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CHAOS AND FREQUENCY TRANSFORMATION IN SYSTEMS OF COUPLED OSCILLATORS

Chaotic instabilities and frequency transformations caused by the interaction of oscillators are important effects for many applications. We review these effects from the point of view of their influence on the dynamics of practical electronic systems. It is demonstrated that the interaction of high-frequency (*HF*) and low-frequency (*LF*) oscillations can result in the development of chaotic oscillations even in the quasilinear limit creating a threat for the stability of many electronic devices. This result is illustrated by considering the destruction of both trains of pulses in a nonlinear *RLC*-circuit and a harmonic oscillation in a two-mode system. In its turn, the *LF* to *HF* transformations that occur in multi-mode systems can be used for the development of novel types of generators. We illustrate this approach by considering the dynamics of an ensemble of linear oscillators with controlled coupling. A possible practical realization of such generator by using an antenna array is proposed. Fig. 4. Ref.: 15 titles.

Key words: electromagnetic oscillation, frequency transformation, chaotic instability, multi-mode systems, generator.

The interaction of two or more oscillations or modes is a well-known effect in the theory of oscillations and its applications. The most studied systems are ones with the resonant interaction of oscillations, when the following condition is met [1, 2]:

$$\omega_1/\omega_2 \approx n/m, \qquad (1)$$

where ω_1 and ω_2 are external frequencies or the natural frequencies of interacting modes, n and m are comparatively small integers. The interaction of oscillations with substantially different frequencies, when

$$\omega_1 >> \omega_2 \qquad (2)$$

has not been studied such deeply. Nevertheless, during the last decades a number of novel results have been obtained in this direction what clearly indicates a crucial role of such interaction on the dynamics of many systems. In this paper, we review these results considering the effects caused by such interaction from the point of view of applications. Moreover, to a large extent arbitrary, we divide these effects into harmful and beneficial. For example, the interaction of high-frequency (*HF*) and low-frequency (*LF*) oscillations can lead to arrising of a chaotic instability in electronic circuits, which is an undesiarable effect for most practical situations. The destruction of modulated oscillations in nonlinear *RLC*-circuits, cavities, or amplifiers [3–6] is an example of the manifestation of such instability. The remarkable feature of the instability is that above devices being stable in the case of a harmonic input signal loose their stability when the signal is modulated at a low frequency. In the next section, we will illustrate this phenomenon by considering a transition of a train of pulses via a nonlinear *RLC*-circuit.

There is a variety of roads to chaotic inabilities in multimode systems due to the considered *HF* and *LF* interaction [7–10]. For example, a harmonic forcing can initiate the chaos onset in a weakly nonlinear two-mode system with the natural frequencies that differ significantly [7]. The corresponding example will be presented in Section 2.

The *LF* to *HF* transformation is another important phenomenon [11, 12]. This transformation is interesting, for example, from the point of view of the development of novel methods of the generation of electromagnetic oscillations. In this paper, we demonstrate that the generation of *HF* electromagnetic oscillations is possible by using coupled linear oscillators, for example, resonant antennas, cavities or *RLC*-circuits, which are excited at a low frequency. The corresponding results are presented in Section 3.

1. Destruction of modulated signals. Let us consider a transition of a modulated signal via a weakly nonlinear oscillator. The dynamics of the oscillator can be described by the following generalized Duffing equation:

$$\frac{d^2x}{dt^2} + \omega_0^2 x = \varepsilon\left[-2\alpha_0\omega_0\frac{dx}{dt} + \frac{8}{3}\omega_0^2\gamma x^3 + A(\varepsilon t)\sin[\omega_1 t + \varphi(\varepsilon t)]\right]. \qquad (1)$$

Here x is the generalized coordinate, ω_0 is the natural frequency, $0 < \varepsilon << 1$ is a small parameter, α_0 is the damping parameter, γ is the parameter of nonlinearity, ω_1 is the carrier frequency of the signal, $A(\varepsilon t)$ and $\varphi(\varepsilon t)$ are slow varying functions (as compared with sin ($\omega_1 t$)). The case of the principal resonance is considered what means that $\omega_1 = \omega_0 + O(\varepsilon\omega_0)$.

The above equation has been used to study the destruction of various types of modulated signals – amplitude-modulated, frequency-modulated, and trains of pulses [3–6]. We will review here the results [5] related to the destruction of trains of pulses what is actual for digital communication lines.

