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Some inverse problem remarks of a continuous-in-time financial model in $L^1([t_{\mathrm{I}}, \Theta_{\max}])$

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In the paper we are going to introduce an operator that is involved in the inverse problem of the continuous-in-time financial model. This framework is designed to be used in the finance for any organization and, in particular, for local communities. It allows to set out annual and multiyear budgets, with describing loan, reimbursement and interest payment schemes. We discuss this inverse problem in the space of integrable functions over $\mathbb{R}$ having a compact support. The concept of ill-posedness is examined in this space in order to obtain interesting and useful solutions. Then, we will give some remarks for not functionality of the model for a given Repayment Pattern Density γ, when this operator is not invertible in the space. Additionally, this inverse problem is illustrated in order to prove its ability to be used in a financial strategy.

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1. Introduction

Over the last two decades, we have seen significant progress in econometric theory [1], computational and estimation methods to test and implement continuous-time models. The continuous-time approach in these areas has produced models with a rich variety of testable implications. It is useful to begin our survey with an overview of some of the major developments in the field. There are many references that deal with research works in finance and the way in which the practitioners viewed the finance research field. The book [2] by Merton laid the foundations for the development of intertemporal asset pricing theory. The purpose of the paper [3] is to build a bridge between continuous time models, which are central in the modem finance literature, and (weak) GARCH processes in discrete time, which often provide parsimonious descriptions of the observed data.

We built in previous work [4, 5], the continuous-in-time model which is designed to be used for the finances of public institutions. This model is based on using measures over time interval to describe the loan scheme, the reimbursement scheme and the interest payment scheme; and, on using mathematical operators. At the same time, some mathematical operators as convolution and primitive are implemented in [6] for continuous-in-time financial model. This development is given in the form of an API (Application Programming Interface). This API is marketed by the company MGDIS. Some aspects of the modeling of the asset prices for the financial market are studied in [7].

This way to proceed gives good results, works well and software tools implementing it help organizations to foresee the consequences of their decisions allowing to elaborate their projects. We highlight that the discrete model utilizes tables and creates outcomes in Excel format. Each value in the tables is a synthetic of a given quantity over a given period of time. In order to provide answers from the discrete model to the implemented one, the key idea is to report these outcomes on any set of periods

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of time without reimplementing it. To build the new framework that is the improvement of the discrete one, measures are managed and defined during the whole time period.

The understanding of most inverse financial problems has a rapid state of development that a review of the subject will inevitably needs to be updated rapidly. Moreover, it is important to quote some contributions in the field. For example, the paper [8] presents a phase retrieval problem and a specific inverse option pricing problem. The paper [9] by Potthast deals with the time-harmonic acoustic obstacle scattering problem with Dirichlet boundary condition. In the paper [10], the authors have developed a series of novel variational source conditions for Linear Inverse Problems in Hilbert Spaces.

We are motivated by a recently developed nonlinear inverse scale in Schwartz space. The questions of the inverse problem in Schwartz space are currently investigated in [11] for ill-posed problems. Further, we mention the paper [12] introduced by Hadamard in the field of ill-posed problems. These problems are ill-conditioned or underdetermined. We say that problem is ill-posed, if and only if, the operator involved in the model denoted by $\mathcal{L}$, is invertible. For completing the history of the inverse problem of the model, the inverse problem stability in Hilbert space $\mathbb{L}^2([t_{\mathrm{I}}, \Theta_{\max}])$ is discussed in [13]. On the other hand, the differential evolution algorithm described in [14] is applied to data inversion to layer geo-electrical models. Since some aspects of the model are explored over Schwartz space and also space of square-integrable functions, this present work aims to enrich the model over another space. We choose it such having less integrability than two, changing consequently, important properties of solution. The main result of the present paper is to find new technique with determining regularized solution to discrete ill-posed in the space of integrable functions $\mathbb{L}^1([t_{\mathrm{I}}, \Theta_{\max}])$ having their support in interval $[t_{\mathrm{I}}, \Theta_{\max}]$. The solvability of this inverse problem is to show existence and uniqueness of the solution.

The rest of this paper is arranged as follows. In Section 2, we introduce the definition of the operator $\mathcal{L}$ and others. We give an overview of its property, described by its injectivity. Next, the regularized solution to discrete ill-posed is investigated in $\mathbb{L}^1([t_{\mathrm{I}}, \Theta_{\max}])$ by determining the inverse of operator $\mathcal{L}$ under some assumptions. In the same part, examples are provided in order to show that Repayment Pattern Density γ does not hold the expectation given in theory. Section 3 makes the model useful within an automatic strategy elaboration to manage financially desired goals and that satisfies imposed constraints.

2. Inverse problem of the model in $\mathbb{L}^1([t_{\mathrm{I}}, \Theta_{\max}])$

The purpose of this section is to build mathematical properties of the financial model. It concerns also some general results of the inverse problem for any density γ. We proceed by denoting $\mathcal{C}_c([t_{\mathrm{I}}, \Theta_{\max}])$ as the continuous functions space defined over non-empty time interval $[t_{\mathrm{I}}, \Theta_{\max}]$ with the usual norm $\| \, \|_{L^\infty([t_{\mathrm{I}}, \Theta_{\max}])}$ which is well known as

$$\|\psi\|_{L^\infty([t_{\mathrm{I}}, \Theta_{\max}])} = \sup_{t \in [t_{\mathrm{I}}, \Theta_{\max}]} \big\{ |\psi(t)| \big\}.$$

We build the continuous-in-time financial model on the Radon measure space $\mathcal{M}([t_{\mathrm{I}}, \Theta_{\max}])$ which is a continuous and linear form acting on the continuous functions space $\mathcal{C}_c([t_{\mathrm{I}}, \Theta_{\max}])$ defined over a time interval $[t_{\mathrm{I}}, \Theta_{\max}]$. The set of Radon measures $\mathcal{M}([t_{\mathrm{I}}, \Theta_{\max}])$ is a Banach space provided with the usual norm

$$\|\mu\|_{\mathcal{M}((t_{\mathrm{I}}, \Theta_{\max}))} = \sup_{\psi \in \mathcal{C}_c([t_{\mathrm{I}}, \Theta_{\max}]), \psi \neq 0} \left\{ \frac{\langle \mu, \psi \rangle}{\|\psi\|_{L^\infty([t_{\mathrm{I}}, \Theta_{\max}])}} \right\}.$$

