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MATHEMATICAL MODELLING OF DISTORTIONS AT WELDING OF LARGE VESSELS OF ALUMINIUM ALLOY

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ABSTRACT

The problem of calculation prediction of the overall distortions of a large vessel made of aluminium alloy during friction stir welding (FSW) is considered. A mathematical model was developed using numerical methods of thermoplasticity analysis for determining the stress-strain state during FSW, by means of which it is possible to obtain residual plastic strains (the inherent strain function parameters) for both types of welded vessel joints (longitudinal and circumferential). This makes it possible to predict the overall distortions of a large cylindrical vessel with a great number of welded joints by the approximated method of inherent strains within the limits of the theory of elasticity. The reliability of the mathematical model for determination of the residual stresses and strains at FSW of aluminium alloy is confirmed by the agreement of the calculated distribution of the residual longitudinal stresses with the data of experimental measurements. This can contribute to ensuring the necessary accuracy of predicting the overall distortions of large vessels. The developed mathematical models and calculation algorithms can be effectively used for in-process prediction of the stress-strain state during assembly welding of large cylindrical vessels made of aluminium alloys.

KEYWORDS: welded vessels, aluminium alloy, friction stir welding, plastic strains, residual stresses, mathematical modelling

INTRODUCTION

Welding technology is widely used to produce joints of large structures made of aluminium alloys, such as fuel tanks for aerospace engineering, bodies for transport mechanical engineering, etc. [1, 2]. To predict the accuracy, strength and fatigue life of such welded structures, the relevant task is to determine residual stresses and strains by calculation. The calculation prediction of the overall distortions of large structures with a great number of welded joints [3] is particularly difficult problem, and the use of aluminium alloys for such structures, as is known, significantly increases welding strains [4]. The solution of such problems requires large computing resources and a long time for computation, which complicates the use of calculation methods in practice during technological preparation of assembling and assembly welding of new structures. Recently, when manufacturing cylindrical vessels of aluminium alloys, a new friction stir welding process (FSW) began to be used. It is believed that FSW provides high quality of welded joints without such defects as pores and hot cracks, as well as a much lower level of residual stresses and strains compared to arc welding processes [1, 5]. Thus, the problem of developing effective calculation methods for in-process prediction of stress-strain state during assembly welding of large cylindrical vessels of aluminium alloys is quite relevant.

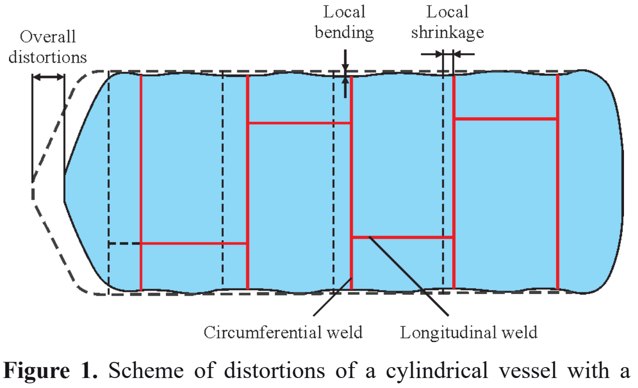

Figure 1. Scheme of distortions of a cylindrical vessel with a great number of longitudinal and circumferential welded joints

PROCEDURE FOR CALCULATED DETERMINATION OF WELDING STRAINS

To solve the problem of predicting strains in large cylindrical vessels of aluminium alloy with a great number of welds (Figure 1), a procedure of modelling welding stresses and strains is used based on the combined use of the general thermoplasticity method [6] and approximated inherent strain method [7]. By means of the thermoplasticity method, temporary and residual stresses and strains for individual welded joints are modelled. It is known that residual welding stresses and plastic strains are formed in a limited area in the welded joint zone. Therefore, the distribution of residual stresses and inherent strain function parameters can be obtained on simplified models of a welded joint of a limited size, which requires much smaller computer resources and time for calculation.

Regarding large cylindrical vessels with longitudinal and circumferential butt joints, it is advis-

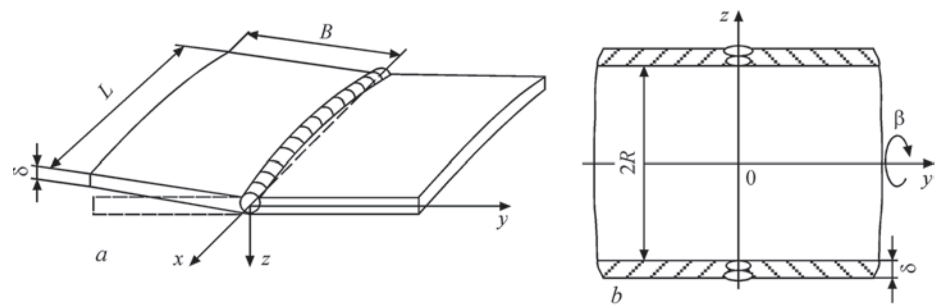

Figure 2. Determination of inherent strain function parameters: *a* — 3D model of a butt welded joint of limited size plates; *b* — 2D model of a circumferential welded joint

able to consider two types of simplified models. To determine the local residual strains (inherent strain function parameters) in the zones of longitudinal joints, the model of a butt joint of plane elements of a limited size can be used (Figure 2, *a*). In the zone of circumferential joints, to determine the residual stresses and strains (inherent strain function parameters), a simplified model in a two-dimensional formulation of a butt circumferential joint of a real size can be used, but with the assumption of a fast-moving welding heating source and "plain strain" hypothesis (Figure 2, *b*).