The application of the secondary averaging technique [13] to (1) enables to find analytical conditions for the chaotic instability onset [5]. For example, the critical value of the pulse intensity A_{0cr}^2, which can cause the destruction of a train of rectangular *RF*-pulses, is determined by the expression

$$A_{0cr}^2 = \frac{\pi^3 \alpha_0}{2\gamma T_p T_m \sin(\pi T_p / T_m)}. \qquad (2)$$

Here A_0 is the amplitude of the pulses, T_p is their duration, T_m is the period of the train. The minimum value of A_{0cr}^2 is achieved when

$$T_p / T_m \cong 2/\pi = 0.64.$$

For such T_p / T_m, the minimum intensity for the chaos to arise is

$$A_{0cr}^2 = 84 \frac{\alpha_0}{\gamma T_m^2}.$$

Therefore, the threshold for the instability is lowering with reducing losses in the system and with increasing both the period of modulation and the nonlinearity parameter.

2. Dynamics of two-mode system. In this section, we review the dynamics of a harmonically forced two-mode system with essentially different natural frequencies [7]. Under rather general conditions, such system can be described by the following equations

$$\frac{d^2 x_{HF}}{d\tau^2} + x_{HF} = -2\varepsilon\mu_1 \frac{dx_{HF}}{d\tau} - \varepsilon(2\gamma x_{HF} x_{LF} - S\cos\nu\tau),$$
$$\frac{d^2 x_{LF}}{d\tau^2} + \varepsilon^2 x_{LF} = -2\varepsilon\mu_2 \frac{dx_{LF}}{d\tau} - \varepsilon^2 \gamma x_{HF}^2. \qquad (3)$$

Here x_{HF} and x_{LF} are variables describing the *HF* and *LF* oscillators with the natural frequencies ω_{HF} and ω_{LF}, correspondingly, μ_1 and μ_2 represent damping in the *HF* and *LF* oscillators, correspondingly, γ is the coefficient of nonlinearity, τ is slow time, S is the amplitude of the external periodic forcing at the carrier frequency ν, which is close to the natural frequency of the *HF* oscillator, $\varepsilon = \omega_{LF} / \omega_{HF} << 1$ is a small parameter.

A possible physical realization of the above system is shown in Fig. 2. Here L_1, R_1, and C_1 represent a *HF* circuit, which is resonantly driven by an external harmonic force. L_2, R_2, and C_2 represent a *LF* circuit. It was generally believed that, if the conditions $L_1 \approx \varepsilon L_2, C_1 \approx \varepsilon C_2$ are met, the influence of the *LF* circuit on the dynamics of the whole system can be neglected. However, as it was shown in [7], such influence can have a crucial effect.

The first equation of (3) can be considered as the motion equation of a quasilinear oscillator. So one can look for a solution in the form:

$$x_{HF} = a(\varepsilon\tau)\cos[\nu\tau + \varphi(\varepsilon\tau)], \qquad (4)$$

where $a(\varepsilon\tau)$ and $\varphi(\varepsilon\tau)$ are the slowly varying amplitude and phase of the *HF* oscillation. After application of the averaging technique [13], we have the following system of the averaged equations:

$$\dot{a} = -\mu_1 a - S\sin\varphi,$$
$$\dot{\varphi} = -\Delta + \gamma u - \frac{S}{a}\cos\varphi,$$
$$\dot{u} = v,$$
$$\dot{v} = -2\mu_2 v - u - \frac{1}{2}\gamma a^2. \qquad (5)$$

The overdot here denotes the differentiation with respect to the slow time $\varepsilon\tau$, $v \equiv \dot{x}_{LF}$, $u \equiv x_{LF}$ are independent variables, and $\Delta = (\nu^2 - 1)/(2\varepsilon\nu)$ is the parameter of the frequency mismatch.

Let us describe qualitative changes in the dynamics of the system (5) with the increase of the forcing amplitude S. When the S-value is small enough, only stable equilibriums exist in the system. The first qualitative change is associated with the Hopf bifurcation occurring in some range of the frequency mismatch Δ, when the amplitude S is above some critical value S_{cr}. Due to this bifurcation, the forced *HF* oscillations become amplitude- and phase-modulated at a frequency, which is close to the natural frequency of the *LF* circuit. At the same time, a *LF* component arises in the system, which corresponds to the energy transfer from the high frequencies to the low ones. The critical value of the amplitude (S_{cr}) can be found from the analysis of the stability conditions. It is possible to show that the minimum value of S_{cr} is

$$S_{\min} \cong \frac{2}{\gamma}\sqrt{\mu_1 \mu_2}, \qquad (6)$$

when $\Delta \approx 1$.