For any interval $[t_1, t_2]$, $t_2 > t_1$, $\mathbb{L}^1([t_1, t_2])$ stands for the space of integrable functions over $\mathbb{R}$ having their support in $[t_1, t_2]$. Similarly, $\mathbb{L}^2([t_1, t_2])$ is the space of square-integrable functions over $\mathbb{R}$ having their support in $[t_1, t_2]$. If measure $\tilde{\mu}$ is absolutely continuous with respect to the Lebesgue measure dt, then, this means that it reads $\mu(t)dt$, i.e. $\tilde{\mu} = \mu(t)dt$, where t is the variable in $\mathbb{R}$. Other measures, are not absolutely continuous with respect to dt, for instance Dirac masses. They illustrated concentrated actions or payments. This density $\mu(t)$ of measure $\tilde{\mu}$ can be seen as a time density. These densities are considered in $\mathbb{L}^1([t_1, t_2])$. In the sequel, we would like to know more about these measures or densities. We will set out to introduce the financial quantities that are involved in the model and the relations between them. For a positive number Θ_γ such that $\Theta_\gamma < \Theta_{\max} - t_{\mathrm{I}}$, we set the Repayment Pattern γ such that

$$\gamma \in \mathbb{L}^\infty([0, \Theta_\gamma]). \tag{1}$$

The Repayment Pattern Measure $\tilde{\gamma}$ is a non-negative measure with a total mass equal to 1, i.e.

$$\int_{-\infty}^{+\infty} \tilde{\gamma} = 1. \tag{2}$$

The equality (2) means that the payment of an amount 1 which is expressed in monetary unit that is borrowed at the initial time $t = 0$. The Loan Measure $\tilde{\kappa}_E$ and the Repayment Measure $\tilde{\rho}_{\mathcal{K}}$ are connected by a convolution operator [15]

$$\tilde{\rho}_{\mathcal{K}} = \tilde{\kappa}_E \star \tilde{\gamma}. \tag{3}$$

The measure $\tilde{\rho}_{\mathcal{K}}^{\mathrm{I}}$ is the repayment of the Current Debt $\mathcal{K}_{RD}$ at initial time t_{I}. It is called an initial Debt Repayment Scheme given by

$$\int_{t_{\mathrm{I}}}^{+\infty} \tilde{\rho}_{\mathcal{K}}^{\mathrm{I}} = \mathcal{K}_{RD}(t_{\mathrm{I}}).$$

Defining the linear operator $\mathcal{L}$: $\mathbb{L}^1([t_{\mathrm{I}}, \Theta_{\max} - \Theta_\gamma]) \cap \mathcal{C}_c([t_{\mathrm{I}}, \Theta_{\max} - \Theta_\gamma]) \to \mathbb{L}^1([t_{\mathrm{I}}, \Theta_{\max}])$ by the operator that is acting on Loan Density κ_E,

$$\mathcal{L}[\kappa_E](t) = \kappa_E(t) - (\kappa_E \star \gamma)(t) - \alpha \int_{t_{\mathrm{I}}}^{t} (\kappa_E - \kappa_E \star \gamma)(s)\, ds. \tag{4}$$

The Algebraic Spending Measure $\tilde{\sigma}$ is defined such that the difference between spendings and incomes required to satisfy the current needs. Its Density σ is decomposed as a sum of operators $\mathcal{L}$ given by equality (4) and $\mathcal{D}$

$$\sigma(t) = \mathcal{L}[\kappa_E](t) + \mathcal{D}[\rho_{\mathcal{K}}^{\mathrm{I}}](t),$$

where the operator $\mathcal{D}$: $\mathbb{L}^1([t_{\mathrm{I}}, \Theta_{\max}]) \cap \mathcal{C}_c([t_{\mathrm{I}}, \Theta_{\max}]) \to \mathbb{L}^1([t_{\mathrm{I}}, \Theta_{\max}])$ is acting on the Initial Debt Repayment Density $\rho_{\mathcal{K}}^{\mathrm{I}}$, defined as

$$\mathcal{D}[\rho_{\mathcal{K}}^{\mathrm{I}}](t) = -\alpha \int_{t}^{\Theta_{\max}} \rho_{\mathcal{K}}^{\mathrm{I}}(s)\, ds - \rho_{\mathcal{K}}^{\mathrm{I}}(t).$$

We denote by $\mathcal{F}$ the Fourier Transform Operator, and by $\mathcal{F}^{-1}$ its operator inverse. The convolution of Loan Density κ_E in the space of integrable functions $\mathbb{L}^1([t_{\mathrm{I}}, \Theta_{\max} - \Theta_\gamma])$ and Pattern Density γ in space $\mathbb{L}^\infty([0, \Theta_\gamma])$ defines a function staying in the space of integrable functions over $\mathbb{R}$ having its support in $[t_{\mathrm{I}}, \Theta_{\max}]$, i.e. $\kappa_E \star \gamma \in \mathbb{L}^1([t_{\mathrm{I}}, \Theta_{\max}])$. From this, the first part in expression (4) defines a density $\kappa_E - \kappa_E \star \gamma$ which belongs also to the same space $\mathbb{L}^1([t_{\mathrm{I}}, \Theta_{\max}])$. Besides of this,

$$\begin{aligned}
\left\| \int_{t_{\mathrm{I}}}^{t} (\kappa_E - \kappa_E \star \gamma)(s)\, ds \right\|_{\mathbb{L}^1([t_{\mathrm{I}}, \Theta_{\max}])} &= \int_{t_{\mathrm{I}}}^{\Theta_{\max}} \left| \int_{t_{\mathrm{I}}}^{t} (\kappa_E - \kappa_E \star \gamma)(s)\, ds \right| dt \\
&\leqslant \int_{t_{\mathrm{I}}}^{\Theta_{\max}} \left| \int_{t_{\mathrm{I}}}^{\Theta_{\max}} (\kappa_E - \kappa_E \star \gamma)(s)\, ds \right| dt \\
&\leqslant (\Theta_{\max} - t_{\mathrm{I}}) \int_{t_{\mathrm{I}}}^{\Theta_{\max}} |(\kappa_E - \kappa_E \star \gamma)(s)|\, ds \\
&\leqslant (\Theta_{\max} - t_{\mathrm{I}}) \|\kappa_E - \kappa_E \star \gamma\|_{\mathbb{L}^1([t_{\mathrm{I}}, \Theta_{\max}])}.
\end{aligned}$$

Consequently, operator $\mathcal{L}$ given by (4) is well defined from space $\mathbb{L}^1([t_{\mathrm{I}}, \Theta_{\max} - \Theta_\gamma]) \cap \mathcal{C}_c([t_{\mathrm{I}}, \Theta_{\max} - \Theta_\gamma])$ to $\mathbb{L}^1([t_{\mathrm{I}}, \Theta_{\max}])$. Noticing that we have studied in previous work [4,5,11,13] the inversion of operator $\mathcal{L}$ in $\mathbb{L}^2([t_{\mathrm{I}}, \Theta_{\max}])$. We have stated its properties, taking advantage of application $\mathcal{F}$ as isometry in this space. Nevertheless, the new difficulty is due to the fact that operator $\mathcal{F}$ is not a bijective application on $\mathbb{L}^1([t_\mathrm{I}, \Theta_\mathrm{max}])$. To tackle this difficult problem, an efficient way is investigated. It consists in using the Gronwall lemma in such an assumption to inverse it in Theorem 1. The aim of expressing this inversion is to show its capability to be used to forecast a financial strategy when these densities have less integrability order than two. In other words, forecast consequences of any financial decision consists of inverting this operator.