Then, by means of a three-dimensional model of a cylindrical vessel of a real size using the approximated method of inherent strain function in the framework of elastic formulation of the problem, the overall distortions of large welded structure from all welds are modelled.

GENERALIZED PRESENTATION OF INHERENT STRAIN FUNCTION AS A TENSOR FUNCTION

Let us present the inherent strain function as a tensor of plastic strains:

$$\Pi_{ij}=\begin{Vmatrix}\Pi_{xx} & \Pi_{xy} & \Pi_{xz}\\ \Pi_{yx} & \Pi_{yy} & \Pi_{yz}\\ \Pi_{zx} & \Pi_{zy} & \Pi_{zz}\end{Vmatrix}=\begin{Vmatrix}\varepsilon^p_{xx} & \varepsilon^p_{xy} & \varepsilon^p_{xz}\\ \varepsilon^p_{yx} & \varepsilon^p_{yy} & \varepsilon^p_{yz}\\ \varepsilon^p_{zx} & \varepsilon^p_{zy} & \varepsilon^p_{zz}\end{Vmatrix}=$$
$$=\begin{Vmatrix}\varepsilon^p_{xx} & 0 & 0\\ 0 & \varepsilon^p_{yy} & 0\\ 0 & 0 & -(\varepsilon^p_{xx}+\varepsilon^p_{yy})\end{Vmatrix}=\begin{Vmatrix}\Pi_{xx} & 0 & 0\\ 0 & \Pi_{yy} & 0\\ 0 & 0 & -(\Pi_{xx}+\Pi_{yy})\end{Vmatrix}, \quad (1)$$

$\varepsilon^p_{xy}=\varepsilon^p_{yx}=\varepsilon^p_{xz}=\varepsilon^p_{zx}=\varepsilon^p_{zy}=\varepsilon^p_{yz}=0$, since the axes X and Y are the main axes for the longitudinal and transverse direction of welded joints; $\varepsilon^p_{zz}=-(\varepsilon^p_{xx}+\varepsilon^p_{yy})$ from the condition of volume preservation. If an inherent strain function along the welded joint is permanent, then $\Pi_{xx}=\varepsilon^p_{xx}(y,z)$, $\Pi_{yy}=\varepsilon^p_{yy}(y,z)$.

FORMULATION OF THE PROBLEM OF DETERMINING STRAINS OF THE CYLINDRICAL SHELL BASED ON THE TENSOR INHERENT STRAIN FUNCTION

In the cylindrical coordinate system, the tensor of full strains ε_{ij} in the welded shell will be determined by the sum of the tensor of elastic strains ε^p_{ij} and the tensor of inherent strain function Π_{ij}:

$$\varepsilon_{ij}=\varepsilon^e_{ij}+\Pi_{ij},\ (i,j=r,\beta,z). \quad (2)$$

The components of the tensor ε_{ij} are expressed at each point (r, β, z) of the shell by three components of the displacement vector U_i (Cauchy distribution) [10]:

$$\varepsilon_{rr}=\frac{\partial U_r}{\partial r},\ \varepsilon_{zz}=\frac{\partial U_z}{\partial z},\ \varepsilon_{\beta\beta}=\frac{\partial U_\beta}{r\partial\beta}+\frac{U_r}{r},$$
$$\varepsilon_{rz}=\frac{1}{2}\left(\frac{\partial U_r}{\partial z}+\frac{\partial U_z}{\partial r}\right),\ \varepsilon_{r\beta}=\frac{1}{2}\left(\frac{\partial U_r}{r\partial\beta}+\frac{\partial U_\beta}{\partial r}-\frac{U_\beta}{r}\right), \quad (3)$$
$$\varepsilon_{z\beta}=\frac{1}{2}\left(\frac{\partial U_\beta}{\partial z}+\frac{\partial U_z}{r\partial\beta}\right)$$

as well as with each other by the strains compatibility equations. On the boundary surfaces, the displacement values U_i are determined by the boundary conditions of the first kind, i.e. the values ΔU_r., ΔU_β, ΔU_z.

Strains compatibility equations [10]:

$$\frac{\partial^2\varepsilon_{rr}}{\partial z^2}+\frac{\partial^2\varepsilon_{zz}}{\partial r^2}=2\frac{\partial^2\varepsilon_{rz}}{\partial r\partial z};$$
$$\frac{1}{r}\frac{\partial^2\varepsilon_{rr}}{\partial\beta\partial z}=-\frac{\partial^2}{\partial r\partial\beta}\left(\frac{\varepsilon_{rz}}{r}\right)-\frac{1}{r^2}\frac{\partial^2(r^2\varepsilon_{r\beta})}{\partial r\partial z}+\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\varepsilon_{\beta z}}{\partial r}\right);$$
$$\frac{1}{r}\frac{\partial^2\varepsilon_{rr}}{\partial\beta^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varepsilon_{\beta\beta}}{\partial r}\right)=\frac{2}{r}\frac{\partial^2(r\varepsilon_{\beta r})}{\partial\beta\partial r}+\frac{\partial\varepsilon_{rr}}{\partial r};$$
$$r\frac{\partial}{\partial z}\left(\varepsilon_{rr}-\frac{\partial(r\varepsilon_{\beta\beta})}{\partial r}\right)=\frac{\partial^2\varepsilon_{rz}}{\partial\beta^2}-\frac{\partial^2(r\varepsilon_{\beta z})}{\partial r\partial\beta}-\frac{\partial^2(r\varepsilon_{\beta r})}{\partial z\partial\beta}; \quad (4)$$
$$\frac{\partial^2\varepsilon_{\beta\beta}}{\partial z^2}+\frac{1}{r^2}\frac{\partial^2\varepsilon_{zz}}{\partial\beta^2}+\frac{1}{r}\frac{\partial\varepsilon_{zz}}{\partial r}=\frac{2}{r}\frac{\partial}{\partial z}\left(\frac{\partial\varepsilon_{\beta z}}{\partial\beta}+\varepsilon_{rz}\right);$$
$$\frac{\partial^2}{\partial r\partial\beta}\left(\frac{\varepsilon_{zz}}{r}\right)=-\frac{\partial^2\varepsilon_{\beta r}}{\partial z^2}+r\frac{\partial^2}{\partial r\partial z}\left(\frac{\varepsilon_{\beta z}}{r}\right)+\frac{1}{r}\frac{\partial^2\varepsilon_{rz}}{\partial z\partial\beta}.$$