The relation $\Delta \approx 1$ can be considered as a resonance condition necessary for the effective interaction between the oscillators. Note also that this condition, written in terms of dimensional frequencies, takes the form: $\omega \approx \omega_1 + \omega_2$. Hence, we can conclude that the instability appears here due to the decay mechanism. In this case the quantum $\hbar\omega$ of the input signal breaks down into two quanta with the frequencies close to the natural frequencies of the circuits (ω_1 and ω_2). The oscillations with the frequencies ω and ω_1 fall within the bandwidth of the *HF* oscillator, resulting in the excitation of a quasiperiodic oscillation. This is the primary reason for the complexity in the system dynamics.

It should be noted that the destruction of the quasiperiodic oscillations, excited in the *HF* circuit, leads to the appearance of chaotic oscillations even under the weakly nonlinear excitation conditions. The distinguishing feature of the considered problem, as compared to two-periodically forced oscillators, is related to the fact that the oscillation with the second incommensurate frequency is induced by the external periodic force. In terms of the averaged equations (5), the destruction of the quasiperiodic oscillation is observed as a cascade of period-doubling bifurcations of the envelope of this oscillation.

Experimental investigations of the circuit shown in Fig. 1 were performed [7]. A reverse-biased varactor was used as the nonlinear element. The natural frequencies of the *HF* and *LF* circuits were: $\omega_1 = 14.47$ MHz, $\omega_2 = 0.187$ MHz, and their *Q*-factors were 130 and 80, correspondingly. It means that $\varepsilon = 0.013$. The values of other dimensionless parameters are: $\mu_1 = 0.7$, $\mu_2 = 0.02$, $\gamma = 1$.

The experimentally obtained bifurcation diagram is shown in Fig. 4. The region of chaos, boundaries of hysteretic areas, and some fragments of period-doubling lines are plotted here. We note that this structure is generally the same as the one following from the theoretical results [7].

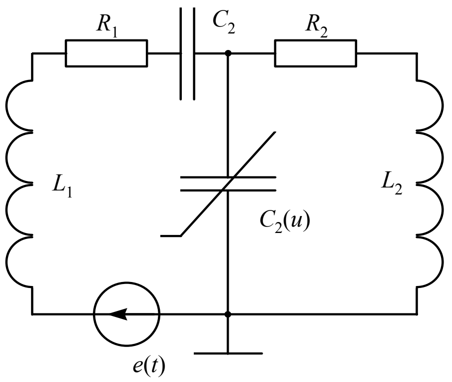

Fig. 1. An example of physical realization of the two-mode externally forced system

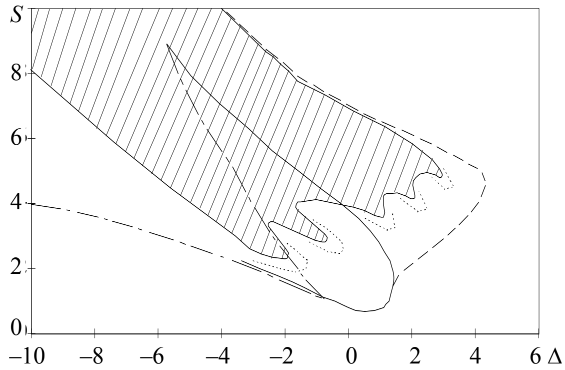

Fig. 2. Bifurcation diagrams on the parameter plane (S, Δ). Regions of chaos are shaded. Solid and dashed lines indicate borders of the soft and hard regimes of the excitation of *LF* oscillations. Dotted lines denote the period-doubling bifurcation

3. *LF* to *HF* transformation. Systems of coupled linear oscillators can be effectively used for the realization of *LF* to *HF* transformation mechanisms. One of such mechanisms was described in [14], where it was shown that the above transformation takes place in a harmonically forced system of two linear non-reciprocally coupled oscillators. In this paper, we consider an ensemble of reciprocally coupled linear oscillators [15]. It is known that such ensemble has a set of normal frequencies which differ from the partial frequencies of the individual oscillator. Provided a large number of the coupled oscillators is used, the minimal value of the normal frequencies can be much less than the partial frequencies of the oscillators. In this case, a *LF* forcing at this minimal normal frequency leads to a resonant excitation of the ensemble of the oscillators. If at some moment of time, the coupling between the oscillators will be broken, the oscillators will start to oscillate at their partial frequencies. Provided these partial frequencies are identical, the energy of the *LF* excitation is transformed to the energy of the *HF* oscillation, determined by the partial frequency.

