u = \begin{bmatrix} 0.5124 & 0.9540 & 0.7436 & 0.6741 \end{bmatrix}.$$

According to this drawing, A_1 originates from zone SIERA2, and, A_2, A_3 and A_4 originate from zone SIERA3 this information must be kept by stocking it in a vector O where each component $O(i)$ is worth either: 0, 1 or 2,

$$O = \begin{bmatrix} 1 & 2 & 2 & 2 \end{bmatrix}.$$

Compute the estimated landing time in hours for each A_i based on the reference order:

$$t = \begin{bmatrix} 0.2575 & 0.3864 & 0.4511 & 0.5026 \end{bmatrix}.$$

Determine the optimal permutation p that minimizes the time delay (order of landing based on the principle of scheduling [6]):

$$p = \begin{bmatrix}1 & 2 & 3 & 4\end{bmatrix}.$$

Determine the permutation q (FCFS landing order [7]):

$$q = \begin{bmatrix}1 & 4 & 2 & 3\end{bmatrix}.$$

We observe that A_4 gets the landing order of A_2 by the FCFS order and A_2 gets the landing order of A_3 by the FCFS order and A_3 gets the landing order of A_4 by the FCFS order.

Compute the estimated touchdown time $test$ following the principle of scheduling (speed limitations) [6] with $test_i$ associated with $A_{p(i)}$,

$$test = \begin{bmatrix}0.3250 & 0.3864 & 0.4511 & 0.5026\end{bmatrix}.$$

Compute the estimated hit time $testf$ using the FCFS principle with $testfi$ corresponding to $A_{q(i)}$,

$$testf = \begin{bmatrix}0.3250 & 0.5026 & 0.5366 & 0.5706\end{bmatrix}.$$

Compute the estimated touchdown time $testv$ based on the scheduling (speed correction) principle [6] with $test_{v_i}$ corresponding to $A_{p(i)}$, by computing the percentages of the corrected speeds:

$$testv = \begin{bmatrix}0.2575 & 0.2915 & 0.3255 & 0.3595\end{bmatrix}.$$

With the adjusted speed percentage $po(i;:)$ associated with $A_{p(i)}$ are:

$$po = \begin{bmatrix}0.1000 & 0.1000 & 0.1000\\ 0.6018 & 0.4346 & 1.0701\\ 0.1675 & 0.7003 & 0.6135\\ 0.9288 & 1.7338 & 0.8504\end{bmatrix}.$$

Compute the time delay R_i and R_{v_i} expressed in hours of each $A_{p(i)}$ based on the scheduling principle with speed limitations and speed corrections [6]:

$$Dsl = \begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}.$$

Notice that the delays of the aircrafts: A_1, A_2, A_3 and A_4 are decreased to zero,

$$Dsc = \begin{bmatrix}-0.0674 & -0.0949 & -0.1256 & -0.1430\end{bmatrix}$$

In some cases, speed corrections will enable us to land aircraft before their estimated landing time, like the A_1, A_2, A_3, and A_4 aircraft.

We compare the accumulated time delay using the FCFS, the accumulated time delay using the scheduling principle with speed limitations and speed corrections;

$$Dcf = 0.0674, \quad Dcsl = 0, \quad Dcsc = -0.1077.$$

The delays presented below are the average delays accumulated in hours by: FCFS, by the scheduling principle with speed limitations and speed corrections,

– $Dcf = 0.0674 \times 4 \times 60 = 16\,\text{min}\,10\,\text{s}$ (delay accumulated by FCFS).
– $Dcsl = 0 \times 4 \times 60 = 0\,\text{s}$ (delay due to speed limitations).
– $Dcsc = -0.1077 \times 4 \times 60 = -25\,\text{min}\,50\,\text{s}$ (gain by speed corrections).

7.2. Comparison

We tried to run the algorithms: Modified Bat algorithm, Standard bat algorithm (BA) and the Particle Swarm Optimization (PSO) applied to two proposed models 10 times. By comparing the results, we prove that our algorithm is better, as illustrated by the following table 1.

Table 1. Comparison between the results of the BA, PSO and Modified BA.

	Best(Dcsc)	Mean	Mean Squared Error	Variance
PSO	-0.0314	-0.0160	2.8641e-04	4.6422e-05
BA	-0.0964	-0.0516	0.0029	9.2176e-04
Modified BA	-0.1077	-0.0634	0.0023	3.7149e-04

8. Conclusion

In this paper, we have proposed a new algorithm, Improved Bat Algorithm, by modifying the speed and frequency update equations of the Standard Bat Algorithm. These modifications have reduced the convergence speed of the algorithm as well as refining the search for the best solutions.

We tested our algorithm on air traffic management by solving two new proposed models (scheduling with speed limitations and speed corrections) [8].

The obtained results demonstrate that our proposed algorithm is better than not only the Standard Bat algorithm but also other algorithms like the particle swarm algorithm [9, 10].

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Моделювання та оптимізація повітряного руху шляхом розв'язування двох нових моделей за модифікованим алгоритмом

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У цій статті здійснено спробу керувати прибуттям у зону заходу на посадку аеропорту Мохаммеда V, розв'язавши два запропонованих алгоритми "Складання розкладу з обмеженням швидкості" та "Складання розкладу з корекцією швидкості" за допомогою модифікованого алгоритму кажана, ця модифікація включає рівняння швидкості, а також рівняння частоти стандартного алгоритму кажана, щоб уникнути явища повільної збіжності до найкращого розв'язку в просторі пошуку цього алгоритму. Запропоновані алгоритми дозволять оптимально планувати рух, збільшити пропускну спроможність аеропорту та краще керувати контрольованою зоною заходу на посадку аеропорту Мохаммеда V.

Ключові слова: *керування повітряним рухом; алгоритм; FCFS; планування; алгоритм кажана.*