Components of the full strain tensor ε_{ij} as part of the elastic formulation of the problem are related to the stress tensor σ_{ij} by the Hooke's law [8]:

$$\begin{aligned}
\varepsilon_{rr} &= \frac{1}{E}\left(\sigma_{rr} - \nu\left(\sigma_{\beta\beta} + \sigma_{zz}\right)\right) + \Pi_{rr};\\
\varepsilon_{\beta\beta} &= \frac{1}{E}\left(\sigma_{\beta\beta} - \nu\left(\sigma_{rr} + \sigma_{zz}\right)\right) + \Pi_{\beta\beta};\\
\varepsilon_{zz} &= \frac{1}{E}\left(\sigma_{zz} - \nu\left(\sigma_{rr} + \sigma_{\beta\beta}\right)\right) + \Pi_{zz};\\
\varepsilon_{r\beta} &= \frac{2(1+\nu)}{E}\sigma_{r\beta} + \Pi_{r\beta};\; \varepsilon_{rz} = \frac{2(1+\nu)}{E}\sigma_{rz} + \Pi_{rz};\\
\varepsilon_{\beta z} &= \frac{2(1+\nu)}{E}\sigma_{\beta z} + \Pi_{\beta z}.
\end{aligned} \tag{5}$$

Components of the stress tensor are related to each other by the equilibrium equations [10]:

$$\begin{aligned}
&\frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{rr}) + \frac{1}{r}\frac{\partial\sigma_{r\beta}}{\partial\beta} + \frac{\partial\sigma_{rz}}{\partial z} + \rho F_r = 0;\\
&\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\sigma_{r\beta}) + \frac{1}{r}\frac{\partial\sigma_{\beta\beta}}{\partial\beta} + \frac{\partial\sigma_{\beta z}}{\partial z} + \rho F_\beta = 0;\\
&\frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{rz}) + \frac{1}{r}\frac{\partial\sigma_{\beta z}}{\partial\beta} + \frac{\partial\sigma_{zz}}{\partial z} + \rho F_z = 0,
\end{aligned} \tag{6}$$

where ρ is the material density; F_r, F_β, F_z are the components of mass force, in the considered problem of determining welding stresses and strains in the cylindrical shell $F_r = F_\beta = F_z = 0$.

The linear problem of elasticity theory can be solved by displacements. From the generalized Hooke's law equations (5), stresses through strains look as follows [8]:

$$\begin{aligned}
\sigma_{rr} &= A_1\varepsilon_{rr} + A_2(\varepsilon_{\beta\beta} + \varepsilon_{zz}) + Y_{rr};\\
\sigma_{\beta\beta} &= A_1\varepsilon_{\beta\beta} + A_2(\varepsilon_{rr} + \varepsilon_{zz}) + Y_{\beta\beta};\\
\sigma_{zz} &= A_1\varepsilon_{zz} + A_2(\varepsilon_{rr} + \varepsilon_{\beta\beta}) + Y_{zz};\\
\sigma_{r\beta} &= A_3\varepsilon_{r\beta} + Y_{r\beta};\quad \sigma_{rz} = A_3\varepsilon_{rz} + Y_{rz};\\
\sigma_{\beta z} &= A_3\varepsilon_{\beta z} + Y_{\beta z};\\
A_1 &= \frac{2K+\psi}{3\psi K};\quad A_2 = \frac{\psi - K}{3\psi K};\quad A_3 = \frac{1}{\psi};\\
K &= \frac{1-2\nu}{E};\quad G = \frac{E}{(1+\nu)},
\end{aligned} \tag{7}$$

where E is the normal elasticity modulus (Young's modulus); K is the bulk pressure modulus; G is the shear modulus; ψ is the material state function, that takes into account the plastic flow for the elastic behaviour of the material $\psi = 0.5G$; additions $Y_{i,j}$ take into account the load of the cylindrical shell by additional strains from welding shrinkage (in general appearance):

$$\begin{aligned}
Y_{rr} &= A_1\Pi_{rr} + A_2(\Pi_{zz} + \Pi_{\beta\beta});\\
Y_{\beta\beta} &= A_1\Pi_{\beta\beta} + A_2(\Pi_{rr} + \Pi_{zz});\\
Y_{zz} &= A_1\Pi_{zz} + A_2(\Pi_{rr} + \Pi_{\beta\beta});\\
Y_{r\beta} &= A_3\Pi_{r\beta};\quad Y_{z\beta} = A_3\Pi_{z\beta};\quad Y_{rz} = A_3\Pi_{rz}.
\end{aligned} \tag{8}$$