Let us confirm the above statement by the corresponding mathematics. An ensemble of N coupled linear identical oscillators is described by the following Hamiltonian:

$$H = \sum_{i=0}^{N}\left(\frac{p_i^2}{2} + \omega_0^2\frac{q_i^2}{2}\right) + \mu q_0 \sum_{j=0}^{N} q_1,$$
$$\dot{p}_i = -\frac{\partial H}{\partial q_i}; \dot{q}_i = -\frac{\partial H}{\partial p_i}. \qquad (7)$$

Here $\mu = const$ is the coupling coefficient, ω_0 is the partial frequency of the oscillators. For simplicity, we consider the system where the oscillators are coupled with each other only across the zero oscillator.

From (7) it is easy to obtain the equations describing dynamics of the oscillators:

$$
\begin{aligned}
&\dot{q}_i = p_i, \dot{p}_i = -\omega_0^2 q_i - \mu q_0, \\
&\dot{p}_0 = -\omega_0^2 q_0 - \mu \sum_{i=0}^{N} q_i, \\
&\ddot{p}_i + \omega_0^2 p_i = -\mu p_0, \\
&\ddot{p}_0 + \omega_0^2 p_0 = -\mu \sum_{i=0}^{N} p_i.
\end{aligned} \tag{8}
$$

This system describes the behavior of the N coupled linear oscillator. Let us find the normal frequencies of such system. For this purpose we look for the solution of (8) in the form:

$$p_i = a_i \exp(i\omega t), \; a_i = \text{const}. \tag{9}$$

Substituting in (8), one can find the following dispersive equation:

$$\left(-\omega^2 + \omega_0^2\right)^2 = \mu^2 N. \tag{10}$$

From this equation we find an expression for the normal frequencies

$$\omega = \pm\omega_0 \sqrt{1 \pm \mu \frac{\sqrt{N}}{\omega_0^2}}. \tag{11}$$

It is seen, that even with a small value of the coupling coefficient, but with a large number of the oscillators, one of the normal frequencies can be very small. If now the ensemble of the oscillators (8) is excited by an external periodic force with the frequency which is close to this minimum normal frequency of the ensemble, then the amplitude of the oscillations will increase in time. In Fig. 3, the dynamics of two oscillators with different initial conditions is illustrated for an ensemble of 100 oscillators with the ratio of the minimum normal frequency to the partial frequency of 0.01. In practical systems, the increase of the amplitude will be limited by dissipation and nonlinear effects.

