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SPACETIME ANALYSIS OF AN ELECTROMAGNETIC AIRY PULSE AFTER ITS INTERACTION WITH A PLANAR BOUNDARY IN UNIFORMLY ACCELERATED RELATIVISTIC MOTION

Subject and Purpose. *The transformation peculiarities that the electromagnetic pulses get when heading towards a boundary that performs uniformly accelerated relativistic motion are the present paper concern. A smooth non-stationarity case when the boundary velocity gradually changes from zero to the pulse velocity value is considered, with a focus on the spacetime distribution and evolution of the electromagnetic Airy pulse field.*

Methods and methodology. *The study and analysis are carried out by the method of Volterra integral equations which can describe electromagnetic wave propagation in a heterogeneous time-varying medium. In terms of this method, the basic initial boundary value electrodynamical problem on the electromagnetic source radiation in a heterogeneous time-varying medium is formulated, taking into account the boundary and initial conditions. The resolvent method for solving the Volterra integral equation of the second kind is described. Its advantage is analytical solution capabilities and a versatility as to the primary field choice.*

Results. *Analytical solutions to the original integral equation have been obtained. By analysis, it has been found that the secondary field expressions have singularities that can be controlled well enough by a proper choice of numerical modeling parameters. The revealed singularities have been analytically studied. Their action on the Airy pulse was examined and illustrated through simulation modeling using the starting parameter that locates the Airy pulse at any moment in time.*

Conclusions. *In this work, the electromagnetic Airy pulse interaction with a boundary perfoming uniformly accelerated relativistic motion was examined using the Volterra integral equations method. The obtained analytical solutions revealed significant spacetime changes in the Airy pulses. Our analysis indicated possibilities for controlling the secondary field characteristics by a proper choice of modeling parameters. The results have been confirmed by numerical simulations. They provide a basis for further research in this area.*

Keywords: *Airy pulse, electromagnetic waves, Volterra integral equations, resolvent, relativistic motion, uniformly accelerated motion.*

Introduction

Significant attention has been recently given to the electromagnetic wave interaction with boundaries performing uniformly accelerated relativistic motion. This topic is crucial for understanding processes going in various physical systems, such as accelerators, plasma colliders, etc. Models of the systems of the kind are considered in works [1—4], demonstrating that a moving boundary can significantly influence the scattering characteristics of electromagnetic waves.

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Set apart are the studies of the Airy pulse propagation in nonstationary media. It is well known that the Airy pulses possess unique properties, such as self-focusing and self-healing, making them promising for quantum optics and photonics [5—7]. However, the Airy pulse interaction with relativistically accelerated boundaries is not understood well enough yet. This sends us to solve the problem of Airy pulse scattering on such boundaries to refine the theoretical basics and expand potential applications in quantum optics, etc.

Considerable recent attention has been increasingly focused on the Airy pulse interaction with boundaries performing uniformly accelerated relativistic motion as well as on the electromagnetic wave propagation in non-stationary media. The studies address challenging problems having potential use in modern technologies. In particular, a numerical investigation has been reported [8] about two Airy pulses travelling at different wavelengths and interacting with a controllable periodic temporal boundary. The intensities of the solitons that emerge from this interaction can form an optical event horizon, whence the weaker Airy pulse is completely reflected. The findings can be welcome to optical manipulation and temporal waveguiding.

Another remarkable work [9] examines asymmetric Airy pulses and their interaction with non-stationary media. This research demonstrates effective control over the intrapulse Raman scattering due to the usage of asymmetric Airy pulses. The methods of the sort can be employed to enhance the pulse propagation control in complex optical media.

Current studies not only highlight the importance of further research into the Airy pulse interaction with relativistic boundaries. They also underline the need for new theoretical and experimental approaches to gain more from the analysis of electromagnetic processes in non-stationary media.

The originality of studies concerning the Airy pulse scattering by a boundary moving with a relativistic uniform acceleration lies in several key aspects. The first is non-standard conditions of the interaction. The relativistic motion creates challenging conditions of the electromagnetic wave interaction with a boundary. They do not comply with conventional scattering scenarios and require novel theoretical approaches and numerical methods for the interaction analysis. The second is that the Airy pulse itself is a unique subject of study. Unusual properties, such as self-amplification and self-acceleration, make Airy pulses particularly intriguing in the context of relativistic dynamics. Studying them under conditions that have been little examined before adds originality to the research.

The third is the interaction with non-stationary media. Analysis of the Airy pulse propagation in non-stationary media, in particular those carrying moving boundaries, can cast new insights into fundamental physical processes and thus extend the range of Airy pulses applications in scientific fields such as optics and photonics.

Lastly, the findings of this research could be valuable for advancing new technologies related to quantum information processing, light control in optical devices, development of devices for electromagnetic propagation control. Thus, the originality of these studies lies in their integrated approach to analyzing the interaction of specific pulses with a dynamic boundary in motion. This makes it possible to discover new physical effects and arrive at innovative technological solutions.

Generally, over the last decades, significant progress has been made in the study, analysis, and shaping of laser beams. Many results mount on a numerical basis, which does not always mean reliability and completeness. Therefore, for synthesis and analysis of new pulse propagation effects, numerical modeling should be combined with mathematical methods which make it possible to obtain electromagnetic-wave analytical expressions with their subsequent analysis.

The electromagnetic field interaction with a heterogeneous non-stationary medium is of fundamental importance for a variety of applied technological processes. The basic process of electromagnetic wave propagation evolves over time in limited spatial areas. The research into such phenomena requires a rigorous mathematical apparatus to build mathematical models of applied problems in electrodynamics. An exact analytical solution is available for many idealized problems describing fairly simple electrodynamical processes. On this basis, more complicated structures and processes can be solved.