Lemma 1. *If Loan Density γ satisfies the assumption* (1) *and the following analytic condition*

$$x \to x\gamma(x) \in \mathbb{L}^1([0, \Theta_\gamma]),$$

then the linear operator $\mathcal{L}$ given by relation (4) *is a one-to-one mapping.*

Proof. We will show that the kernel of the operator $\mathcal{L}$ is defined as

$$\mathrm{Ker}(\mathcal{L}) = \left\{\kappa_E \in \mathbb{L}^1([t_\mathrm{I}, \Theta_\mathrm{max} - \Theta_\gamma]), \kappa_E - (\kappa_E \star \gamma) = 0\right\}. \tag{5}$$

We recall in [4] that we have proven over $\mathbb{L}^2([t_\mathrm{I}, \Theta_\mathrm{max}])$ the following equality

$$\kappa_E(t) - \kappa_E \star \gamma(t) = \mathcal{L}[\kappa_E](t) + \alpha \int_{t_\mathrm{I}}^{t} \mathcal{L}[\kappa_E](s) \mathrm{e}^{\alpha(t-s)}\, ds, \tag{6}$$

which stays true in $\mathbb{L}^1([t_\mathrm{I}, \Theta_\mathrm{max}])$ due to the integration by parts (see equality (3.12) of Lemma 3.2). The way to prove that two sets are equal as mentioned in relation (5), we will prove each of two sets is a subset of the other set. In particular, according to (6), if the Loan Density κ_E is supposed to be in the kernel $\mathrm{Ker}(\mathcal{L})$, then it satisfies

$$\kappa_E - \kappa_E \star \gamma = 0, \tag{7}$$

achieving the direct inclusion. Conversely, injecting equality (7) in (4), confirms that density κ_E stays in the kernel. Consequently, equality (5) holds. In what follows, the Fourier transform is applied to equality (7) to get

$$\mathcal{F}(\kappa_E) \times (1 - \mathcal{F}(\gamma)) = 0.$$

We now use the method of proof by contradiction to prove that the function $\mathcal{F}(\gamma)$ does not coincide with the value 1. Let us assume, for the sake of contradiction, that there exists a Pattern Density γ in $\mathbb{L}^\infty([0, \Theta_\gamma])$ which satisfies

$$\mathcal{F}(\gamma) = 1. \tag{8}$$

The derivative operator is applied to equality (8) which takes the following form

$$\mathcal{F}'(\gamma) = 0.$$

From this, the derivative property of the Fourier Transform [16] implies

$$\mathcal{F}(-x\gamma(x)) = 0.$$

Since the null Pattern γ is not responding to definition (2), a contradiction is obtained. Then, the operator $\mathcal{L}$ is a one-to-one application, completing the proof of the lemma. Now, we will announce Theorem 1, which is a generic result that indicates the way to reverse the process that consists in computing the Loan Density κ_E from the Algebraic Spending Density σ into a process that involves building this density. In order to prove this theorem, the previous Lemma 1 is needed. ■

Theorem 1. *Assuming that density γ satisfies* (1), *and following assumption, we have*

$$\left(\frac{1}{1 - \mathcal{F}(\gamma)}\right) \in \mathbb{L}^\infty(] - \infty, -\varepsilon[\cup]\varepsilon, +\infty[), \tag{9}$$

for any positive real ε. Let $\mathcal{L}[\kappa_E] = \sigma - \mathcal{D}[\rho_\mathcal{K}^\mathrm{I}]$ be a positive function and the rate α be a negative real that satisfies the following balanced equation

$$\int_{t_\mathrm{I}}^{\Theta_\mathrm{max}} \left(\mathcal{L}[\kappa_E](y) + \alpha \int_{t_\mathrm{I}}^{y} \mathcal{L}[\kappa_E](s) \mathrm{e}^{\alpha(y-s)}\, ds\right) dy = 0. \tag{10}$$

Then, the Density κ_E is expressed in terms of σ as

$$\kappa_E = \mathcal{F}^{-1}\left(\frac{\mathcal{F}\Big(\sigma - \mathcal{D}[\rho_{\mathcal{K}}^{\mathrm{I}}] + \alpha \int_{t_{\mathrm{I}}}^{\bullet}(\sigma(s) - \mathcal{D}[\rho_{\mathcal{K}}^{\mathrm{I}}](s))\, \mathrm{e}^{\alpha(\bullet - s)}\, ds\Big)}{1 - \mathcal{F}(\gamma)}\right), \tag{11}$$

where

$$\mathcal{F}\left(\sigma - \mathcal{D}[\rho_{\mathcal{K}}^{\mathrm{I}}] + \alpha \int_{t_{\mathrm{I}}}^{\bullet} \big(\sigma(s) - \mathcal{D}[\rho_{\mathcal{K}}^{\mathrm{I}}](s)\big)\, \mathrm{e}^{\alpha(\bullet - s)}\, ds\right),$$

stands for the Fourier Transform of function

$$t \mapsto \mathcal{F}\left(\sigma(t) - \mathcal{D}[\rho_{\mathcal{K}}^{\mathrm{I}}](t) + \alpha \int_{t_{\mathrm{I}}}^{t} \big(\sigma(s) - \mathcal{D}[\rho_{\mathcal{K}}^{\mathrm{I}}](s)\big)\, \mathrm{e}^{\alpha(t - s)} ds\right).$$

Proof. First, employing definition (2) of the Repayment Pattern Density γ coupled with the convolution operator $\star$ to get

$$\kappa_E(t) - \kappa_E \star \gamma(t) = \int_0^{\Theta_\gamma} \gamma(y)\big(\kappa_E(t) - \kappa_E(t-y)\big) dy.$$

