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Modeling and mathematical analysis of drug addiction with the study of the effect of psychological and biological treatment

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In this article, we propose a discrete mathematical model which describes the propagation of the drug phenomenon in a human population. The population is unscrewed in five compartments: "S" People likely to become drug addicts, "M" Moderate drug addicts, "H" Heavy drug addicts, "T" People receiving drug addiction treatment, "R" The recovered people who have completely abstained from drug addiction. Our goal is to find a better strategy to reduce the number of heavy addicts and to maximize the number of people receiving full treatment. The tools of optimal control theory were used in this study, in particular the Pontryagin maximum principle.

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1. Introduction

Addiction is a compulsive behavior that makes the addict lives at the mercy of the subject of his her addiction. Drug addiction is considered a disease due to the compulsive habituation to taking narcotic substances and the behavioral disorders, besides the affection of the psychological and social life of the addict.

Drug addiction and its negative repercussions and effects on the addict and society alike push us to search for the causes of this dangerous scourge and ways to prevent and treat it, then the extent of the effectiveness of the role of associations and social institutions in helping drug addicts free themselves from their addiction and facilitate their integration into society, as well as the preventive measures of governments by virtue of their possession of the various media, and what they have of the control devices and the laws they enact.

The incidence of addiction varies from one person to another according to the circumstances surrounding that slip, which is often gradual towards addiction to a narcotic or intoxicating substance. Getting that pleasure and that feeling of happiness as the drug abuser knows it pushes him her to search for it again whenever the drugs effect ends. Then the search for that illusion of happiness converges. Then it happens that the person discovers his her complete immersion and the impossibility of life for him without that drug. As a result he fells into the dilemma of addiction, and here begins the suffering that varies in severity and violence according to the strength of the narcotic substance that the addict is taking.

Addiction in our time is not linked to a specific age. Among the addicts are children and young adults, middle-aged and old people, and it does not distinguish between male or female. It has become accessible to all types of drugs that may be among the most destructive to humans, such as heroin and cocaine, opium and other chemically compounded drugs, including hallucinogenic pills, after a period of time, the addict becomes a ghost of a mindless body.

A society that argues what happens in his heedlessness finds itself in every case of addiction that pays a heavy price, from within that same society those who only care about the profit they make from promoting those narcotic poisons, those who have drunk in their souls crime do not hesitate to planting the seeds of addiction in the doors of schools. They promoted it and made it a piece of candy.

We may be mistaken if we say that fragility or poverty is the main major of becoming a drug abuses. Here we have reduced the entire problem to poverty, knowing that there are addicts who have a financially affordable situation, but we must not deny that economic problems among the motives for this scourge, while we find other motives related to psychological fragility, and there are those who resort to drugs just out of curiosity to discover something new, but when addiction occurs we are now facing compulsive behavior as mentioned previously. Therefore, we have reached a satisfactory condition that requires the search for treatment.

The drug addict eventually suffers from complications at the physical, psychological, social and relational level, as his her academic level and professional performance decline, and he starts a state of depression and isolation, and begins to search for that dose that restores him her a kind of satisfactory balance.

On the other hand, we find that the ease of obtaining this narcotic substance is a direct reason for the rise in the number of addicts, which is noticed strongly in front of schools, as drugs are distributed inside or outside the school walls at cheap prices or sometimes for free, given that the school is a fertile place for the spread of this infection, and bringing the largest number of victims, and here comes the role of awareness and upbringing of the family and educational institutions as a proactive step and a preventive way to combat this scourge, and this is reflected in the development of health educational programs within educational institutions and recreational artistic programs such as theater, music and activities, primarily sports.

From another perspectives, we find the main role played by associations and social institutions in helping these addicts free themselves from this drug, by going to the place where drugs are located and listening to addiction victims, encouraging them to change their behavior and then directing them to health centers after they were psychologically and behaviorally prepared to start receiving treatment sessions by an integrated medical staff consisting of a doctor, a psychologist and a social worker, but in some cases we notice that there are addicts who are more prepared than others, meaning that there are disparities at the level of psychological readiness and ability to complete treatment Some of them cannot complete the therapeutic classes to till the end, that is, until they are completely healthy, which means the end of their addiction, so it is necessary to support and accompany psychologically these patients so that they can get rid of their addiction.