To model welding strains in a cylindrical shell from circumferential and longitudinal welded joints, the following basic parameters of the inherent strain function can be specified: plastic shrinkage strains in the axial Π_{zz} and circumferential $\Pi_{\beta\beta}$ directions. Plastic shrinkage strains in the radial direction Π_{rr} (across the thickness of the shell wall) cause local strains and do not significantly affect the overall distortions of the welded cylindrical shell. The tangent components of plastic shrinkage strains Π_{rz}, $\Pi_{r\beta}$, $\Pi_{z\beta}$ can be neglected, since the directions of welds in the cylindrical shell coincide with the longitudinal and circumferential direction of the cylindrical coordinate system.

$$\begin{aligned}
Y_{rr} &= A_2(\Pi_{zz} + \Pi_{\beta\beta});\\
Y_{\beta\beta} &= A_1\Pi_{\beta\beta} + A_2\Pi_{zz};\\
Y_{zz} &= A_1\Pi_{zz} + A_2\Pi_{\beta\beta};\\
Y_{r\beta} &= 0;\quad Y_{z\beta} = 0;\quad Y_{rz} = 0.
\end{aligned} \tag{9}$$

With the use of the Cauchy distributions (3) binding the components of the full strain tensor ε_{ij} with the components of the displacement vector U_i, the stresses (7) can be presented through the displacements:

$$\begin{aligned}
\sigma_{rr} &= A_1\frac{\partial U_r}{\partial r} + A_2\left(\frac{\partial U_z}{\partial z} + \frac{\partial U_\beta}{r\partial\beta} + \frac{U_r}{r}\right) + Y_{rr};\\
\sigma_{\beta\beta} &= A_1\left(\frac{\partial U_\beta}{r\cdot\partial\beta} + \frac{U_r}{r}\right) + A_2\left(\frac{\partial U_r}{\partial r} + \frac{\partial U_z}{\partial z}\right) + Y_{\beta\beta};\\
\sigma_{zz} &= A_1\frac{\partial U_z}{\partial z} + A_2\left(\frac{\partial U_r}{\partial r} + \frac{\partial U_\beta}{r\cdot\partial\beta} + \frac{U_r}{r}\right) + Y_{zz};\\
\sigma_{r\beta} &= A_3\frac{1}{2}\left(\frac{\partial U_r}{r\partial\beta} + \frac{\partial U_\beta}{\partial r} - \frac{U_\beta}{r}\right);\\
\sigma_{rz} &= A_3\frac{1}{2}\left(\frac{\partial U_r}{\partial z} + \frac{\partial U_z}{\partial r}\right);\; \sigma_{\beta z} = A_3\frac{1}{2}\left(\frac{\partial U_\beta}{\partial z} + \frac{\partial U_z}{r\partial\beta}\right);
\end{aligned} \tag{10}$$

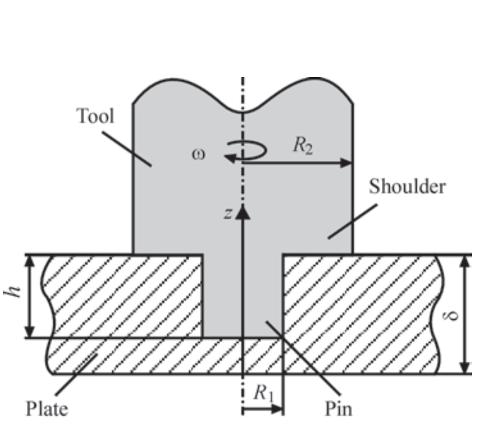

Figure 3. Scheme of working tool and joint plates at FSW

Let us substitute the obtained expressions for the components of the stress tensor into the equilibrium equation (6):

$$
\begin{aligned}
&\frac{1}{r}\frac{\partial}{\partial r}\left(rA_1\frac{\partial U_r}{\partial r}+rA_2\left(\frac{\partial U_z}{\partial z}+\frac{\partial U_\beta}{r\partial\beta}+\frac{U_r}{r}\right)\right)+\\
&+\frac{1}{r}\frac{\partial}{\partial\beta}\left(A_3\frac{1}{2}\left(\frac{\partial U_r}{r\partial\beta}+\frac{\partial U_\beta}{\partial r}-\frac{U_\beta}{r}\right)\right)+\\
&+\frac{\partial}{\partial z}\left(A_3\frac{1}{2}\left(\frac{\partial U_r}{\partial z}+\frac{\partial U_z}{\partial r}\right)\right)=-\frac{1}{r}\frac{\partial}{\partial r}(rY_{rr});\\
&\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2A_3\frac{1}{2}\left(\frac{\partial U_r}{r\partial\beta}+\frac{\partial U_\beta}{\partial r}-\frac{U_\beta}{r}\right)\right)+\\
&+\frac{1}{r}\frac{\partial}{\partial\beta}\left(A_1\left(\frac{\partial U_\beta}{r\partial\beta}+\frac{U_r}{r}\right)+A_2\left(\frac{\partial U_r}{\partial r}+\frac{\partial U_z}{\partial z}\right)\right)+\\
&+\frac{\partial}{\partial z}\left(A_3\frac{1}{2}\left(\frac{\partial U_\beta}{\partial z}+\frac{\partial U_z}{r\partial\beta}\right)\right)=-\frac{1}{r}\frac{\partial Y_{\beta\beta}}{\partial\beta};\\
&\frac{1}{r}\frac{\partial}{\partial r}\left(rA_3\frac{1}{2}\left(\frac{\partial U_r}{\partial z}+\frac{\partial U_z}{\partial r}\right)\right)\sigma_{rz}+\\
&+\frac{1}{r}\frac{\partial}{\partial\beta}\left(A_3\frac{1}{2}\left(\frac{\partial U_\beta}{\partial z}+\frac{\partial U_z}{r\partial\beta}\right)\right)+\frac{\partial}{\partial z}\times\\
&\times\left(A_1\frac{\partial U_z}{\partial z}+A_2\left(\frac{\partial U_r}{\partial r}+\frac{\partial U_\beta}{r\partial\beta}+\frac{U_r}{r}\right)\right)=-\frac{\partial Y_{zz}}{\partial z}.
\end{aligned}
\qquad (11)
$$