Singularities in the electromagnetic pulse conversion occur not only in the case of sharply non-stationary boundary motion but for smooth non-stationarity, too, when the boundary velocity tends to the phase velocity of the pulse. This is well illustrated by the uniformly accelerated relativistic motion of the boundary whose velocity continuously varies from zero to the relativistic values so that it goes through all possible critical points.

The visual sign of the critical point presence is that the scattered wave amplitudes tend to infinity and there is ambiguity in determining the number of scattered waves. The remedy can be by searching for appropriate analytical solutions within qualitative considerations, as was done in [10] for the case of a plane monochromatic wave. However, based on experience, we acknowledge that there is a high likelihood of losing some solutions.

In this paper, we will explore a qualitatively new approach to solving the problem of electromagnetic pulse interaction with a boundary that is moving at relativistic speeds. The problem will be addressed using the exact method of integral equations to find appropriate analytical solutions.

1. Methods and methodology

When the electromagnetic wave and medium parameters depend only on time and one spatial coordinate, the problem is one-dimensional and described by the Volterra integral equation of the second kind in one space dimension [11]:

$$E = E_0 - \frac{v - v_1^2}{2vv_1^2}\frac{\partial^2}{\partial t^2}\times$$
$$\times\int\limits_{-\infty}^{\infty} dt' \int\limits_{-\infty}^{\infty} dx'\theta\left(t - t' - \frac{|x-x'|}{v}\right)\chi(t',x')E(t',x'), \tag{1}$$

where E_0 is the wave field coming from the half-space $x<0$, the values $v = c/\sqrt{\varepsilon}$ and $v_1 = c/\sqrt{\varepsilon_1}$ are the phase velocities on both sides of the boundary, and $\chi(t,x)$ is the characteristic function that equals zero in the region with the medium $\varepsilon(\varepsilon_1)$ and equals unity outside it. A light cone whose vertex is at the observation point (t,x) is given by the Heaviside function $\theta(\tau)$ and determines the kernel of integral Eq. (1) so that the kernel is not equal to zero only inside the cone. Equation (1) considers the initial and boundary conditions and describes the electromagnetic signal evolution over time $t\in[0;\infty)$.

The given type of equations is advisable to use for describing processes going on in media whose properties vary over time, making the ongoing processes non-stationary. It renders a possibility to take into account how variations in medium parameters affect electromagnetic wave propagation. An important condition for employing the Volterra integral equations is the causality of the system. This means that at any time, the system response only depends on the system previous rather than future states. In physical problems such as pulse scattering in non-stationary media, the use of the Volterra equation is justified. It allows taking into account the historical development of the process and the effects accumulated over time. This is important for conducting a detailed analysis of temporary processes and assessing how the initial conditions influence the future evolution of the system.

Let at time zero, the boundary "switch on" its uniformly accelerated motion. Consider the case when at time zero the boundary starts moving, $\chi^{+}(t',x') = \theta(x - x_s(t))$, while the media on both sides of the boundary remain stationary. If so, a part of the integration domain in Eq. (1) will change to become part of the region $O = \{x = (t,x) : 0 \le t < \infty, -\infty < x < \infty\}$. Thus, in this half-space, Eq. (1) for the secondary field becomes

$$E = E_1 - \frac{v_1^2 - v^2}{2v_1v^2}\frac{\partial^2}{\partial t^2}\times$$
$$\times\int\limits_{0}^{\infty} dt' \int\limits_{-\infty}^{\infty} dx'\theta\left(t - t' - \frac{|x-x'|}{v}\right)\chi^{+}(t',x')E(t',x'), \tag{2}$$

where

$$E_1 = E_0 - \frac{v^2 - v_1{}^2}{2vv_1{}^2}\frac{\partial^2}{\partial t^2}\times$$
$$\times\int\limits_{-\infty}^{0} dt' \int\limits_{-\infty}^{\infty} dx'\theta\left(t - t' - \frac{|x-x'|}{v}\right)\chi^{-}(t',x')E_1(t',x')$$

describes the historical development of the field before the boundary starts moving. It is evaluated by substituting into Eq. (2) the primary field in the Airy pulse appearance

$$E_0(t,x) = \mathrm{Ai}\left(-\frac{t}{T} + \frac{x}{vT}\right)e^{\alpha(-t/T + x/vT)},$$

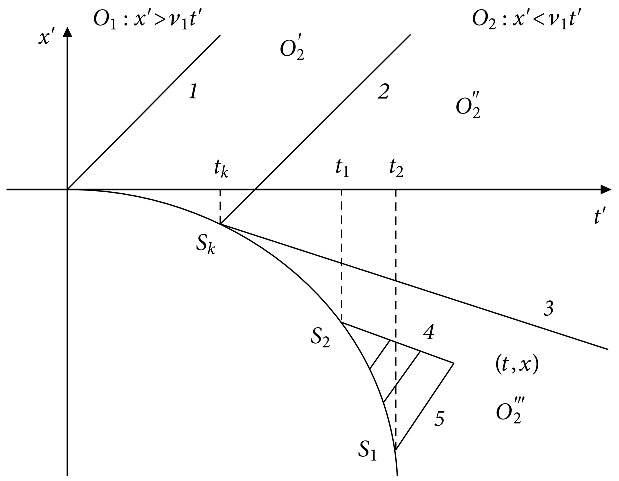

Fig. 1. Electromagnetic field determination in the uniformly accelerated motion case: *1* — $x' = v_1 t'$, *2* — $\tau_H = t' - x'/v_1$, *3* — $\tau_k = -t' + x'/v_1$, *4* — $x' = -v_1(t' - t) - x$, *5* — $x' = v_1(t' - t) - x$, $S_1 \ldots S_k$ are the points with the abscissas $t_1 \ldots t_k$, and O_2', O_2'', O_2''' are subregions of the region O_2

where $\mathrm{Ai}(t,x)$ is the Airy function, T is the time scale, and α is the parameter describing the dissipative properties of the medium.