Next, using Heine theorem [17] which states that a continuous function κ_E on compact interval $[t_{\mathrm{I}}, \Theta_{\max} - \Theta_\gamma]$ is uniformly continuous, allowing the existence of a positive constant C such that

$$|\kappa_E(t) - \kappa_E \star \gamma(t)| \leqslant C \sup_{y \in [0, \Theta_\gamma]} \big\{|\gamma(y)|\big\} \int_0^{\Theta_\gamma} y\, dy \leqslant \frac{C\Theta_\gamma^2}{2} \|\gamma\|_{L^\infty([0,\Theta_\gamma])}. \tag{12}$$

Inequality (12) means that density $\kappa_E - \kappa_E \star \gamma$ is bounded on interval $[t_{\mathrm{I}}, \Theta_{\max}]$. From this, and according to equality (6), it follows that

$$\sigma(t) - \mathcal{D}[\rho_{\mathcal{K}}^{\mathrm{I}}](t) + \alpha \int_{t_{\mathrm{I}}}^{t} \left(\sigma - \mathcal{D}[\rho_{\mathcal{K}}^{\mathrm{I}}]\right)(s) \mathrm{e}^{\alpha(t-s)} ds \leqslant \frac{C\Theta_\gamma^2}{2} \|\gamma\|_{L^\infty([0,\Theta_\gamma])}. \tag{13}$$

Multiplying inequality (13) by $\mathrm{e}^{-\alpha t}$, and applying Gronwall lemma to obtain following inequality

$$\begin{aligned}
\left(\sigma - \mathcal{D}[\rho_{\mathcal{K}}^{\mathrm{I}}]\right)(t)\mathrm{e}^{-\alpha t} &\leqslant \frac{C\Theta_\gamma^2}{2} \|\gamma\|_{L^\infty([0,\Theta_\gamma])} \mathrm{e}^{-\alpha t} - \frac{\alpha C\Theta_\gamma^2}{2} \|\gamma\|_{L^\infty([0,\Theta_\gamma])} \int_{t_{\mathrm{I}}}^{t} \mathrm{e}^{-\alpha s} \mathrm{e}^{-\int_s^t \alpha\, du}\, ds \\
&\leqslant \frac{C\Theta_\gamma^2}{2} \|\gamma\|_{L^\infty([0,\Theta_\gamma])} \mathrm{e}^{-\alpha t} - \frac{\alpha C\Theta_\gamma^2}{2} \|\gamma\|_{L^\infty([0,\Theta_\gamma])} \mathrm{e}^{-\alpha t}(t - t_{\mathrm{I}}) \\
&\leqslant \frac{C\Theta_\gamma^2}{2} \|\gamma\|_{L^\infty([0,\Theta_\gamma])} \mathrm{e}^{-\alpha t}\big(1 - \alpha(t - t_{\mathrm{I}})\big).
\end{aligned}$$

That is, for all variable time t in interval $[t_{\mathrm{I}}, \Theta_{\max}]$,

$$\left(\sigma - \mathcal{D}[\rho_{\mathcal{K}}^{\mathrm{I}}]\right)(t) \leqslant \frac{C\Theta_\gamma^2}{2} \|\gamma\|_{L^\infty([0,\Theta_\gamma])} \big(1 - \alpha(t - t_{\mathrm{I}})\big). \tag{14}$$

The function defined by $t \to 1 - \alpha(t - t_{\mathrm{I}})$ is in $\mathbb{L}^2([t_{\mathrm{I}}, \Theta_{\max}])$ because of

$$\begin{aligned}
\|t \to 1 - \alpha(s - t_{\mathrm{I}})\|_{\mathbb{L}^2([t_{\mathrm{I}},\Theta_{\max}])} &= \sqrt{\int_{t_{\mathrm{I}}}^{\Theta_{\max}} \big(1 - \alpha(s - t_{\mathrm{I}})\big)^2 ds} \\
&= \sqrt{-\frac{1}{3\alpha}\left[\big(1 - \alpha(\Theta_{\max} - t_{\mathrm{I}})\big)^3 - 1\right]},
\end{aligned}$$

is a finite quantity. From this and according to inequality (14), the operator $\mathcal{L}$ is in $\mathbb{L}^2([t_{\mathrm{I}}, \Theta_{\max}])$. As $\mathcal{L} \in \mathbb{L}^2([t_{\mathrm{I}}, \Theta_{\max}])$, we will show that:

$$\mathcal{L}[\kappa_E] + \alpha \int_{t_{\mathrm{I}}}^{\bullet} \mathcal{L}[\kappa_E](s)\, \mathrm{e}^{\alpha(\bullet - s)}\, ds \in \mathbb{L}^2([t_{\mathrm{I}}, \Theta_{\max}]). \tag{15}$$

Indeed, for all reals $t \leqslant \Theta_{\max}$, we have

$$\left\| \int_{t_{\mathrm{I}}}^{\bullet} \mathcal{L}[\kappa_E](s)\, \mathrm{e}^{\alpha(\bullet - s)}\, ds \right\|_{\mathbb{L}^2([t_{\mathrm{I}},\Theta_{\max}])} = \sqrt{\int_{t_{\mathrm{I}}}^{\Theta_{\max}} \left(\int_{t_{\mathrm{I}}}^{t} \mathcal{L}[\kappa_E](s)\, \mathrm{e}^{\alpha(t-s)}\, ds\right)^2 dt}$$

$$\leqslant \sqrt{\Theta_{\max}-t_{\rm I}} \times \left| \int_{t_{\rm I}}^{\Theta_{\max}} \mathcal{L}[\kappa_E](s)\, \mathrm{e}^{\alpha(\Theta_{\max}-s)}\, ds \right|.$$

From this, and according to following inequality $|\mathrm{e}^{\alpha(\Theta_{\max}-s)}| \leqslant \mathrm{e}^{|\alpha|(\Theta_{\max}-t_{\rm I})}$ for all $s \in [t_{\rm I}, \Theta_{\max}]$ yields,

$$\left\| \int_{t_{\rm I}}^{\bullet} \mathcal{L}[\kappa_E](s)\mathrm{e}^{\alpha(\bullet-s)}\, ds \right\|_{\mathbb{L}^2([t_{\rm I},\Theta_{\max}])} \leqslant \sqrt{\Theta_{\max}-t_{\rm I}} \times \mathrm{e}^{|\alpha|(\Theta_{\max}-t_{\rm I})} \times \|\mathcal{L}[\kappa_E]\|_{\mathbb{L}^1([t_{\rm I},\Theta_{\max}])}. \tag{16}$$