The obtained system of equations relative to the three components of the displacement vector U_x, U_β, U_z of an arbitrary point (x, β, z) of the cylindrical shell during loading by additional strains from the welding shrinkage (components Y_{rr}, $Y_{\beta\beta}$, Y_{zz} in the right part of the equations) and boundary conditions on the shell surface determine the general formulation of the boundary problem.

DETERMINATION OF INHERENT STRAIN FUNCTION FOR WELDED JOINTS

To determine the inherent strain function, the methods of mathematical modelling of temperature distributions and stress-strain state of a butt joint of plates at welding heating were used.

The peculiarity of the developed model of the heating source at FSW is the heat dissipation due to the friction of the tool relative to the joint material [6]. The tool (Figure 3) rotates around a vertical axis at a certain angular speed ω, rev/s and is pressed to the plate with an axial distributed force P_n, Pa, which causes the heat flux into the joint material on the contact surface of the tool:

$$
\lambda\frac{\partial T}{\partial n}=\mu P_n\omega r,\ \mathrm{W/m^2};\qquad Q=\mu P_n\omega\iint_s r\,dS, \qquad (12)
$$

where μ is the friction coefficient, whose value can be accepted as permanent at a level of 0.3–0.4 or variable depending on the temperature [11]; Q is the power of heat dissipation, W; S is the area of the corresponding contact surfaces, $Q_1=2\pi/3\,\mu P_n\omega(R_2^3-R_1^3)$ on the shoulder ($z=\delta$, $R_1<r<R_2$), $Q_2=2\pi\mu P_n\omega R_1^2h$ on the side surface of the pin ($r=R_1$, $\delta-h>z>\delta$), $Q_3=2\pi/3\,\mu P_n\omega R_1^3$ on the lower end surface of the pin ($z=\delta-h$, $0>r>R_1$); δ is the thickness of welded plates, m; h is the length of the pin, m. The power of bulk heat dissipation $W(x, y, z, t)$, W/m^3 consists of heat dissipation in the volume V_1 on the upper surface of the joint plates under the tool shoulder and in the volume of the pin V_2 (dz is the size of the finite element):

$$
\begin{aligned}
W(x,y,z,t)&=W_1+W_2=\frac{Q_1}{V_1}+\frac{Q_2+Q_3}{V_2}=\\
&=\frac{2\mu P_n\omega(R_2^2+R_2R_1+R_1^2)}{3(R_2+R_1)dz}+\frac{2\mu P_n\omega}{h}\left(h+\frac{R_1}{3}\right).
\end{aligned}
\qquad (13)
$$

The model of thermoplastic deformation of the welded joint material provides that overall distortion tensor is the sum of elastic and plastic strains [8]:

$$
\varepsilon_{ij}=\varepsilon_{ij}^e+\varepsilon_{ij}^p,\qquad \varepsilon_{ij}=\frac{\sigma_{ij}-\delta_{ij}\sigma}{}+\delta_{ij}(K\sigma+\varphi)\quad (i,j=x,y,z), \qquad (14)
$$

where δ_{ij} is the Cronecker symbol; σ is the ball tensor; $\varphi=\alpha(T-T_0)$, where α is the ratio of relative temperature elongation of the material. Plastic strains are associated with the stresses by the equation of the theory of plastic non-isothermal flow (von Mises condition) [8].

RESULTS OF RESIDUAL STRESSES AND PLASTIC STRAINS CALCULATION

The calculated model for determination of residual stresses and strains at FSW of plates made of aluminium alloy (Al–6.5 % Cu) was validated by comparing the results of calculation by residual stresses with experimental measurement data. The calculated residual longitudinal stresses at FSW of a butt joint of an aluminium alloy with 10 mm thickness were determined as a result of the solution of the thermoplasticity problem for 3D model of a butt joint of plates of the limited size of 300×300 mm (Figure 2, *a*). Thermophysical and mechanical properties of aluminium alloy, depending on the temperature are shown in Figure 4 [9]. FSW parameters are R_2 = 10 mm, R_1 = 5 mm, ω = 700 rpm, P_n = 70 MPa, linear welding speed is 160 mm/min. The maximum calculated temperature at the set FSW parameters does not exceed 470 °C (Figure 5).