The solution of integral equation (2) is constructed via the resolvent $\hat{R}$ and will be shown in further calculations.

Let us consider relativistic uniformly accelerated motion, the simplest and most natural form of non-inertial motion. The choice of this motion law in analyzing the interaction of an Airy pulse with a moving boundary is justified for several reasons, both physical and mathematical. For the interaction between an electromagnetic pulse and a moving boundary under relativistic conditions, it is crucial to account for effects that occur at velocities approaching the speed of light. In such cases, classical kinematics is insufficient, as it fails to incorporate relativistic effects like length contraction and time dilation. The relativistic uniformly accelerated motion provides an accurate description of boundary kinematics, enabling precise modeling of interactions with the electromagnetic field. This law is particularly advantageous for analytical studies, as it facilitates stable and physically accurate solutions for the electromagnetic field, which is vital in problems involving Volterra integral equations. The possibility of constructing precise solutions hinges on the analytical properties of the input parameters. From a physical perspective, relativistic uniformly accelerated motion can serve as a model for describing the motion of particles or boundaries in a plasma accelerated by electromagnetic fields. This adds practical relevance to the use of this motion law, allowing for the extrapolation of study results to real-world physical systems. Let us consider uniformly accelerated relativistic motion as the simplest and the most natural form of non-inertial motion. Behind the choice, there are several reasons connected with physical and mathematical aspects of the problem of Airy pulse interaction with a moving boundary. The question of the electromagnetic pulse interaction with a moving boundary in relativistic terms requires considering effects that occur at great velocities approaching the speed of light. In the relativistic case, the boundary movement cannot be described within the limits of classical kinematics whose laws fail to deal with relativistic effects like length contraction and time dilation. The concept of uniformly accelerated relativistic motion provides a correct description of the boundary kinematics and make it possible to model the boundary interaction with electromagnetic field. This law of uniformly accelerated motion is particularly advantageous for analytical studies, as it facilitates stable and physically true solutions for the electromagnetic field. This is especially important in problems involving Volterra integral equations, where the availability of an exact solution hinges on the analytical properties of the input parameters. Physically, the relativistic uniformly accelerated motion can be thought of as a model describing the motion of particles or boundaries in an electromagnetically accelerated plasma. This interpretation stuffs the motion law employed in the given problem with a physical sense and extrapolates the study results to physical systems of the real world.

In the present scenario, the boundary velocity continuously changes, gradually increasing from zero to relativistic values and running through all critical points. The critical points are characterized by abrupt perturbations in wave scattering processes with the result that the scattered field amplitudes sharply increase, the number of scattered waves is determined with ambiguities. A mention should be made that the discussed type of motion makes the problem fundamentally non-stationary, which significantly complicates its analysis and solution.

The counter-motion between the boundary and the electromagnetic wave obeys the expression [12]

$$x_s(t) = -v(\sqrt{\xi^2 + t^2} - \xi),$$

where $\xi = c / w$, with w being the acceleration in its own frame of reference. As $t \to \infty$, the boundary velocity varies by the law $u(t) = -vt / \sqrt{\xi^2 + t^2}$ until approaches the wave phase velocity, $u(t) \to -v$.

The properties of the medium do not change after the boundary starts moving at time zero. So, in the uniform motion case, the secondary field in the region $O_1 : x > v_1 t$ (see Fig. 1) will remain the same whether before or after time zero. In the region $O_2 : x < v_1 t$, the resolvent looks different for different values of the refractive index v / v_1.

In the case $v_1 / v > 1$, the resolvent characteristic reflected from the light line of the boundary always resides in the region $\chi^{(+)} = \theta(x - x_s(t)) = 1$:

$$\left\langle \mathbf{x} \middle| \hat{R} \middle| \mathbf{x}' \right\rangle = -\theta(x - x_s(t)) \times \\ \times \frac{\partial}{\partial t} \frac{v^2 - v_1^2}{2v^2 v_1} \left\{ \frac{\partial}{\partial t} \theta\left(t - t' - \frac{|x - x'|}{v_1} \right) + \right. \\ \left. + R_u(\tau^-) \frac{\partial}{\partial t} \theta\left(\varphi\left(\tau^-\right) - t' - \frac{x'}{v_1} \right) \right\} \theta(x' - x_s(t')), \tag{3}$$

where $\varphi(\tau) = 2t_1 - \tau$, with t_1 being the point where the lower characteristic of the resolvent $\hat{R}$ meets the light line of the boundary, $t_{1,2}(\tau) = v_1 v / (v^2 - v_1^2) \times \times \left(\pm\xi - \frac{v_1}{v}\tau + \sqrt{\xi^2 \pm 2\frac{v_1}{v}\xi\tau + \tau^2} \right)$. The coefficient $R_u(\tau) = (v - v_1) / (v + v_1)(v_1 - u_1(\tau)) / (v_1 + u_1(\tau))$ is the observation point function (the boundary velocity is chosen at the moment it approaches the resolvent characteristic, $u_1(\tau) = u(t_1)$).

In the opposite case $v_1 / v < 1$, the reflected characteristic belongs to the region $\chi^+ = 1$ only until the moment of contact $t_k = v_1 \xi / \sqrt{v^2 - v_1^2}$, when the boundary velocity approaches the pulse velocity v_1.