2. Background

In this paper, we propose an epidemiological approach (for example see [1–8] to describe and study the propagation of the drug phenomenon in a human population.

In epidemiology, we generally use compartment model to describe the spread of an infectious disease. In these epidemiological models, the population is divided into different classes according to people's status versus the disease (susceptible to catch the disease, infected, or removed) and the infection process depends on the contact with infectious individuals. Using this approach, many researches have focused on the topic of drug phenomenon and other related topics [7, 9–17]. For example in [18], the authors tried to find an effective strategy to reduce the number of light drug users, heavy drug users, heavy drug users-dealers and providers, and temporary quitters of drug consumption, they used four control strategies which are awareness programs through media and education, preventing contact through security campaigns, treatment, and psychological support along with a follow-up.

In [19], the authors modeled the interaction between the classes of drinkers, namely, potential drinkers P, moderate drinkers M, heavy drinkers H, poor heavy drinkers Tp, rich heavy drinkers Tr, and quitters of drinking Q. They looked at the importance of treatment within addiction treatment centers aiming to find the optimal strategies to minimize the number of drinkers and maximize the number of heavy drinkers who join addiction treatment centers.

In this paper, our work is a little different because we will try to reduce the number of heavy drugs addicts by raising awareness as a proactive step and encouraging the largest possible number of addiction victims to join treatment centers, in addict to that, as mentioned in the presentation, there are those who cannot complete treatment, we will see in this work the importance of psychological support and accompaniment of this category until they recover permanently. The aim is to find an optimal strategy that will reduce the number of victims of drug addiction, and at the same time sensitize people of the seriousness of this dangerous scourge.

The mathematical processing employed calls upon the tools of the theory of optimal control and more precisely the Pontryagin's maximum principle [20–30].

3. Model formulation

The model SMHTR that we propose below makes it possible to describe the dynamics of population and analyze interactions between classes of addicts.

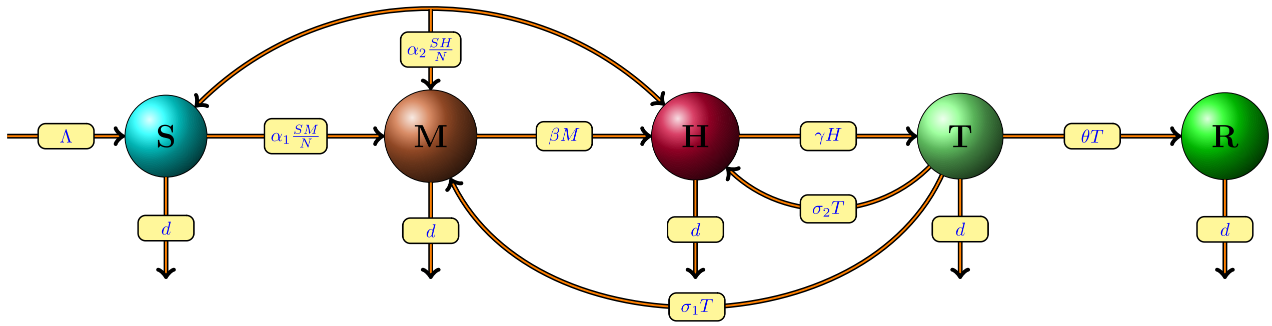

Fig. 1. Diagram of the evolution of drug addiction in a population.

The different compartments of our model are described below:

Compartment S represents the number of People likely to become drug addicts. It is increased by adding a number Λ each day (the time unit used in this study) and decreases by an effective contact with moderate drug addicts and others heavy drug addicts respectively with reports $\alpha_1\frac{SM}{N}$ and $\alpha_2\frac{SH}{N}$ and also it is decreased by a natural mortality rate d.

Compartment M represents the number of moderate addicts who are able to control their drug consumption during some events and occasions, but they inevitably develop the situation and become adult addicts. It increased by potential addicts who turn to be moderate addicts respectively with reports $\alpha_1\frac{SM}{N}$ and $\alpha_2\frac{SH}{N}$ and also by addicts who could not complete treatment at the rate σ_1. It is decreased when moderate addicts become heavy addicts at a rate β and also by natural death at rate d.