Experimental measurements of residual stress distribution were carried out on the samples of butt joints

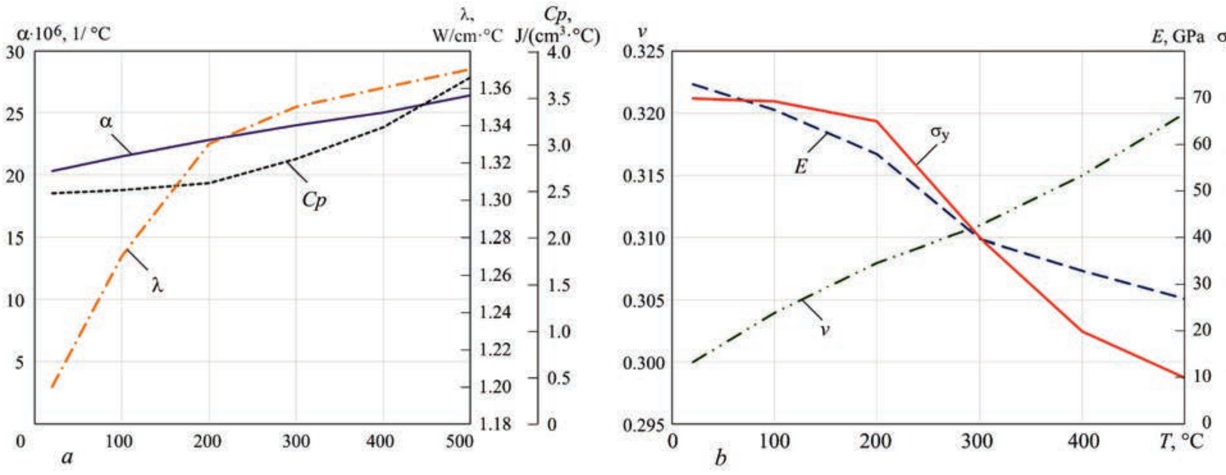

Figure 4. Thermophysical (*a*) and mechanical (*b*) properties of aluminium alloy (Al–6.5 % Cu)

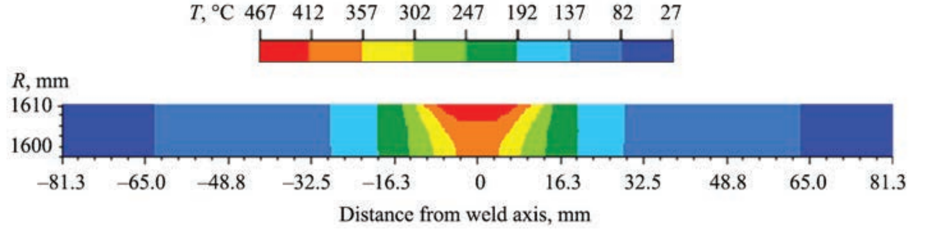

Figure 5. Maximum calculated temperature at FSW of a butt joint

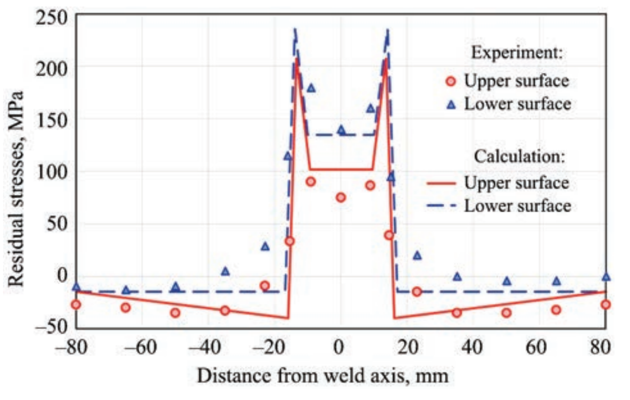

Figure 6. Distribution of residual longitudinal stresses

made by FSW by the method of full cutting into templates and measurement of elastic strains by means of mechanical strain gauge.

The nature of the distribution of calculated residual stresses is close to the experimental (Figure 6). The maximum level of calculated tensile stresses (up to 200 MPa) is slightly higher than experimental values up to 180 MPa). In the welded joint center, a zone of significant reduction in the tensile longitudinal stresses is located, which is associated with the effect of aluminium alloy softening. Also for experimental and calculated data, a significant difference between stresses on the upper and lower surfaces of welded

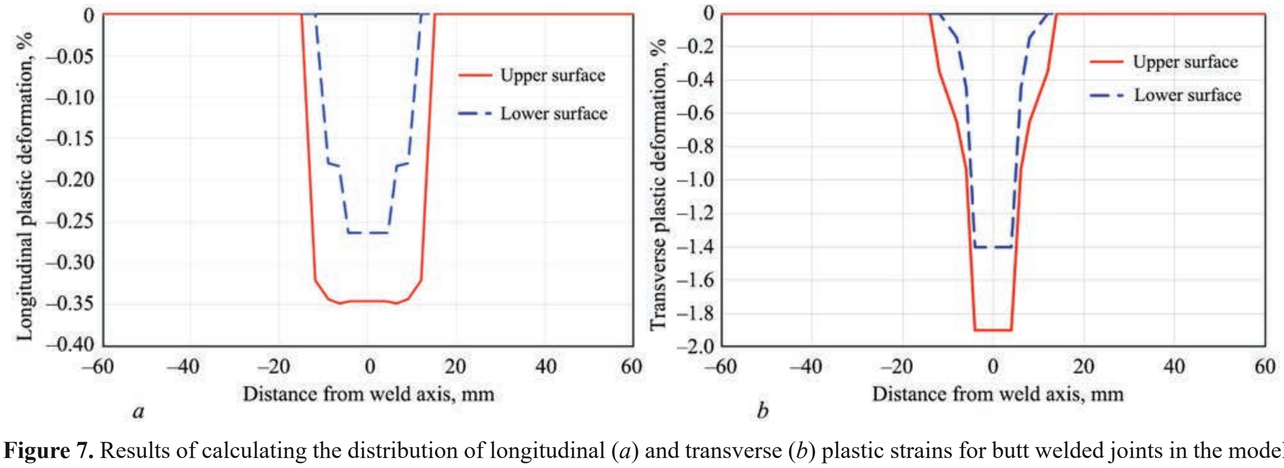

Figure 7. Results of calculating the distribution of longitudinal (*a*) and transverse (*b*) plastic strains for butt welded joints in the model of FSW of plates (300×300 mm, δ = 10 mm)

joint is typical — on the lower surface, residual tensile stresses are higher.