The ray $\tau^- = \tau_H = \xi \left(\frac{v^2 + {v_1}^2}{v_1 \sqrt{v^2 - {v_1}^2}} - \frac{v}{v_1} \right)$ separates the subregion O_2'', in which the reflected characteristic no longer contributes to the resolvent, from the region O_2. The tangent ray $\tau^+ = \tau_{t_n} = \xi\left(\left(v - \sqrt{v^2 - v_1^2}\right) / v_1\right)$ separates the subregion O_2''' in which both lines of the resolvent characteristic meet the light line of the boundary. In this connection, in formula (3) describing the resolvent, the factor R_u should be set equal to zero in the subregions O_2'' and O_2'''.

2. Results and discussion

Where the ratio $v_1 / v < 1$ is fulfilled in the subregion O_2', the secondary field only consists of the single Airy pulse

$$E_{O_2'}(t, x) = \frac{2v_1^2}{(v + v_1)^2}(1 + \Omega^-) \times \\ \times \mathrm{Ai}\left[-\frac{v_1}{v + v_1}\left(t - \frac{x}{v_1} - \xi + \right.\right. \\ \left.\left. + \sqrt{\xi^2 + 2\xi\frac{v}{v_1}(t - \frac{x}{v_1}) + (t - \frac{x}{v_1})^2} \right) \right], \tag{4}$$

where

$$\Omega^- = \frac{\left(\frac{v}{v_1}\xi + t - \frac{x}{v_1} \right)}{\sqrt{\xi^2 + 2\xi\frac{v}{v_1}(t - \frac{x}{v_1}) + (t - \frac{x}{v_1})^2}}.$$

Where the same ratio $v_1 / v < 1$ is true in the O_2'' subregion, the secondary field consists of two pulses as follows

$$E_{O_2''}(t, x) = \frac{v_1}{2v}(1 + \Omega^-)\mathrm{Ai}\left[-\frac{v_1}{v + v_1}\left(t - \frac{x}{v_1} - \xi + \right.\right. \\ \left.\left. + \sqrt{\xi^2 + 2\xi\frac{v}{v_1}(t - \frac{x}{v_1}) + (t - \frac{x}{v_1})^2} \right) \right] - \\ - \frac{v_1}{2v}(1 - \Omega^-)\mathrm{Ai}\left[-\frac{v_1}{v - v_1}\left(t - \frac{x}{v_1} + \xi + \right.\right. \\ \left.\left. + \sqrt{\xi^2 + 2\xi\frac{v}{v_1}(t - \frac{x}{v_1}) + (t - \frac{x}{v_1})^2} \right) \right]. \tag{5}$$

In this case, the primary waves determined by the free term E_1 are split at the boundary, and the backward waves appear in the O_2''' subregion as shown below

$$E_{O_2'''}(t, x) = \frac{v_1}{2v}(1 + \Omega^-)\mathrm{Ai}\left[-\frac{v_1}{v + v_1}\left(t - \frac{x}{v_1} - \xi + \right.\right. \\ \left.\left. + \sqrt{\xi^2 + 2\xi\frac{v}{v_1}(t - \frac{x}{v_1}) + (t - \frac{x}{v_1})^2} \right) \right] - \\ - \frac{v_1}{2v}(1 - \Omega^-)\mathrm{Ai}\left[-\frac{v_1}{v - v_1}\left(t - \frac{x}{v_1} + \xi + \right.\right. \\ \left.\left. + \sqrt{\xi^2 + 2\xi\frac{v}{v_1}(t - \frac{x}{v_1}) + (t - \frac{x}{v_1})^2} \right) \right] -$$