Compartment H represents the number of heavy drug addicts those who face a great difficulty to control their symptoms of drug. It increases by the rate β of moderate addicts and also by addicts who could not complete treatment at the rate σ_2. The heavy drug addict will usually need to get help at a rehab to overcome their addiction, the comportment of those individuals decreases by the rate of hospitalized people γ and by the mortality rate d.

Compartment T represents the number of hospitalized People receiving the drug addiction treatment. It increases by the rate γ of heavy drug addicts and decreases by addicts who could not complete treatment and then returned to moderate and heavy addicts respectively with rates σ_1 and σ_2 and also by the rate θ of recovered persons as well as by a mortality rate of d.

Compartment R represents the number of the recovered people. It increases by the rates θ of hospitalized people who have finished the treatment till the end and decreases by the rate of natural mortality d.

The previous diagram is mathematically translated by the following nonlinear system:

$$\begin{cases} S_{k+1} = \Lambda - \alpha_1 \dfrac{S_k M_k}{N_k} - \alpha_2 \dfrac{S_k H_k}{N_k} + (1-d) S_k, \\ M_{k+1} = \alpha_1 \dfrac{S_k M_k}{N_k} + \alpha_2 \dfrac{S_k H_k}{N_k} + \sigma_1 T_k + (1-\beta-d) M_k, \\ H_{k+1} = \beta M_k + \sigma_2 T_k + (1-\gamma-d) H_k, \\ T_{k+1} = \gamma H_k + (1-\sigma_1-\sigma_2-\theta-d) T_k, \\ R_{k+1} = \theta T_k + (1-d) R_k. \end{cases} \tag{1}$$

Where $S_0 \geqslant 0$, $M_0 \geqslant 0$, $H_0 \geqslant 0$, $T_0 \geqslant 0$, and $R_0 \geqslant 0$. The total companies size at time k is denoted by N_k with $N_k = S_k + M_k + H_k + T_k + R_k$, and it is supposed to be constant.

4. The optimal control problem

In this section, we aim to introduce three controls that enable us to minimize the number of heavy drug addicts, and maximize the number of people who have finished the treatment till the end and who are out of drug addiction. The first control u actually represents awareness campaigns on media and education or through direct contact with people on the street. As for the second control v represents the effort to encourage the heavy drug addicts to join addiction treatment centers. The third control w represent the psychological support for heavy drug addicts in order to complete the treatment classes till the end. So, the controlled mathematical system is given by the following system of difference equations:

$$\begin{cases} S_{k+1} = \Lambda - \alpha_1 (1-u_k) \dfrac{S_k M_k}{N_k} - \alpha_2 (1-u_k) \dfrac{S_k H_k}{N_k} + (1-d) S_k, \\ M_{k+1} = \alpha_1 (1-u_k) \dfrac{S_k M_k}{N_k} + \alpha_2 (1-u_k) \dfrac{S_k H_k}{N_k} + \sigma_1 (1-w_k) T_k + (1-\beta-d) M_k, \\ H_{k+1} = \beta M_k + \sigma_2 (1-w_k) T_k + (1-\gamma-d-v_k) H_k, \\ T_{k+1} = (\gamma+v_k) H_k + (1+w_k-\sigma_1-\sigma_2-\theta-d) T_k, \\ R_{k+1} = \theta T_k + (1-d) R_k. \end{cases} \tag{2}$$

Where $S_0 \geqslant 0$, $M_0 \geqslant 0$, $H_0 \geqslant 0$, $T_0 \geqslant 0$, and $R_0 \geqslant 0$.

Then, the problem is to minimize the objective functional is given by

$$J(u,v) = M_T + H_T - T_T + \sum_{k=0}^{T-1} \left(M_k + H_k - T_k + \frac{A_k u_k^2}{2} + \frac{B_k v_k^2}{2} + \frac{C_k w_k^2}{2} \right). \tag{3}$$

Where the parameters $A_k > 0$, $B_k > 0$, and $C_k > 0$ are selected to weigh, respectively, the relative importance of the cost of awareness campaigns and preventive measures to reduce social contact.