Next, we use Cauchy–Schwarz inequality to obtain

$$\|\mathcal{L}[\kappa_E]\|_{\mathbb{L}^1([t_{\rm I},\Theta_{\max}])} \leqslant \sqrt{\Theta_{\max}-t_{\rm I}} \times \|\mathcal{L}[\kappa_E]\|_{\mathbb{L}^2([t_{\rm I},\Theta_{\max}])}. \tag{17}$$

Inequalities (16) and (17) imply

$$\left\|\mathcal{L}[\kappa_E] + \alpha \int_{t_{\rm I}}^{\bullet} \mathcal{L}[\kappa_E](s)\, \mathrm{e}^{\alpha(\bullet-s)}\, ds \right\|_{\mathbb{L}^2([t_{\rm I},\Theta_{\max}])} \leqslant (1+|\alpha|(\Theta_{\max}-t_{\rm I})\, \mathrm{e}^{|\alpha|(\Theta_{\max}-t_{\rm I})})\|\mathcal{L}[\kappa_E]\|_{\mathbb{L}^2([t_{\rm I},\Theta_{\max}])},$$

achieving the proof of (15). In the sequel, noticing that the equality (11) is proved in [4] under the following assumption $\kappa_E \in \mathbb{L}^2([t_{\rm I}, \Theta_{\max}-\Theta_\gamma])$, however, we will continue to prove it with the initial one that is $\kappa_E \in \mathbb{L}^1([t_{\rm I}, \Theta_{\max}-\Theta_\gamma]) \cap \mathcal{C}_c([t_{\rm I}, \Theta_{\max}-\Theta_\gamma])$. Since the Fourier transform map $\mathcal{F}$ is an isometry with respect to the $\mathbb{L}^2$ norm, hence we may apply this application to relation (6), the Loan Density κ_E is given in terms of the Algebraic Spending Density σ as

$$\mathcal{F}(\kappa_E) = \frac{\mathcal{F}\left(\sigma - \mathcal{D}[\rho_{\mathcal{K}}^{\rm I}] + \alpha \int_{t_{\rm I}}^{\bullet} (\sigma(s) - \mathcal{D}[\rho_{\mathcal{K}}^{\rm I}](s))\, \mathrm{e}^{\alpha(\bullet-s)} ds\right)}{1-\mathcal{F}(\gamma)}. \tag{18}$$

The meaning of the equality (18) is interpreted as follows. The product of function $\mathcal{F}\big(\sigma - \mathcal{D}[\rho_{\mathcal{K}}^{\rm I}] + \alpha \int_{t_{\rm I}}^{\bullet} (\sigma(s) - \mathcal{D}[\rho_{\mathcal{K}}^{\rm I}](s))\, \mathrm{e}^{\alpha(\bullet-s)}\, ds\big)$ in $\mathbb{L}^2(\mathbb{R})$ by a function $\frac{1}{1-\mathcal{F}(\gamma)}$ in $\mathbb{L}^\infty(]-\infty,-\varepsilon[\cup]\varepsilon,+\infty[)$, is function $\mathcal{F}(\kappa_E)$ which stays in the space $\mathbb{L}^2(]-\infty,-\varepsilon[\cup]\varepsilon,+\infty[)$ for any positive real ε. From this, and according to the assumption (9), the Lemma is proved only over the interval $]-\infty,-\varepsilon[\cup]\varepsilon,+\infty[$. In order to complete the proof of the Lemma, we will show that the equality (11) stays true over an interval containing the singularity point [18, 19], which is naturally zero.

The equality (10) is coupled with the Taylor development of function $\mathcal{F}(\mathcal{L}[\kappa_E] + \alpha \int_{t_{\rm I}}^{\bullet} \mathcal{L}[\kappa_E](s)\, \mathrm{e}^{\alpha(\bullet-s)}\, ds)$ around zero to give

$$\mathcal{F}\left(\mathcal{L}[\kappa_E] + \alpha \int_{t_{\rm I}}^{\bullet} \mathcal{L}[\kappa_E](s)\, \mathrm{e}^{\alpha(\bullet-s)}\, ds\right)(\xi)$$
$$= -i\,\xi \int_{t_{\rm I}}^{\Theta_{\max}} y\left(\mathcal{L}[\kappa_E](y) + \alpha \int_{t_{\rm I}}^{y} \mathcal{L}[\kappa_E](s)\, \mathrm{e}^{\alpha(y-s)} ds\right) dy + O(\xi^2). \tag{19}$$

Similary, the property of the Repayment Pattern Density γ given by the definition (2) is coupled with the Taylor development of function $\frac{1}{1-\mathcal{F}(\gamma)}$ around zero in order to yield the following equality

$$\frac{1}{1-\mathcal{F}(\gamma)(\xi)} = \frac{-i}{\xi \int_0^{\Theta_\gamma} y\, \gamma(y)\, dy + O(\xi^2)}. \tag{20}$$

According to these developments given by equalities (19) and (20), $\mathcal{F}(\kappa_E)$ is a finite term around the singularity point $\xi = 0$. In other words, we can say in this case that the function $\mathcal{F}(\kappa_E)$ is in $\mathbb{L}^\infty(\mathbb{R})$, which gives $\mathcal{F}(\kappa_E)$ in $\mathbb{L}^2(\mathbb{R})$. Here, we remark that if equality given by relation (10) does not hold for the operator $\mathcal{L}$, then the function $\mathcal{F}(\kappa_E)$ is not in $\mathbb{L}^\infty(\mathbb{R})$. Indeed, when the variable ξ is close to zero, the equivalent function $\mathcal{F}(\kappa_E)$ is defined by

$$\mathcal{F}_{R_0}(\kappa_E) - \frac{i}{\xi}\mathcal{F}_{I_0}(\kappa_E). \tag{21}$$

In which, operators $\mathcal{F}_{R_0}$ and $\mathcal{F}_{I_0}$ depend on the Loan Density κ_E are defined as

$$\mathcal{F}_{R_0}(\kappa_E) = \frac{\int_{t_{\rm I}}^{\Theta_{\max}} y\left(\mathcal{L}[\kappa_E](y) + \alpha \int_{t_{\rm I}}^{y} \mathcal{L}[\kappa_E](s)\mathrm{e}^{\alpha(y-s)} ds\right) dy}{\int_0^{\Theta_\gamma} y\, \gamma(y)\, dy},$$

$$\mathcal{F}_{I_0}(\kappa_E) = \frac{\int_{t_I}^{\Theta_{\max}} \left(\mathcal{L}[\kappa_E](y) + \alpha \int_{t_I}^{y} \mathcal{L}[\kappa_E](s) e^{\alpha(y-s)} ds \right) dy}{\int_0^{\Theta_\gamma} y \, \gamma(y) \, dy}.$$