The estimate of the error of calculation data shows that in the zone of tensile stresses (3 central points, from –16 to +16 mm from the joint axis), the mean-square deviation from the experimental values on the lower surface of the sample is 21 %, and on the upper surface it is up to 31 %. Such an error can be accepted as satisfactory, considering the complex nature of residual stress distribution.

The results of residual distributions of longitudinal $\Pi_{xx} = \varepsilon^{p}_{xx}(y,z)$ and transverse $\Pi_{yy} = \varepsilon^{p}_{yy}(y,z)$ plastic strains at FSW were obtained (Figure 7). It should be noted that transverse and longitudinal plastic strains on the upper surface of the welded plate are approximately by 25 % higher in absolute value than those on the lower surface. This is predetermined by the nonuniform distribution of the maximum temperature over the welded joint thickness. It should also be noted that transverse plastic strains in terms of absolute value are approximately five times higher than longitudinal, but the zone of longitudinal plastic strain formation is 1.5–2 times wider.

RESULTS OF WELDING STRAIN MODELLING

The obtained distributions of residual longitudinal $\Pi_{xx} = \varepsilon^{p}_{xx}(y,z)$ and transverse $\Pi_{yy} = \varepsilon^{p}_{yy}(y,z)$ plastic strains were used in the prediction of the overall distortions of the long cylindrical shell by the inherent strain function method (2) (Figure 8, *a*, $2R$ = = 3200 mm, L = 6000 mm, δ = 10 mm), which contains six longitudinal and five circumferential welded joints (location coordinates of the circumferential welded joints z = 1, 2, 3, 4, 5 m, longitudinal welded

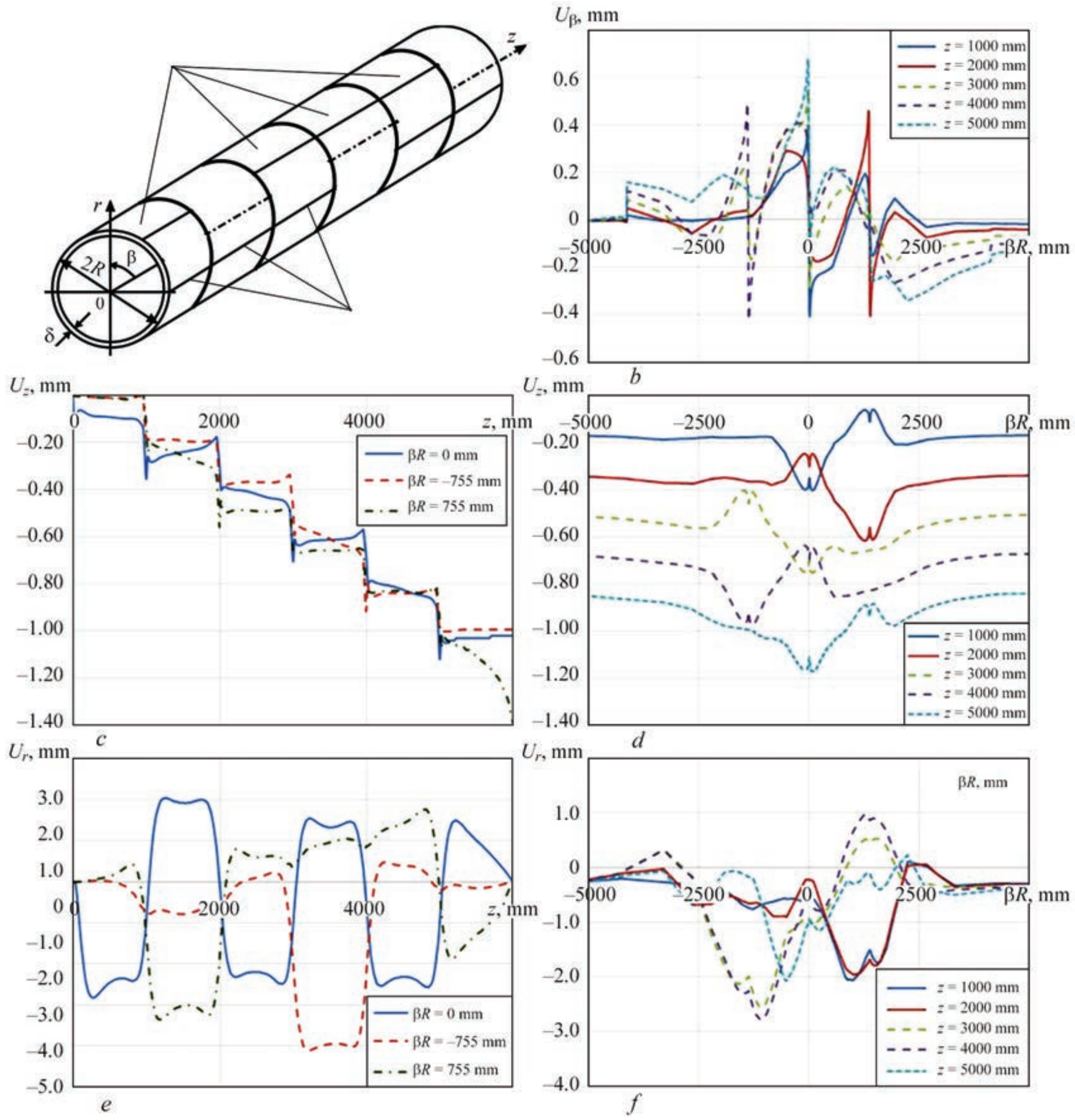

Figure 8. Welding distortions of the cylindrical shell: *a* — scheme of welds location; *b* — displacements U_β along the shell circle; *c* — displacement U_z along the shell axis; *d* — displacements U_z in different sections along the length of the shell; *e* — radial bending U_r along the shell axis; *f* — radial bending U_r along the circle in different sections along the length of the shell