The aim is to find an optimal control, (u^*, v^*, w^*) such that

$$J(u^*, v^*, w^*) = \min_{(u,v,w) \in U_{ad}^3} J(u,v,w),$$

where U_{ad} is the set of admissible controls defined by

$$U_{ad} = \big\{ X = (X_0, X_1, \ldots, X_{T-1}) /\ 0 \leqslant X_{\min} \leqslant X_k \leqslant X_{\max} \leqslant 1;\ \text{for} \\ X \in \{u, v, w\} \text{ and } k = 0, 1, \ldots, T-1. \big\}$$

The sufficient condition of the existence of an optimal control (u^*, v^*, w^*) for problem (2) and (3) comes from the following theorem.

Theorem 1. *There exists the optimal control (u^*, v^*, w^*) such that*

$$J(u^*, v^*, w^*) = \min_{(u,v,w) \in U_{ad}^3} J(u,v,w),$$

subject to the control system (2) with initial conditions.

Proof. Since the coefficients of the state equations are bounded and there are a finite number of time steps, $S = (S_0, S_1, S_2, \ldots, S_T)$, $M = (M_0, M_1, M_2, \ldots, M_T)$, $H = (H_0, H_1, H_2, \ldots, H_T)$, $T =$

$(T_0, T_1, T_2, \dots, T_T)$, $R = (R_0, R_1, R_2, \dots, R_T)$ are uniformly bounded for all (u, v, w) in the control set U_{ad} thus $J(u, v, w)$ is bounded for all $(u, v, w) \in U_{ad}^3$. Since $J(u, v, w)$ is bounded, $\inf_{(u,v,w)\in U_{ad}^3} J(u, v, w)$ is finite, and there exists a sequence $(u^j, v^j, w^j) \in U_{ad}^3$ such that $\lim_{j\to+\infty}(u^j, v^j, w^j) = \inf_{(u,v,w)\in U_{ad}^3} J(u, v, w)$ and corresponding sequences of states S^j, M^j, H^j, T^j, and R^j.

Since there is a finite number of uniformly bounded sequences, there exist (u^*, v^*, w^*) and S^*, M^*, H^*, T^*, and $R^* \in \mathbb{R}^{T-1}$ such that, on a subsequence, $\lim_{j\to+\infty}(u^j, v^j, w^j) = (u^*, v^*, w^*)$, $\lim_{j\to+\infty} S^j = S^*$, $\lim_{j\to+\infty} M^j = M^*$, $\lim_{j\to+\infty} H^j = H^*$, $\lim_{j\to+\infty} T^j = T^*$, and $\lim_{j\to+\infty} R^j = R^*$.

Finally, due to the finite dimensional structure of system (2) and the objective function $J(u, v, w)$, (u^*, v^*, w^*) is an optimal control with corresponding states S^*, M^*, H^*, T^*, and R^*. Therefore, $\inf_{(u,v,w)\in U_{ad}^3} J(u, v, w)$ is achieved. ∎

5. Characterization of the optimal control

To characterize the optimal controls, we apply the discrete version of Pontryagin's maximum principle [20–30].

The key idea is introducing the adjoint function to attach the system of difference equations to the objective functional resulting in the formation of a function called the Hamiltonian.

This principle converts the problem of finding the control to optimize the objective functional subject to the state of difference equation with initial condition into finding the control to optimize Hamiltonian pointwise (with respect to the control).

Now, we have the Hamiltonian $\widehat{H}$ at time step k, defined by

$$\widehat{H}_k = M_k + H_k - T_k + \frac{A_k u_k^2}{2} + \frac{B_k v_k^2}{2} + \frac{C_k w_k^2}{2} + \sum_{i=1}^{5} \lambda_{i,k+1} f_{i,k+1},$$

where $f_{i,k+1}$ is the right side of the system of difference equation (2) of the i^{th} state variable at time step $k+1$.