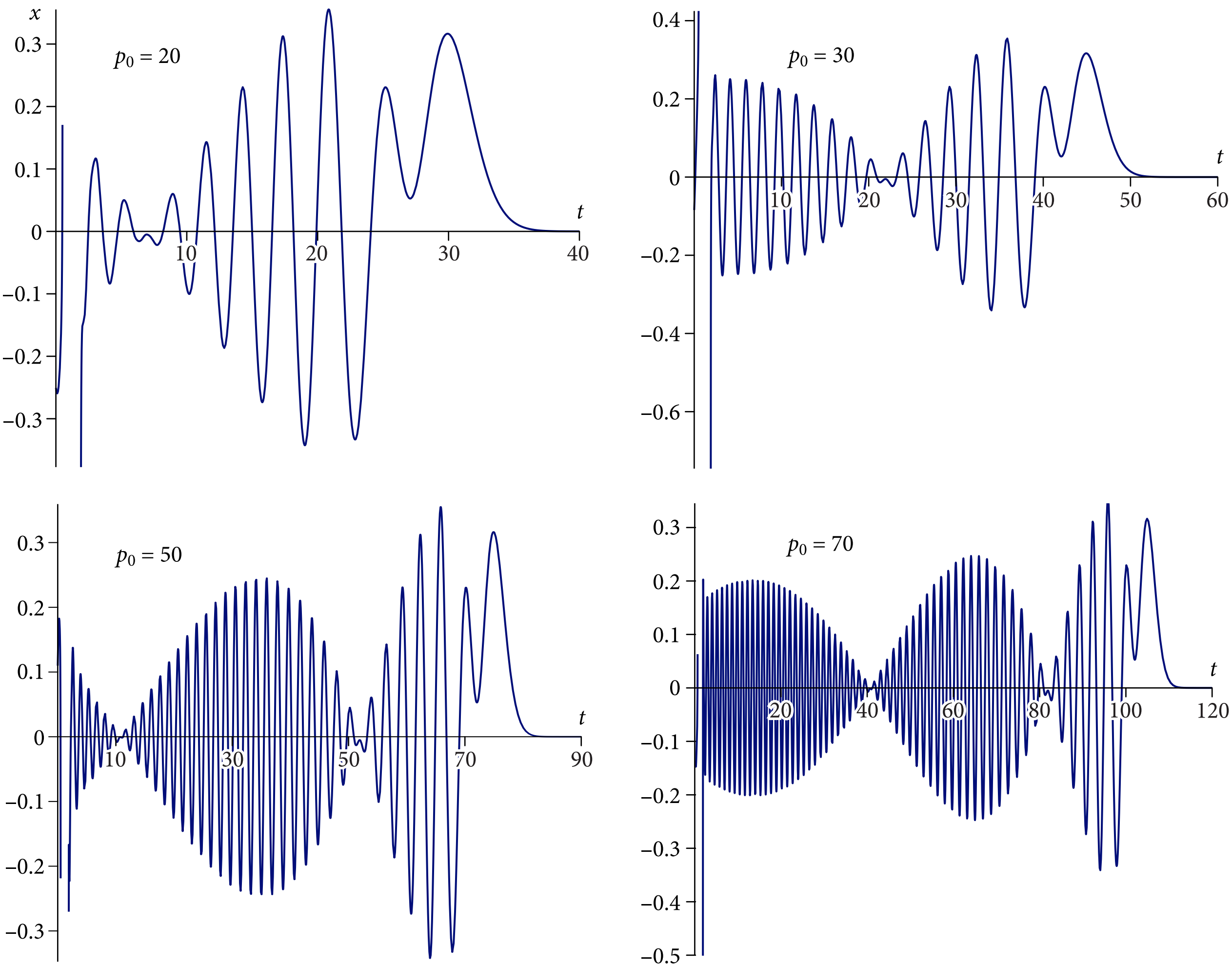

Fig. 2. Evolution of the secondary Airy pulses at different values of the starting parameter p_0 for $x/vT = 1$, $\xi = 0.1$, and $v_1/v = 0.5$

$$-\frac{v_1}{2v}(1-\Omega^+)\mathrm{Ai}\left[\frac{v_1}{v-v_1}\left(t+\frac{x}{v_1}-\xi-\right.\right.$$
$$\left.\left.-\sqrt{\xi^2-2\xi\frac{v}{v_1}(t+\frac{x}{v_1})+(t+\frac{x}{v_1})^2}\right)\right]+$$
$$+\frac{v_1}{2v}(1+\Omega^+)\mathrm{Ai}\left[\frac{v_1}{v+v_1}\left(t+\frac{x}{v_1}+\xi+\right.\right.$$
$$\left.\left.+\sqrt{\xi^2-2\xi\frac{v}{v_1}(t+\frac{x}{v_1})+(t+\frac{x}{v_1})^2}\right)\right], \qquad (6)$$

where

$$\Omega^+ = \frac{-\frac{v}{v_1}\xi+t+\frac{x}{v_1}}{\sqrt{\xi^2-2\xi\frac{v}{v_1}(t+\frac{x}{v_1})+(t+\frac{x}{v_1})^2}}.$$

The radicands in the fraction denominators in the Ω^+ and Ω^- expressions show where in corresponding formulas (4)—(6) the critical points are possible to reside. At the two points

$$\tau_1^+ = t+\frac{x}{v_1} = \frac{\xi}{v_1}\left(v+\sqrt{v^2-v_1^2}\right)$$

and

$$\tau_2^+ = t+\frac{x}{v_1} = \frac{\xi}{v_1}\left(v-\sqrt{v^2-v_1^2}\right),$$

the denominator in the Ω^+ expression turns to zero.

The denominator in the Ω^- expression turns to zero at the two points

$$\tau_1^- = t - x/v_1 = -\xi/v_1\left(v+\sqrt{v^2-v_1^2}\right)$$

and

$$\tau_2^- = t - x/v_1 = -\xi/v_1\left(v-\sqrt{v^2-v_1^2}\right).$$

The ratio $v_1/v < 1$ determines which of the two expressions to use. A careful examination of the exact