According to decomposition (21), the function $\mathcal{F}(\kappa_E)$ diverges when the variable ξ goes to zero. ■

Example 1. Considering a fixed Pattern Repayment Density γ given by

$$\gamma(t) = \left(\frac{t}{\Theta_\gamma^2} + \frac{1}{2\Theta_\gamma} \right) \mathbb{1}_{[0,\Theta_\gamma]}(t).$$

The Fourier Transform of this density γ is computed as

$$\forall \xi \in \mathbb{R}^*, \quad 1 - \mathcal{F}(\gamma)(\xi) = 1 - \left(\frac{1}{(\xi\Theta_\gamma)^2} + \frac{i}{2\xi\Theta_\gamma} \right) \left(e^{-i\xi\Theta_\gamma} - 1 \right) - \frac{i}{\xi\Theta_\gamma} e^{-i\xi\Theta_\gamma}. \tag{22}$$

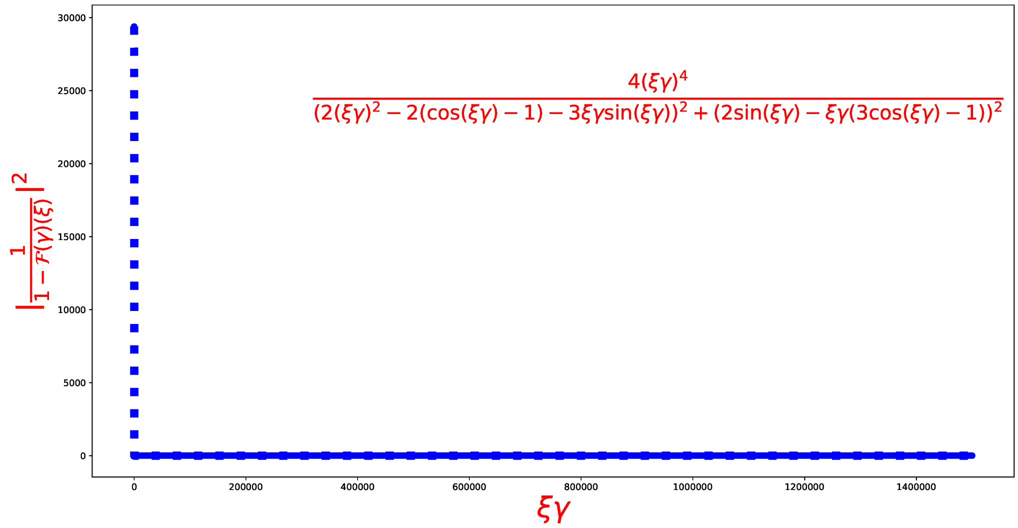

Fig. 1. Graph of the function defined in the equality (1) over the interval $[0.05, 15.10^5]$, showing that is $\mathbb{L}^\infty(]-\infty, -\varepsilon[\cup]\varepsilon, +\infty[)$.

From (22), the inverse of the modulo squared yields

$$\left| \frac{1}{1 - \mathcal{F}(\gamma)(\xi)} \right|^2 = \frac{4(\xi\Theta_\gamma)^4}{\mathcal{T}^2(\xi) + \mathcal{Y}^2(\xi)},$$

where functions $\mathcal{T}$ and $\mathcal{Y}$ are defined as follows

$$\mathcal{T}(\xi) = 2(\xi\Theta_\gamma)^2 - 2(\cos(\xi\Theta_\gamma) - 1) - 3\xi\Theta_\gamma \sin(\xi\Theta_\gamma),$$
$$\mathcal{Y}(\xi) = 2\sin(\xi\Theta_\gamma) - \xi\Theta_\gamma(3\cos(\xi\Theta_\gamma) - 1).$$

Remark 1. We will show in this remark that even the Repayment Pattern Density γ has a lower bound, we could not apply Lemma 1. Indeed, if the density γ admits a lower bound over its support, then there exists a positive real M satisfying

$$\inf_{z\in[0,\Theta_\gamma]} \{|\gamma(z)|\} = M. \tag{23}$$

We will show that under assumption (23), relation (9) is not satisfied. Otherwise, we will show

$$\left(\frac{1}{1 - \mathcal{F}(\gamma)} \right) \notin \mathbb{L}^\infty(\mathbb{R}). \tag{24}$$

By ordered compactness, we have,

$$\left| \frac{1}{1 - \mathcal{F}(\gamma)(\xi)} \right|^2 = \frac{1}{\left| \int_0^{\Theta_\gamma} \gamma(x)(1 - e^{-ix\xi}) \, dx \right|^2}$$

$$\leqslant \frac{1}{M^2} \times \frac{1}{\left|\int_0^{\Theta_\gamma}(1-\mathrm{e}^{-ix\xi})\,dx\right|^2}.$$

The integration of the exponential function $x \to \mathrm{e}^{-ix\xi}$ states that

$$\int_0^{\Theta_\gamma}(1-\mathrm{e}^{-ix\xi})\,dx = \Theta_\gamma + \frac{(\mathrm{e}^{-i\Theta_\gamma\xi}-1)}{i\xi}.$$

We take the square root of the modulus

$$\left|\int_0^{\Theta_\gamma}(1-\mathrm{e}^{-ix\xi})\,dx\right|^2 = \Theta_\gamma^2 + \frac{2(1-\cos(\Theta_\gamma\xi))}{\xi^2} - \frac{2\Theta_\gamma\sin(\Theta_\gamma\xi)}{\xi}.$$

Making out that this function

$$\xi \to \frac{1}{\Theta_\gamma^2 + \frac{2(1-\cos(\Theta_\gamma\xi))}{\xi^2} - \frac{2\Theta_\gamma\sin(\Theta_\gamma\xi)}{\xi}}$$

is pair. For that we will show that is $\notin \mathbb{L}^\infty(\mathbb{R}^+)$ in order to prove (24). The functions $\sin(\Theta_\gamma\xi)$ and $\cos(\Theta_\gamma\xi)$ are Taylor expanded in zero respectively up to the third-order to obtain following inequalities

$$\sin(\Theta_\gamma\xi) \leqslant \Theta_\gamma\xi - \frac{(\Theta_\gamma\xi)^3}{6}, \quad \cos(\Theta_\gamma\xi) \leqslant 1 - \frac{(\Theta_\gamma\xi)^2}{2}.$$

From this, we get for all positive real numbers ξ

$$\frac{1}{\Theta_\gamma^2 + \frac{2(1-\cos(\Theta_\gamma\xi))}{\xi^2} - \frac{2\Theta_\gamma\sin(\Theta_\gamma\xi)}{\xi}} \leqslant \frac{3}{\Theta_\gamma^4\xi^2}.$$

Since the function $\xi \to \frac{3}{\Theta_\gamma^4\xi^2}$ is not bounded, then it is not in $\mathbb{L}^\infty(\mathbb{R}^+)$, completing the proof of (24).