Theorem 2. *Given an optimal control $(u^*, v^*, w^*) \in U_{ad}^3$ and solutions S_k^*, M_k^*, H_k^*, T_k^*, and R_k^* of corresponding state system there exist adjoint functions λ_1, λ_2, λ_3, λ_4, and λ_5 satisfying the following equations:*

$$\begin{aligned}
\lambda_{1,k} &= \frac{\partial \widehat{H}_k}{\partial S_k} = (\lambda_{2,k+1} - \lambda_{1,k+1})(1 - u_k)\left(\alpha_1 \frac{M_k}{N_k} + \alpha_2 \frac{H_k}{N_k}\right) + \lambda_{1,k+1}(1 - d), \\
\lambda_{2,k} &= \frac{\partial \widehat{H}_k}{\partial M_k} = 1 + (\lambda_{2,k+1} - \lambda_{1,k+1})(1 - u_k)\alpha_1 \frac{S_k}{N_k} + (\lambda_{3,k+1} - \lambda_{2,k+1})\beta + \lambda_{2,k+1}(1 - d), \\
\lambda_{3,k} &= \frac{\partial \widehat{H}_k}{\partial H_k} = 1 + (\lambda_{2,k+1} - \lambda_{1,k+1})(1 - u_k)\alpha_2 \frac{S_k}{N_k} + (\gamma + v_k)(\lambda_{4,k+1} - \lambda_{3,k+1}) + \lambda_{3,k+1}(1 - d), \\
\lambda_{4,k} &= \frac{\partial \widehat{H}_k}{\partial T_k} = \sigma_1(1 - w_k)\lambda_{2,k+1} + \sigma_2(1 - w_k)\lambda_{3,k+1} + \lambda_{4,k+1}(1 + w_k - \sigma_1 - \sigma_2 - d) + \theta_k \lambda_{5,k+1}, \\
\lambda_{5,k} &= \frac{\partial \widehat{H}_k}{\partial R_k} = \lambda_{6,k+1}(1 - d)
\end{aligned}$$

with the transversal conditions at time T:

$$\begin{aligned}
\lambda_1(T) &= 0, \\
\lambda_2(T) &= 1, \\
\lambda_3(T) &= 1, \\
\lambda_4(T) &= -1, \\
\lambda_5(T) &= 0.
\end{aligned}$$

Furthermore, for $k = 0, 1, \ldots, T-1$, we obtain the optimal control (u^*, v^*, w^*) as

$$\begin{cases} u_k^* = \min\left\{\max\left\{\frac{(\lambda_{2,k+1}-\lambda_{1,k+1})\left(\alpha_1\frac{S_kM_k}{N_k}+\alpha_2\frac{S_kH_k}{N_k}\right)}{A_k}, u_{\min}\right\}, u_{\max}\right\}, \\ v_k^* = \min\left\{\max\left\{\frac{H_k(\lambda_{3,k+1}-\lambda_{4,k+1})}{B_k}, v_{\min}\right\}, v_{\max}\right\}, \\ w_k^* = \min\left\{\max\left\{\frac{T_k(\sigma_1\lambda_{2,k+1}+\sigma_2\lambda_{3,k+1}-\lambda_{4,k+1})}{C_k}, w_{\min}\right\}, w_{\max}\right\}. \end{cases} \tag{4}$$

Proof. The Hamiltonian $\widehat{H}_k$ at time step k is given by

$$\begin{aligned} \widehat{H}_k &= H_k - T_k + \frac{A_k u_k^2}{2} + \frac{B_k v_k^2}{2} + \frac{C_k w_k^2}{2} \\ &+ \lambda_{1,k+1}\left[\Lambda - \alpha_{1,k}(1-u_k)\frac{S_kM_k}{N_k} - \alpha_{2,k}(1-u_k)\frac{S_kH_k}{N_k} + (1-d)S_k\right] \\ &+ \lambda_{2,k+1}\left[\alpha_{1,k}(1-u_k)\frac{S_kM_k}{N_k} + \alpha_{2,k}(1-u_k)\frac{S_kH_k}{N_k} + \sigma_1(1-w_k)T_k + (1-\beta-d)M_k\right] \\ &+ \lambda_{3,k+1}\left[\beta M_k + \sigma_2(1-w_k)T_k + (1-\gamma-d-v_k)H_k\right] \\ &+ \lambda_{4,k+1}\left[(\gamma+v_k)H_k + (1+w_k-\sigma_1-\sigma_2-\theta-d)T_k\right] + \lambda_{5,k+1}\left[\theta T_k + (1-d)R_k\right]. \end{aligned}$$