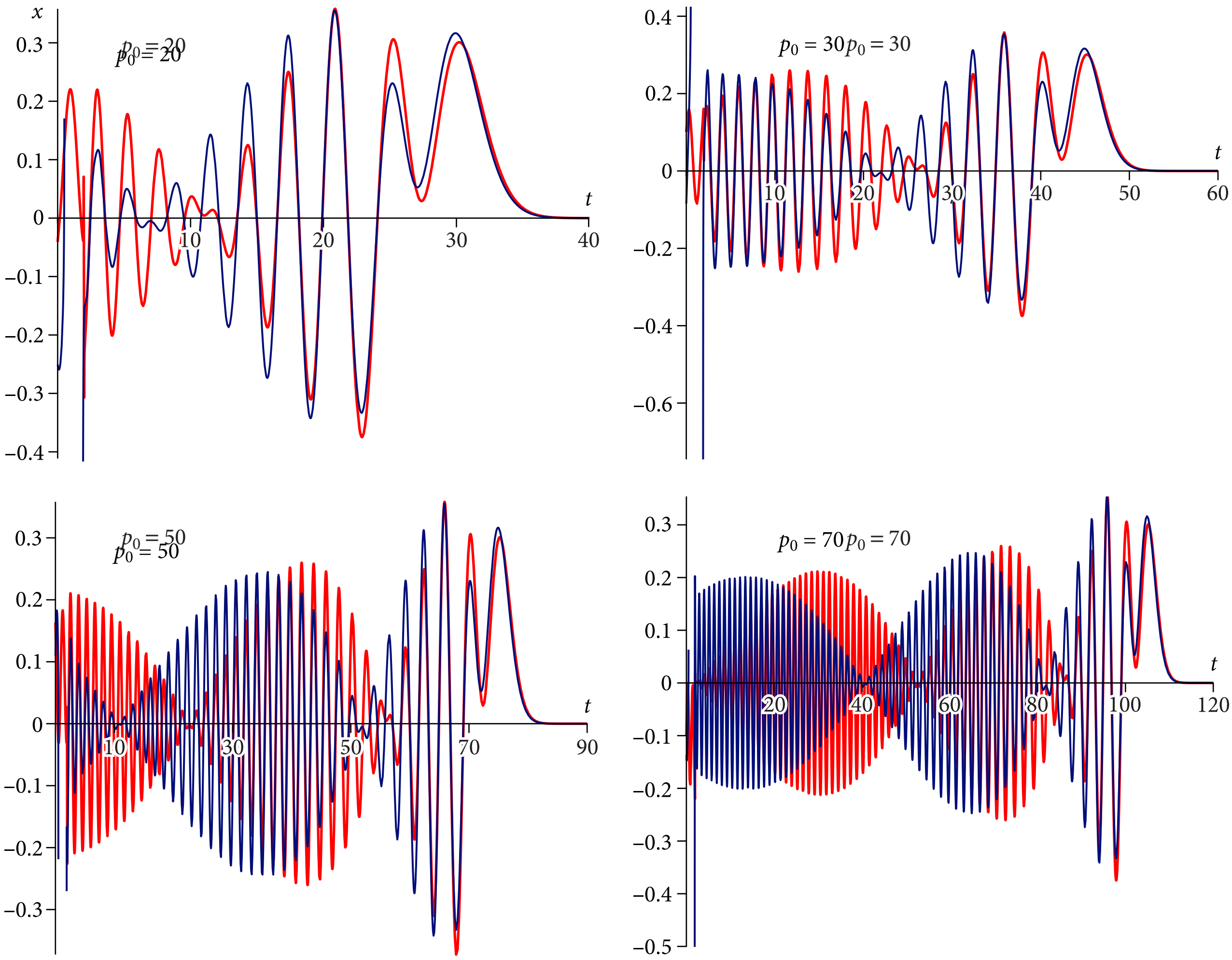

Fig. 3. Comparison of the secondary Airy pulses at different boundary accelerations and starting parameter p_0 values: $x / vT = 1$, $\xi_1 = 0.01$ (grey line), $\xi = 0.5$ (black line), and $v_1 / v = 0.5$

expressions picks up all critical points related to the problem.

The simulation modeling of the obtained results involves the so-called starting parameter $p_0 = (x_0 / v - t_0) / T$ which chases the spacetime position of the Airy pulse [13]. The starting parameter p_0 is characterized by the location x_0 of the Airy pulse generation source at a given time t_0. With the p_0 parameter negative, the pulse leading edge does not reach the boundary until time zero. We should only focus on positive p_0 values, as the main part of the Airy pulse is already deep within a medium whose dielectric constant changes at time zero. Of most interest is analysis of the Airy pulse field after the interaction with the boundary in the subregion O_2''', where backward pulses appear. The evolution of the Airy pulse propagation in the subregion O_2''' is shown in Fig. 2 for different values of the starting parameter p_0. One can see the deformation of the Airy pulse leading edge at the moment it interacts with a moving boundary. Whatever the starting parameter value, the main lobe gets bifurcation. Besides, the oscillation character of the pulse tail changes.

Figure 3 shows the Airy pulse evolution depending on the boundary velocity and for different values of the starting parameter p_0.

The simulation of the pulse field using formula (6) shows that in addition to the main lobe bifurcation and modified oscillatory behavior of the tail, the Airy pulse also experiences a phase shift. In Fig. 3, the black diagram illustrates the Airy pulse field after interacting with a moving boundary that is traveling at a velocity close to the pulse velocity. The black diagram shows a more pronounced phase shift compared to

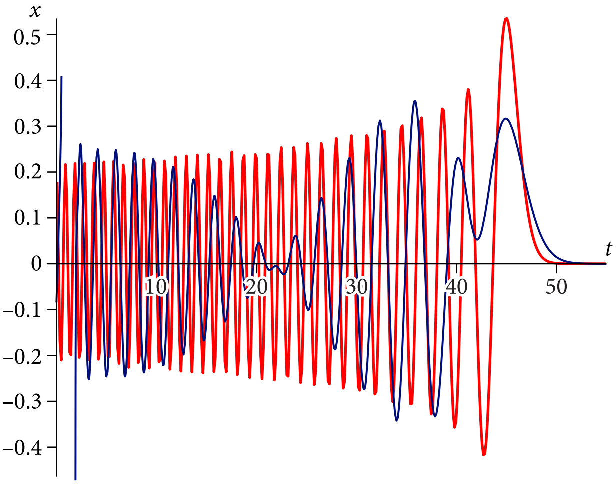

Fig. 4. Comparison of the secondary and primary fields of the Airy pulse: $p_0 = 30$, $\xi = 0.4$, $x/vT = 1$, and $v_1/v = 0.5$