To enrich the model, a new idea consists in imposing a novel assumption on the relationship of γ, which is usable in this form. It allows to keep the same Loan Density κ_E in term of the Algebraic Spending Density σ. Following this idea leads to obtain the uniqueness of κ_E given in Theorem 2.

Theorem 2. *Assuming that the density γ satisfies* (1), *and the following expression*

$$\left(\frac{1}{1-\mathcal{F}(\gamma)}\right) \in \mathbb{L}^2(]-\infty,-\varepsilon[\cup]\varepsilon,+\infty[), \tag{25}$$

for any positive real ε such that (10) *holds for the operator $\mathcal{L}$. Then, the Loan Density κ_E can be expressed in terms of σ as*

$$\kappa_E = \mathcal{F}^{-1}\left(\frac{\mathcal{F}\big(\sigma - \mathcal{D}[\rho_\mathcal{K}^\mathrm{I}] + \alpha\int_{t_\mathrm{I}}^{\bullet}(\sigma(s)-\mathcal{D}[\rho_\mathcal{K}^\mathrm{I}](s))\,\mathrm{e}^{\alpha(\bullet-s)}\,ds\big)}{1-\mathcal{F}(\gamma)}\right).$$

Proof. In the first place, we will show that function $\int_{t_\mathrm{I}}^{\bullet}(\sigma(s)-\mathcal{D}[\rho_\mathcal{K}^\mathrm{I}](s))\,\mathrm{e}^{\alpha(\bullet-s)}ds$ is integrable over $\mathbb{R}$ having their support in $[t_\mathrm{I},\Theta_{\max}]$. In other words,

$$\int_{t_\mathrm{I}}^{\bullet}\big(\sigma(s)-\mathcal{D}[\rho_\mathcal{K}^\mathrm{I}](s)\big)\,\mathrm{e}^{\alpha(\bullet-s)}ds \in \mathbb{L}^1(\mathbb{R}).$$

For all reals $t \leqslant \Theta_{\max}$, we have

$$\begin{aligned}\left\|\int_{t_\mathrm{I}}^{\bullet}\mathcal{L}[\kappa_E](s)\,\mathrm{e}^{\alpha(\bullet-s)}ds\right\|_{\mathbb{L}^1([t_\mathrm{I},\Theta_{\max}])} &= \int_{t_\mathrm{I}}^{\Theta_{\max}}\left|\int_{t_\mathrm{I}}^{t}\mathcal{L}[\kappa_E](s)\,\mathrm{e}^{\alpha(t-s)}ds\right|dt \\ &\leqslant (\Theta_{\max}-t_\mathrm{I}) \times \mathrm{e}^{|\alpha|(\Theta_{\max}-s)}\|\mathcal{L}[\kappa_E]\|_{\mathbb{L}^1([t_\mathrm{I},\Theta_{\max}])}.\end{aligned}$$

Detecting that the Linear operator $\mathcal{L}$ is acting on the Loan Density $\kappa_E \in \mathbb{L}^1([t_\mathrm{I},\Theta_{\max}-\Theta_\gamma]) \cap \mathcal{C}_c([t_\mathrm{I},\Theta_{\max}-\Theta_\gamma])$ which is defined on $\mathbb{L}^1([t_\mathrm{I},\Theta_{\max}-\Theta_\gamma])$. From this the Fourier transform of function in $\mathbb{L}^1([t_\mathrm{I},\Theta_{\max}-\Theta_\gamma])$ is in $\mathbb{L}^\infty(\mathbb{R})$. Formally, we get

$$\mathcal{F}\left(\sigma - \mathcal{D}[\rho_\mathcal{K}^\mathrm{I}] + \alpha\int_{t_\mathrm{I}}^{\bullet}\big(\sigma(s)-\mathcal{D}[\rho_\mathcal{K}^\mathrm{I}](s)\big)\mathrm{e}^{\alpha(\bullet-s)}ds\right) \in \mathbb{L}^\infty(\mathbb{R}). \tag{26}$$

Under assumption (25), since the product of function

$$\mathcal{F}\bigg(\sigma - \mathcal{D}[\rho_{\mathcal{K}}^{\mathrm{I}}] + \alpha \int_{t_{\mathrm{I}}}^{\bullet} \big(\sigma(s) - \mathcal{D}[\rho_{\mathcal{K}}^{\mathrm{I}}](s)\big) \mathrm{e}^{\alpha(\bullet - s)} ds\bigg)$$

in $\mathbb{L}^{\infty}(\mathbb{R})$ by function $\frac{1}{1-\mathcal{F}(\gamma)}$ which is in $\mathbb{L}^2(\mathbb{R})$, is in space $\mathbb{L}^2(\mathbb{R})$. Hence, the proof of the lemma is achieved. ■

Remark 2. The aim of this remark is to check Lemmas 1 and 2 for a constant piecewise density γ, which is equal to $\frac{1}{\Theta_\gamma}$ over $[0, \Theta_\gamma]$ and is equal to zero outside, i.e.

$$\gamma = \frac{1}{\Theta_\gamma} \mathbb{1}_{[0,\Theta_\gamma]}. \tag{27}$$

The Repayment Pattern Density γ given by (27) satisfies (2), because it is a non-negative density with total mass which equals 1. The aim here is to show that this density γ does not satisfy assumption (9). Since the modulus of $\frac{1}{1-\mathcal{F}(\gamma)}$ is computed as

$$\left|\frac{1}{1-\mathcal{F}(\gamma)(\xi)}\right| = \frac{|\xi\Theta_\gamma|}{\sqrt{(\xi\Theta_\gamma - \sin(\xi\Theta_\gamma))^2 + (\cos(\xi\Theta_\gamma) - 1)^2}},$$

then, when $\xi \to 0$, this modulus is equivalent to

$$\frac{1}{\frac{(\xi\Theta_\gamma)^2}{4}\left(1 + \frac{(\xi\Theta_\gamma)^4}{9}\right)}.$$