For $k = 0, 1, \ldots, T-1$, the adjoint equations and transversal conditions can be obtained by using Pontryagin's maximum principle, in discrete time, given in [20–30]. Such that

$$\begin{aligned} \lambda_{1,k} &= \frac{\partial \widehat{H}_k}{\partial S_k}, \quad \lambda_1(T) = 0, \\ \lambda_{2,k} &= \frac{\partial \widehat{H}_k}{\partial M_k}, \quad \lambda_2(T) = 1, \\ \lambda_{3,k} &= \frac{\partial \widehat{H}_k}{\partial H_k}, \quad \lambda_3(T) = 1, \\ \lambda_{4,k} &= \frac{\partial \widehat{H}_k}{\partial T_k}, \quad \lambda_4(T) = -1, \\ \lambda_{5,k} &= \frac{\partial \widehat{H}_k}{\partial R_k}, \quad \lambda_5(T) = 0. \end{aligned}$$

For $k = 0, 1, \ldots, T-1$, the optimal control (u^*, v^*, w^*) can be solved from the optimality condition

$$\begin{aligned} \frac{\partial \widehat{H}_k}{\partial u_k} &= A_k u_k + (\lambda_{1,k+1} - \lambda_{2,k+1})\left(\alpha_1\frac{S_kM_k}{N_k} + \alpha_2\frac{S_kH_k}{N_k}\right) = 0, \\ \frac{\partial \widehat{H}_k}{\partial v_k} &= B_k v_k + H_k(\lambda_{4,k+1} - \lambda_{3,k+1}) = 0, \\ \frac{\partial \widehat{H}_k}{\partial w_k} &= C_k w_k + T_k(\lambda_{4,k+1} - \sigma_1\lambda_{2,k+1} - \sigma_2\lambda_{3,k+1}) = 0. \end{aligned}$$

So, we have

$$\begin{aligned} u_k &= \frac{(\lambda_{2,k+1} - \lambda_{1,k+1})\left(\alpha_1\frac{S_kM_k}{N_k} + \alpha_2\frac{S_kH_k}{N_k}\right)}{A_k}, \\ v_k &= \frac{H_k(\lambda_{3,k+1} - \lambda_{4,k+1})}{B_k}, \\ w_k &= \frac{T_k\left(\sigma_1\lambda_{2,k+1} + \sigma_2\lambda_{3,k+1} - \lambda_{4,k+1}\right)}{C_k}. \end{aligned}$$

By the bounds in U_{ad} of the control, it is easy to obtain (u_k^*, v_k^*, w_k^*) in the form of (4). ■

6. Numerical simulation and presentation of results

We carried out a numerical simulation to test the reliability of our mathematical model with and without controls. The results will be presented in the following.

We start by presenting the results with the different controls by adopting several strategies that will be explained in details (see Table 1).

Table 1. Parameters and initial conditions for the model with and without control.

Parameters	Values
$(S_0, M_0, H_0, T_0, R_0, \Lambda)$	$(27000, 6000, 3000, 1800, 600, 2496)$
$(\alpha_1, \alpha_2, \beta, \gamma, \sigma_1, \sigma_2, \theta, d)$	$(0.52, 0.48, 0.6, 0.25, 0.5, 0.5, 0.15, 0.065)$

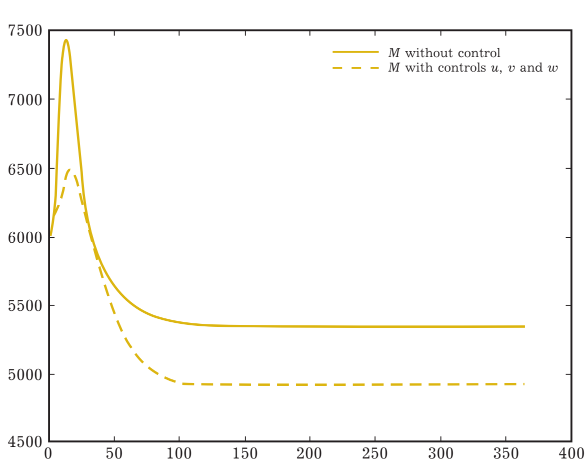

Fig. 2. Model results of moderate drug addicts.