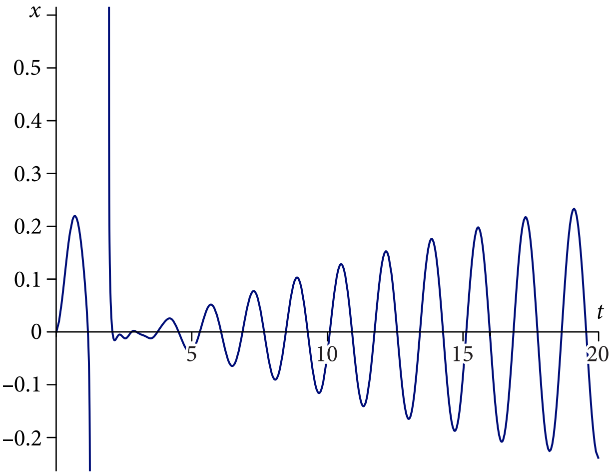

Fig. 5. Spacetime propagation of the secondary Airy pulse at the moment its velocity equals the boundary velocity, $p_0 = 30$, $x/vT = 1$, $\xi = 0.5$, and $v_1/v = 0.5$

the grey diagram, where the boundary moves much slower than the pulse. The difference in phase shifts is explained by the different (in these two, grey and black, cases) conditions of the interaction between the pulse and the moving boundary. The boundary movement, especially if at relativistic velocities, alters the nature of the wave as a source of excitation by changing the phase of the wave within the spacetime. These changes can be caused by the Doppler effect which alters the frequency, wavelength, and wave phase when the pulse interacts with a moving boundary. When the boundary moves at a relativistic velocity, additional relativistic effects occur, leading to spatial and temporal changes in the reference frame of the pulse, which further shifts the phase. Moreover, the unique dispersive properties of the Airy pulse cause different frequency components to travel at different speeds. The result is that various parts of the pulse interact with the boundary at different times, making the phase shift pattern more complex. Beyond that, when the boundary moves at a relativistic velocity, the angle at which the pulse interacts with the boundary varies and additionaly affects the phase.

Thus, the interaction of an Airy pulse with a moving boundary involves relativistic and dispersive processes that are considered crucial in causing the wave phase shift. Studying these processes will provide a deeper insight into the mechanisms of wave propagation and transformation under challenging conditions.

Figure 4 compares the initial Airy pulse and the pulse field due to the interaction with a uniformly accelerated boundary according to formula (6). The black line represents the post-interaction field of the pulse, while the grey line illustrates the initial field of the Airy pulse in the absence of the boundary.

Figure 4 shows the change of the oscillation amplitude of the pulse lobe in comparison with the primary Airy pulse. Despite the bifurcation, the size of the main lobe of the pulse remains almost unchanged.

Let us dwell on the moment the boundary velocity approaches the pulse velocity. At that instant, the field of the pulse gets a discontinuity attributed, as shown before, to the existence of the critical points. The number of the critical points within the pulse domain is connected with the numerical simulation parameters. Thus, for the simulation parameters in Figs 2—4, the observation domain (after time zero) contains a single critical point $\tau_1^+ > 0$. The other three are negative-valued and, therefore, are not included in the secondary pulse domain.

Figure 5 illustrates a discontinuity of the pulse field with the critical point presence. The result is a spacetime band where the pulse does not exist at all, the main lobe is on one side of the band, the oscillating tail is on the other. This effect can be employed to truncate the pulse tail and put to use the principal amount of the power the main lobe carries. The process is easily controlled by picking an optimal value of the starting parameter.

Thus, at the moment the boundary velocity approaches the pulse velocity, $-u = v_1$, the amplitudes of the backward waves make a jump for infinity along the line $\tau^{+} = \tau_{H}$. This is a consequence of the idealizing assumption of an infinite-power source of the boundary movement. It indicates that the actual motion cannot be at the velocity mentioned above. Therefore, the infinite power source approximation during unsteady motion of the boundary causes infinite discontinuities of the electromagnetic field. Work [14] studies peculiarities of the plane monochromatic wave conversion by a moving boundary in a non-stationary medium and also confirms that it is a non-stationarity of the boundary that causes field discontinuities and leads to the formation of new waves. When the primary field is represented by the Airy pulse, the problem is challenged by the asymmetric shape of the pulse.

Conclusions

In this study, the unique transformation features of the electromagnetic Airy pulse were examined during the pulse interaction with a relativistically moving and uniformly accelerated boundary. This study has provided us with deeper insights into the electromagnetic field behavior in non-stationary conditions which were simulated through a smooth change of the boundary velocity over time. The method of Volterra integral equations was implemented to model the interaction between Airy pulses and a relativistically moving with uniform acceleration boundary. The method admits an analytical solution to the fundamental equation. This solution is essential to further analyze and penetrate the problem. The choice of uniformly accelerated relativistic motion aims to accurately model the electromagnetic pulse interaction with a moving boundary. This approach ensures accuracy, physical sense, and practical value of the research results. Our analysis shows that when a pulse interacts with a moving boundary, substantial changes occur in spacetime and can be influenced by the parameters governing the boun-dary motion.

The obtained results provide a detailed understanding of how an electromagnetic pulse propagates and evolves under complex conditions. Furthermore, the analysis of the secondary field suggests that the field characteristics can be controlled by adjusting numerical modeling parameters, such as initial pulse position, boundary velocity, etc. This control opens new opportunities for controlling electromagnetic wave propagation across different media. The analytical solution results have been validated by numerical modeling, which confirms their validity and highlights potential applications in future research and practical scenarios. A deeper level of understanding of the Airy pulse interaction with a relativistically moving boundary has been gained, laying the groundwork for further research and practical applications in optics, photonics, and theoretical physics.

