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APPLICATION OF DIFFERENTIAL-TAYLOR TRANSFORMATION FOR MODELING PROCESSES IN RESONANCE POWER SOURCES

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To model electrical processes in resonance-type arc welding sources, it was proposed to use differential-Taylor transformation, which essentially simplifies computational procedures for analysis of the modes and determination of the main parameters of the secondary circuit. The essence of this method consists in conversion of the time continuous function of the original into the image function from discrete argument, the coefficients of which are called discretes. The accuracy of the derived results is determined by the number of discretes used at the stage of image analysis. 6 Ref., 1 Figure.

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It is known [1] that differential-Taylor transformation (DTT) proposed and studied by G.E. Pukhov, has become widely accepted in recent years in the problems of mathematical modeling of non-linear electric circuits, which include also arc welding sources. The essence of DTT method consists in conversion of the function of the original of some continuous argument, for instance, time, into the function of the image of discrete argument, the coefficients of which are called discretes. This way transition from differential equations of electric circuits to algebraic ones is performed, that essentially simplifies the processes of modeling and analysis of the derived results.

Here direct and inverse transformation of function $x(t)$ of continuous argument t into discrete function $X(k) = C_k$ of discrete argument $k = 0, 1, 2 – n$ is performed. The above-mentioned pair of transformations is usually presented in the form of the following expressions:

$$X(k) = \frac{H^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0} \leftrightarrow x(t) = \sum_{k=0}^{k=\infty}\left(\frac{1}{H}\right)^k X(k),$$

where direct transformation of $x(t)$ original into $X(t)$ transform is on the left, and inverse transformation of $X(k)$ into $x(t)$ is on the right.

Values of function $X(k)$ at concrete values of argument k are called discretes: ($X(0)$ is the zero discrete; $X(1)$ is the first discrete, etc.).

Using the method of analysis and synthesis of non-linear electric circuits proposed by G.E. Pukhov, we will study operating modes of LC-type sources. In the linear approximation their operation is described in sufficient detail in [2, 3]. In the proposed report the task of analysis of operation of these devices, allowing for non-linear nature of reactive and ohmic resistances, forming the secondary circuit, is posed. Figure 1, *a*, shows the simplified schematic image of the considered device, and the equivalent secondary circuit is given in Figure 1, *b*. Here the following designations are used: L_S is the scattered inductance; C is the electric capacitance of the capacitor unit; R_a is the non-linear resistance of the arc gap and voltage drop on these elements, U_L, U_C and U_a, respectively.

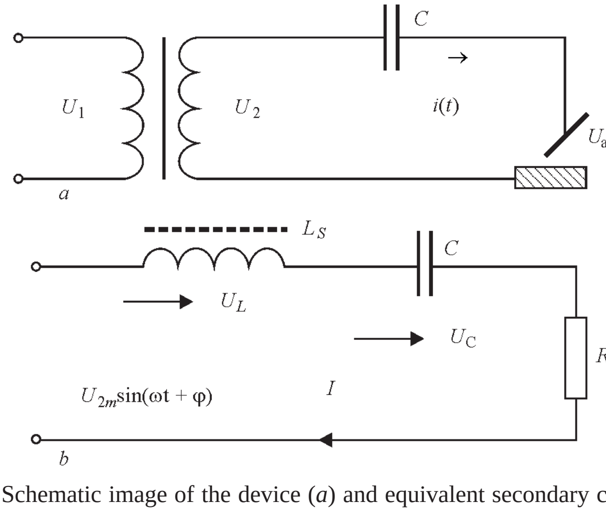
Equation, describing the state of such a circuit, as is known [4], has the following form:

$$U_2(t) = U_L + U_C + U_a, \quad (1)$$

where

$$U_L = \frac{d\psi}{dt} = L_S(I)\frac{dI}{dt};$$

$$U_C = \frac{1}{C}\left[g_0 + \int_0^t I(t)dt\right], \quad U_a = R_a(I)I(t).$$



Schematic image of the device (*a*) and equivalent secondary circuit (*b*)

If approached strictly, and if in equation (1) all the three terms are considered as non-linear relative to welding current $I(t)$, we obtain the non-linear differential equation in the following representation:

$$\frac{d^2I}{dt^2}+k_1\left(\frac{dI}{dt}\right)+k_2I=0, \qquad (2)$$

where

$$k_1=\frac{R_a}{L_S(I)},\quad k_2=\frac{1}{CL_S(I)}.$$

As is known [5], analysis of forced oscillations in such non-linear *RCL*-circuits is performed by the methods of chaotic dynamics. Depending on selection of circuit parameters, manifestations of deterministic chaos are possible here, which, however, is beyond the scope of this report.

Since, as shown by experience, the value of electric capacity C practically does not depend on current in the range of working modes, we will eliminate this non-linearity from further analysis. Therefore, allowing for the fact that initial charge is equal to zero ($g_0 = 0$) by condition, voltage on C is determined by the following expression

$$U_C=x_CI(t)=\frac{I(t)}{314C}.$$

Value of scattered inductance can be obtained experimentally, and it can be assigned in the tabular form, or approximated by a quadratic polynomial

$$L_S(I)=k_0+k_1l+k_2I^2.$$

As regards arc gap resistance, it can also be obtained experimentally for a specific design of the welding source, or from the known relationship $U = U_0 + 0.04$ (GOST 95–77), where $U_0 = 20$ V for manual arc welding. Dividing the right and left part by $I(t)$, we obtain $R_a(I) = U_0/I(t) + 0.04$.

Thus, non-linear equation of welding circuit can be reduced to the following form:

$$\frac{dI}{dt}\left(k_0+k_1I+k_2I^2\right)+I.\left(0.04+\frac{1}{314C}\right)+ +U_0=U_{2m}\sin(\omega t+\varphi) \qquad (3)$$

We will solve this non-linear differential equation by DTT method [1].

Translation of the equation allowing for the specifics of welding source operation in the region of T-image for time variable t on segment $0 \le t \le H$ gives the following DT-model [1].

$$\frac{k+1}{H}I(k+1)\left(k_0Ъ(k)+k_1I(k)+k_2\sum_{l=0}^{l=k}I(k-l)I(l)\right)+ +I(k)\left(0.04+\frac{1}{314C}\right)+U_0Ъ(k)=\frac{(\omega H)^k}{k!}U_{2m}\times \times\left(\cos\varphi\sin\frac{\pi k}{2}+\sin\varphi\cos\frac{\pi k}{2}\right),\quad k=0,1,2,...,\infty,$$

where Ъ(k) is the Taylor unit.

Knowing the initial discrete $I(0) = i(0)$, this formula can be used to successively find discretes $I(1)$, $I(2)$, … , $I(n)$, then present the solution in the form of a finite segment of a power series:

$$i(t)=\sum_{k=0}^{k=n}\left(\frac{t}{H}\right)^2I(k).$$

The accuracy of the result will depend on the number of counted discretes, on *RLC*-circuit parameters, as well as on the initial phase φ of applied voltage.

It should be noted that G.E. Pukhov table, as well as its refinement, derived in the dissertation work of E.D. Golovin [6], was used at translation of initial equations into the region of images [1].

Thus, the described method can be used at the final stage of circuit design of arc welding resonance sources, when *LC*-circuit elements, providing the requirements of the statement of work, are selected and calculated by the results of circuit analysis. The advantage of the considered method is that it enables welding equipment developer creating algebraic models of the same accuracy, as the initial models-originals.

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