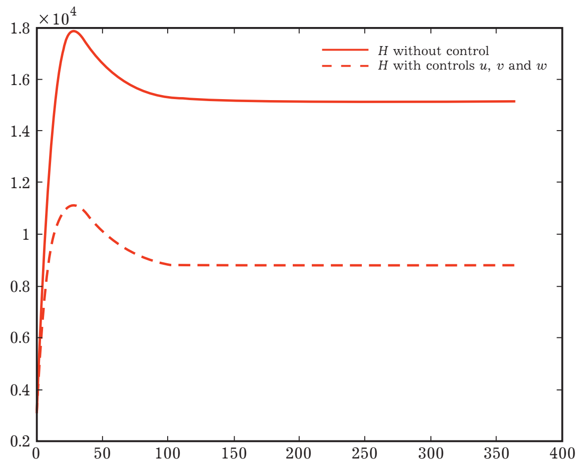

Fig. 3. Model results of heavy drug addicts.

We noticed a fall in the number of moderate drug addicts after applying the controls. This result was to be expected, especially given the deployment of public awareness campaigns and the promotion of comprehensive medical treatment.

Awareness and education initiatives, as well as encouraging addicts to join hospital for treatment and psychological support efforts for patients resulted in a significant decrease in the number of addicts, which was expected.

We see a rise in the number of persons undergoing treatment sessions who are determined and motivated to finish them, thanks to raising awareness of the dangers of drug addiction, encouraging addicts to join the hospital, and mentally and morally supporting them.

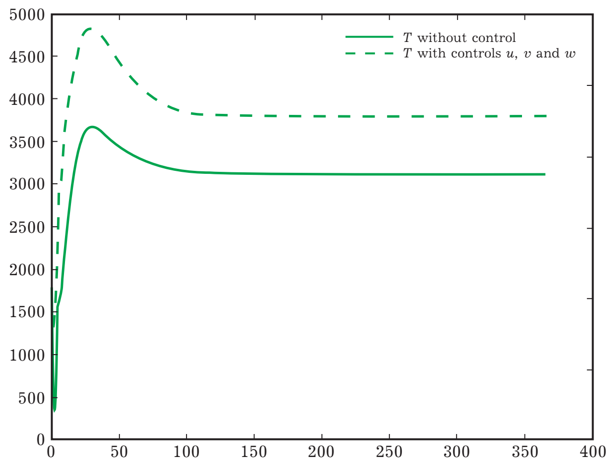

Fig. 4. Model results of people who have finished the treatment till the end and who are out of drug addiction.

7. Conclusion

The global drug problem continues to be a source of concern. According to the 2019 World Drug Report, the global population of drug users has increased by 22% in the last ten years. Last year, around 275 million people took drugs, up from 226 million in 2010. Healthcare systems are also facing an increasingly difficult challenge, with 36 millions individuals suffering from drug use disorders in 2019, up from 27 million in 2010.

All these numbers indicate that the world needs an effective strategy to combat this dangerous scourge. According to our study, we are convinced that this strategy must include three basic controls:

- Using media and educational activities, as well as direct contact with people on the street to promote awareness of the pest's significance.
- Persuading heavy drug users to join addiction treatment centers.
- Providing psychological assistance to heavy drug users in order for them to finish treatment rationing programs.

This is the strategy that we have came up with in our article and we were able to see its positive results.

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Моделювання та математичний аналіз наркоманії з вивченням ефекту психологічного та біологічного лікування

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У цій статті пропонується дискретна математична модель, яка описує поширення явища наркоманії серед людської популяції. Населення розділене на п'ять сегментів: "S" — люди, які можуть стати наркоманами, "M" — помірні наркомани, "H" — важкі наркомани, "T" — люди, які проходять лікування від наркозалежності, "R" — люди, які одужали та повністю утрималися від наркотичної залежності. Мета статті полягає в тому, щоб знайти кращу стратегію, щоб зменшити кількість важких наркозалежних і максимально збільшити кількість людей, які отримують повне лікування. У статті використовувався інструментарій теорії оптимального керування, зокрема принцип максимуму Понтрягіна.

Ключові слова: *оптимальне керування; математична модель; дискретна модель; наркотична залежність.*