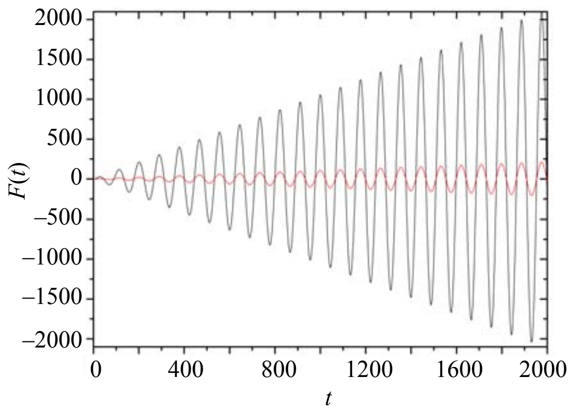

Fig. 3. Dynamics of the ensemble of oscillators with external periodic forcing

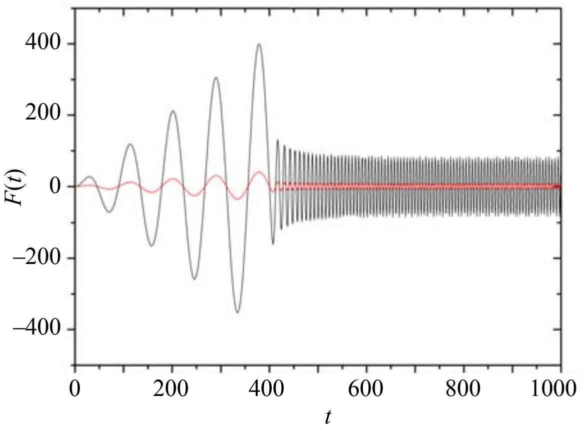

Fig. 4. Dynamics of oscillators after breaking the coupling

The system dynamics after breaking of the coupling is illustrated in Fig. 4. The oscillators start to oscillate at their partial frequencies. So, we observe the *LF* to *HF* transformation. The important feature is that after the coupling breaking the oscillators oscillate coherently. This enables to organize a coherent radiation and an efficient summation of the power of individual oscillators.

One of possible practical realizations of the described frequency transformation mechanism is as follows. Let us use a resonant antenna as the oscillator. By creating an antenna array with such coupled antennas we will have an ensemble of oscillators. In order to realize the coupling breaking, and electrically controlled switches can be introduced. Exciting such antenna array with low frequency pulses it is possible to produce and directly radiate *HF* pulses by switching off and on the switches.

Conclusions. The results of the theoretical and experimental investigations accumulated to date support the viewpoint that that interaction of low- and high frequency oscillations can exert a strong influence on the dynamics of various systems. Such interaction can initiate arising of chaotic instabilities in single- and multi-mode systems. These instabilities arise even in the weakly nonlinear limit. Mathematically, the mechanism of the interaction of high- and low-frequencies is described in terms of second-order resonances. Such description provides theoretical results that are in good agreement with the corresponding experimental data.

Coupled linear oscillators can be effectively for the development novel types of generators of *RF*-oscillations by means of a *LF* to *HF* transformation. Such transformation can be realized by using nonreciprocally coupled oscillators [14] or by using reciprocally coupled oscillators with controlled coupling.

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ХАОС И ПРЕОБРАЗОВАНИЕ ЧАСТОТЫ В СИСТЕМАХ СВЯЗАННЫХ ОСЦИЛЛЯТОРОВ

Хаотические неустойчивости и преобразования частоты, обусловленные взаимодействиями осцилляторов, являются важными эффектами для многих приложений. Мы проводим обзор этих эффектов с точки зрения их влияния на динамику реальных электронных систем. Показано, что взаимодействие высокочастотных (ВЧ) и низкочастотных (НЧ) колебаний может приводить к возникновению хаотических колебаний даже в квазилинейном пределе, что представляет угрозу для многих электронных приборов. Этот результат обусловлен анализом разрушения последовательности импульсов в нелинейном колебательном контуре, а также гармонического колебания в двухмодовой системе. В свою очередь, преобразования НЧ- в ВЧ-колебания, которые происходят в многомодовых системах, могут использоваться для создания новых типов генераторов, что иллюстрирует анализ динамики ансамбля линейных осцилляторов с контролируемой связью. Предложена возможная практическая реализация такого генератора на основе антенной решетки.

Ключевые слова: электромагнитные колебания, преобразование частоты, хаотическая неустойчивость, многомодовые системы, генератор.

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ХАОС ТА ПЕРЕТВОРЕННЯ ЧАСТОТИ В СИСТЕМАХ ЗВ'ЯЗАНИХ ОСЦИЛЯТОРІВ

Хаотичні нестійкості та перетворення частоти, які зумовлені взаємодіями осциляторів, є важливими ефектами для багатьох застосувань. Ми проводимо огляд цих ефектів з точки зору їхнього впливу на динаміку реальних електронних систем. Показано, що взаємодія високочастотних (ВЧ) і низькочастотних (НЧ) коливань може призводити до виникнення хаотичних коливань навіть у квазілінійній межі, що становить загрозу для багатьох електронних приладів. Цей результат зумовлено аналізом руйнування послідовності імпульсів у нелінійному коливальному контурі, а також гармонічного коливання у двомодовій системі. У свою чергу, перетворення НЧ- у ВЧ-коливання, які відбуваються в багатомодових системах, можуть використовуватися для створення нових типів генераторів, що ілюструє аналіз динаміки ансамблю лінійних осциляторів з контрольованим зв'язком. Запропоновано можливу практичну реалізацію такого генератора на основі антенної решітки.

Ключові слова: електромагнітні коливання, перетворення частоти, хаотична нестійкість, багатомодові системи, генератор.