REFERENCES

1. Duggan, R.P., & Woodhouse, J.E., 2022. Relativistic wave scattering in curved spacetime: applications to cosmology. *J. Cosmol. Astropart. Phys.,* **35**(10), pp. 135—140. DOI: 10.1088/1475-7516/2022/10/010
2. Ban, Y., & Chen, X., 2023. Scattering of relativistic electrons and analogies with optical phenomena: A study of longitudinal and transverse shifts at step potentials. *Phys. Rev.,* **108**(4), pp. 1—9. DOI: 10.1103/PhysRevA.108.042218
3. King, M., Wilson, R., Bacon, E.F.J., Dolier, E.J., Frazer, T.P., Goodman, J., Gray, R.J., & McKenna, P., 2023. Perspectives on laser-plasma physics in the relativistic transparency regime. *Eur. Phys. J. A*, **59**(6), 132, 17 p. DOI: 10.1140/epja/s10050-023-01043-2
4. Whittam, M.R., Lamprianidis, A.G., Augenstein, Y., & Rockstuhl, C., 2023. Identifying regions of minimal backscattering by a relativistically moving sphere. *Phys. Rev.,* **108**(4), pp. 456—465. DOI: 10.1103/PhysRevA.108.043510
5. Xiao, Y., Zhang, C., & Zhu, Y., 2021. Propagation and control of Airy pulses in nonlinear inhomogeneous media. *Phys. Rev.,* **103**(2), pp. 1056—1065. DOI: 10.1103/PhysRevA.103.023801
6. Agarwal, P., Shrivastava, R., Kumar, S., & Sinha, S., 2020. Airy wavepackets and their interactions with dispersive and inhomogeneous media. *J. Opt. Soc. Am. B,* **37**(5), pp. 1453—1461. DOI: 10.1364/JOSAB.37.001453
7. Felsen, L.B., & Whitman, G.M., 2020. Wave propagation in time-varying media. *IEEE Trans. Antennas Propag.*, **18**(2), pp. 242—253. DOI: 10.1109/TAP.1970.1139657
8. Gaur, D.S., Mishra, A.K., 2024. Reflection and Transmission of Airy Pulse from Controllable Periodic Temporal Boundary. *Ann. Phys.*, **536**(9). DOI: 10.1002/andp.202400141
9. Hu, Y., Tehranchi, A., Wabnitz, St., Kashyap, R., Chen, Zh., and Morandotti, R., 2015. Improved Intrapulse Raman Scattering Control via Asymmetric Airy Pulses. *Phys. Rev. Lett.,* **114**(7), 073901. DOI: 10.1103/PhysRevLett.114.073901
10. Nerukh A.G., Khizhnyak, N.A., 1979. Modern problems of non-stationary macroscopic electrodynamics. *Sov. Phys. Tech. Phys.*, **49**(2), pp. 225—230.
11. Khizhnyak, N.A., 1958. Green's function of Maxwell's equations for inhomogeneous media. *Sov. Phys. Tech. Phys.*, **28**(7), pp. 1592—1609.

12. Landau, L.D., Lifshitz, E.M., 1967. *The Field Theory*. Moscow: Nauka Publ. (in Russian).
13. Nerukh, A.G., and Kuryzheva, O.V., 2018. Transformation of the airy pulse by a jump-like change of the medium permittivity in time. *Telecommunications and Radio Engineering*, **77**(12), pp. 1017—1028. DOI: 10.1615/TelecomRadEng.v77.i12.10
14. Nerukh, A., Sakhnenko, N., Benson, T., Sewell, Ph., 2013. *Non-stationary electromagnetics*. Singapore: Pan Stanford Publishing Pte., pp. 315—325.

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ПРОСТОРОВО-ЧАСОВИЙ АНАЛІЗ ЕЛЕКТРОМАГНІТНОГО
ІМПУЛЬСУ ЕЙРІ ПІСЛЯ ЙОГО ВЗАЄМОДІЇ З ПЛОСКОЮ МЕЖЕЮ,
ЩО ЗДІЙСНЮЄ РЕЛЯТИВІСТСЬКИЙ РІВНОПРИСКОРЕНИЙ РУХ

Предмет і мета роботи. Метою роботи є дослідження особливостей перетворення електромагнітного імпульсу в результаті зустрічного руху з межею, яка здійснює релятивістський рівноприскорений рух. Це випадок плавної нестаціонарності, коли швидкість межі поступово змінюється від нульового значення до значення швидкості імпульсу. Предметом дослідження є просторово-часовий розподіл і еволюція поля електромагнітного імпульсу Ейрі.

Методи та методологія. Дослідження та аналіз вищеописаного явища проводиться методом інтегральних рівнянь Вольтерра, який дозволяє описувати поширення електромагнітних хвиль у неоднорідному нестаціонарному середовищі. У рамках цього методу сформульовано базову початково-граничну електродинамічну задачу про випромінювання джерела електромагнітних хвиль у неоднорідному нестаціонарному середовищі з урахуванням відповідних початкових і граничних умов. Описано метод резольвенти для розв'язання інтегрального рівняння Вольтерра другого роду, перевага використання якого полягає в отриманні аналітичного розв'язку рівняння та в універсальності відносно вибору первинного поля.

Результати. У результаті проведених досліджень отримано аналітичні розв'язки вихідного інтегрального рівняння, аналіз яких показує наявність особливостей у виразах для вторинного поля та можливість їхнього контролю шляхом підбору параметрів чисельного моделювання. Також проведено аналітичний аналіз отриманих особливостей і проілюстровано їхню наявність і вплив на імпульс за допомогою імітаційного моделювання з використання стартового параметру, який характеризує розташування імпульсу Ейрі в певний момент часу.

Висновки. У цій роботі досліджено взаємодію електромагнітного імпульсу Ейрі з межею, що рухається з релятивістським рівноприскоренням. Використано метод інтегральних рівнянь Вольтерра, що дозволяє отримати аналітичні розв'язки. Аналіз показав суттєві зміни в просторі й часі з можливістю контролю характеристик вторинного поля через налаштування параметрів моделювання. Результати підтверджено чисельним моделюванням, що закладає основу для подальших досліджень у цій галузі.

Ключові слова: *імпульс Ейрі, електромагнітні хвилі, інтегральні рівняння Вольтерра, резольвента, рівноприскорений рух, релятивістський рух.*