Consequently, $|\frac{1}{1-\mathcal{F}(\gamma)(\xi)}|$ diverges around origin, proving that

$$\left(\frac{1}{1-\mathcal{F}(\gamma)}\right) \notin \mathbb{L}^{\infty}(\mathbb{R}).$$

Taking now the same Repayment Pattern Density γ defined by equality (27). This density γ does not satisfy assumption (9). Indeed, around infinity, we get the following equivalence:

$$\left|\frac{1}{1-\mathcal{F}(\gamma)(\xi)}\right|^2 = \frac{(\xi\Theta_\gamma)^2}{(\xi\Theta_\gamma - \sin(\xi\Theta_\gamma))^2 + (\cos(\xi\Theta_\gamma) - 1)^2} \simeq 1,$$

which is not integrable between a positive number A and infinity. Consequently, we get

$$\left(\frac{1}{1-\mathcal{F}(\gamma)}\right) \notin \mathbb{L}^2(\mathbb{R}).$$

3. Setting out a financial strategy

The aim of this section is to show how continuous-in-time model is used for our financial strategy. Any organization and, in particular, local communities needs to establish a budget project for its own strategy. It consists in predicting all consequences for future finance, which is followed by establishing a new one if and only if the first one does not give satisfaction. Forecasting this strategy allows the organization to make the best decisions in order to elaborate the adapted one.

Here, we give a concrete example in order to illustrate this strategy. Running the expected model shows promising results, meaning of correspondence with respect to realistic values. This example is presented in Figure 2, shared into seven diagrams. The first one is the Repayment Pattern Density γ expresses the way a unit amount borrowed at the initial time is repaid. The second one is Loan Density κ_E. It means how public institution defines loan scheme, stated by their financial plan, taking into account all the process needs. Next, the third one shows the Repayment Density $\rho_{\mathcal{K}}$. It translates the result of the action of γ on κ_E via the convolution operator (see equality (3)). Assuming here that this Repayment Density is not satisfactory for elaborating a given financial strategy. Then, the organization chooses a new suitable targeted Repayment shown in the fourth diagram. In what follows, the Fourier Transform Operator is involved in the computation. Indeed, the direct and inverse ones are used to

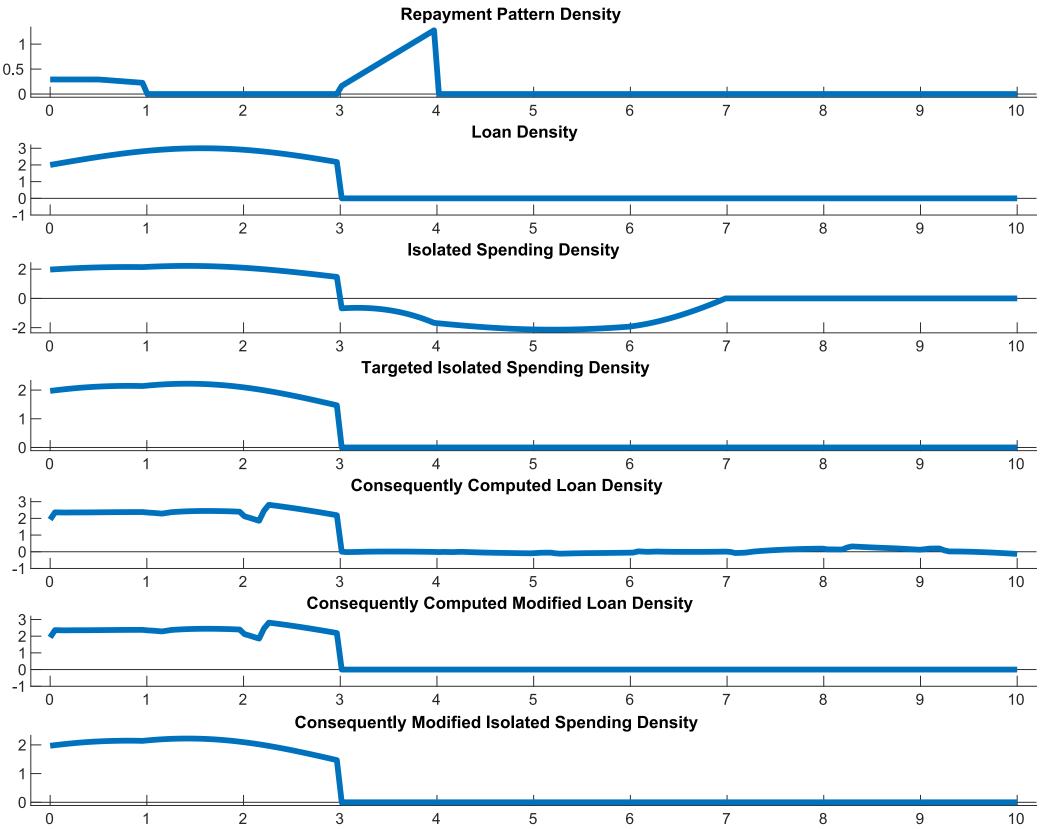

Fig. 2. Financial strategy elaboration controlling constrains and goals.

calculate Loan Density in the fifth diagram. Next, this obtained Loan Density is adjusted in order for decision-makers to be efficiently arranged. This modified Loan Density responds to encountered problems and is generated in the sixth diagram. From this Loan, Repayment Density is computed and stated in the last diagram.

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Деякі зауваження щодо оберненої задачі фінансової моделі з неперервним часом у $L^1([t_{\text{I}}, \Theta_{\max}])$

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У статті ми збираємося ввести оператор, який бере участь в оберненій задачі фінансової моделі з неперервним часом. Ця структура призначена для використання у фінансах для будь-якої організації та, зокрема, для місцевих громад. Це дозволяє складати річні та багаторічні бюджети з описом схем позики, відшкодування та виплати відсотків. Обговорюємо цю обернену задачу в просторі інтегровних функцій над $\mathbb{R}$ з компактним носієм. У цьому просторі розглядається концепція некоректності, щоб отримати цікаві та корисні розв'язки. Потім даємо деякі зауваження щодо нефункціональності моделі для заданої щільності схеми погашення γ, коли цей оператор не є оборотним у просторі. Крім того, ця обернена задача проілюстрована, щоб довести її здатність використовуватися у фінансовій стратегії.

Ключові слова: *обернена задача; інтегральні оператори; фінансова модель; щільності та міри.*