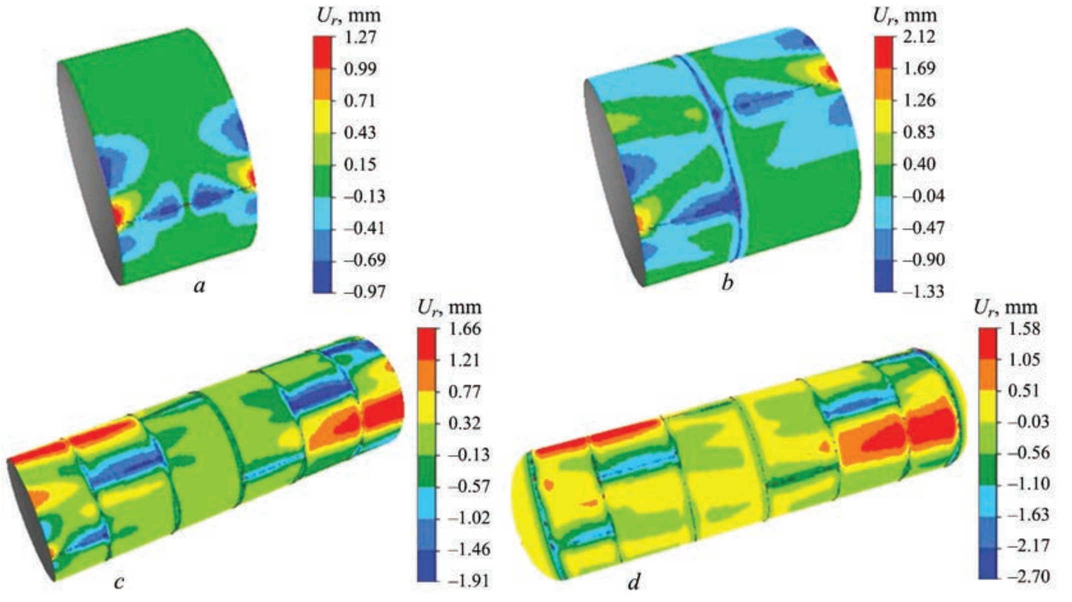

Figure 9. Radial displacements of the cylindrical vessel at different stages of assembly and assembly welding: *a* — longitudinal joint of the shell ring; *b* — circumferential joint of two rings; *c* — cylindrical shell with circumferential and longitudinal welds; *d* — cylindrical vessel with welded-on bottoms

butts β = 0, ±755 mm). The solution of the problem by displacements of the surface points of the cylindrical shell was obtained by the finite element method on a full-size model for each welded joint separately and then overall displacements were added by the principle of superposition.

The presence of a great number of circumferential joints leads to the accumulation of axial (longitudinal) displacements U_z along the shell (Figure 8, *c*, *e*). The rest of the strain components like circumferential displacements U_β (Figure 8, *b*) and radial bending U_r (Figure 8, *e*, *f*) have a complex local nature of distribution. This example demonstrates the efficiency of the developed calculation algorithm when predicting welding strains of large cylindrical shells with a great number of welded joints based on the approximated inherent strain function method.

The developed mathematical model and calculation algorithm based on the inherent strain function method can be effectively used in the technological preparation of manufacturing large cylindrical vessels of aluminium alloys for in-process prediction of stress-strain state. As an example, Figure 9 presents the results of predicting the distribution of radial displacements at different stages of assembly and assembly welding of a large cylindrical vessel. Radial displacements have a complex nature of distribution, deflections of up to ≈ –3 mm in the zones of circumferential and longitudinal welded joints are changed with the shell bends of up to ≈ 1.5 mm in the gaps between the welded joints. Given the large sizes of the considered welded vessel (3200 mm diameter), the maximum obtained displacement values are not high, due to the low level of residual strains at FSW.

CONCLUSIONS

1. A mathematical model and calculation algorithm have been developed based on the inherent strain function method for numerical determination of welding strains of large cylindrical shells with a great number of circumferential and longitudinal welded joints.

2. The inherent strain function parameters of welded joints are plastic strains in the axial Π_{zz} and circumferential $\Pi_{\beta\beta}$ directions that can be determined by numerical methods of thermoplasticity on simplified models of longitudinal and circumferential welded joints. These models also allow determining local distributions of maximum welding temperatures and residual stresses in the joint zone.

3. The reliability of the mathematical model of determining residual stresses and plastic strains at FSW of aluminium alloy is confirmed by the agreement of the calculated data relative to the distribution of residual longitudinal stresses with the data of experimental measurements.

4. The developed mathematical models and calculation algorithms can be effectively used for in-process prediction of a stress-strain state in assembly welding of large cylindrical vessels of aluminium alloys.

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CONFLICT OF INTEREST

The Authors declare no conflict of interest

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