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Generalization of the mode-matching technique to the problems of scattering by semi-infinite slow-wave structures

Subject and Purpose. *The scattering matrix of a semi-infinite slow-wave structure formed by grooves in a rectangular waveguide is investigated with a view to developing a calculation technique for a semi-infinite periodic grating.*

Methods and Methodology. *The mode-matching technique is generalized to semi-infinite periodic structures. The fields of the periodic part of the structure are series expanded in terms of periodic structure eigenmodes, which on imposing the boundary condition at infinity yields the linear matrix equation for finding the scattering matrix. Only propagating modes of the periodic structure are considered. To be sure that the field representations are reliable, the field matching is performed at a period somewhat distant from the junction of the regular and periodic waveguides.*

Results. *Matrix equations have been obtained for determining the scattering matrix blocks corresponding to the semi-infinite grating. The reliability of these equations has been checked through a number of investigations, including tests for convergence, reciprocity, energy balance and a test for scattering matrix conservation once one period is added to the semi-infinite structure. For the main confirmation, the scattering matrix of a finite fragment of the slow-wave structure was calculated in two ways to compare: through the scattering matrices of the semi-infinite slow-wave structure and through a cascade assembly of the scattering matrices of the waveguide elements making up the structure.*

Conclusion. *An algorithm of the scattering matrix calculation for a semi-infinite grating has been obtained. It can be used in building a rigorous hot model for vacuum electronics devices with slow-wave structures involved. Fig. 8. Ref.: 9 items.*

Key words: *semi-infinite grating, slow-wave structure, mode-matching technique.*

The problem of finding the scattering matrix of a semi-infinite periodic structure is relevant in the calculation of complex resonators of vacuum devices using slow-wave structures. It is important here to have the field expansion of the field in eigenmodes of the slow-wave structure since the electron beam interacts only with one of them. Despite the fact that the field for any finite fragment of the slow-wave structure can be found by widely used mode-matching technique and the method of generalized scattering matrices [1–3], this information is insufficient to build a hot model of the device. It is also not enough to expand this field in series of the eigenmodes of the slow-wave structure since this approach does not allow taking into account the change in the amplitude of such a mode when interacting with the electron beam. It is necessary to know the scattering matrix of the semi-infinite slow-wave structure. It will allow one to take into account the reflection, transmission and transformation of the modes of the periodic structure at the boundaries of the slow-wave structure.

This waveguide problem is equivalent to the problem of scattering by a periodic half-space. A few solution approaches to the problem are known.

In [4], the problem of scattering of a plane wave by a half-space periodically filled with plasmonic nanospheres is considered. The problem is solved by the discretized Wiener-Hopf method.

In works [5, 6], an approach to solve the problem of scattering by a half-space filled with strip gratings is proposed. The problem is reduced to a nonlinear matrix equation that has a non-unique solution. This problem arises in connection with the absence of the requirement of the condition at infinity for the solution of the nonlinear equation. Only one of the solutions satisfies the condition at infinity. The choice of the required physical solution turns out to be difficult when considering the structure in multimode regime.

In [7], the problem is solved more rigorously. The field in the periodic half-space is expanded in series of the pre-calculated eigenmodes of the periodic structure which transfer energy from the junction with the free half-space. This provided the only solution satisfying the condition at infinity.

The present work uses an approach similar to [7]. The difference is that, first, the waveguide problem for a semi-infinite slow-wave structure is considered. Second, the method proposed here is constructed in such a way that it is sufficient to take into account only the propagating modes of the slow-wave structure, which reduces the size of the inverted matrices.

The method uses the scattering matrix of a finite fragment of the slow-wave structure. It is based on the application of the mode-matching technique and the generalized scattering matrix method. It was implemented and used [8] to build a hot model of vacuum electronics devices without a detailed description. This work is devoted to the description of the method and some features of its implementation.

1. Description of the method. The structure under investigation is shown schematically in Fig. 1. It is a junction of a regular waveguide with a semi-infinite periodic structure with a period l. Generally speaking, the regular waveguide can be arbitrary, as well as the elements of the periodic structure. In this paper, as an example, we consider the junction of a rectangular waveguide with a semi-infinite lamellar grating.

Let the basis of eigenmodes of a periodic structure be known. The method for finding them is described in [9]. And let a mode with index p be incident from port 0 corresponding to the regular waveguide. Then at some distance equal to an integer number of the periods $L = ql$ from the junction, the field can be expanded in the series of modes

$$\vec{E}(x,y,z) = \sum_{n=1}^{N} A_{np}\vec{E}_n(x,y,z). \qquad (1)$$

Here $\vec{E}_n(x,y,z)$ is the field of the n^{th} eigenmode of the periodic structure and A_{np} are the unknown coefficients.

The following nuance is important here. An eigenmode of the slow-wave structure can be backward. The sign of the propagation constants should be chosen so that the corresponding modes transfer energy from the junction with the regular waveguide to infinity. This choice determines the solution that satisfies the condition at infinity.

If q is large enough, then it is possible to leave only propagating modes in the basis. The addition of evanescent modes to the basis increases the accuracy of the field expansion at small q.

Let us introduce the following designations: S is the scattering matrix of the semi-infinite slow-wave structure, P is the number of waves taken into account at port 0 corresponding to the regular waveguide, N is the number of waves at port 1 (modes of the slow-wave structure), M is the number of waves in the groove.

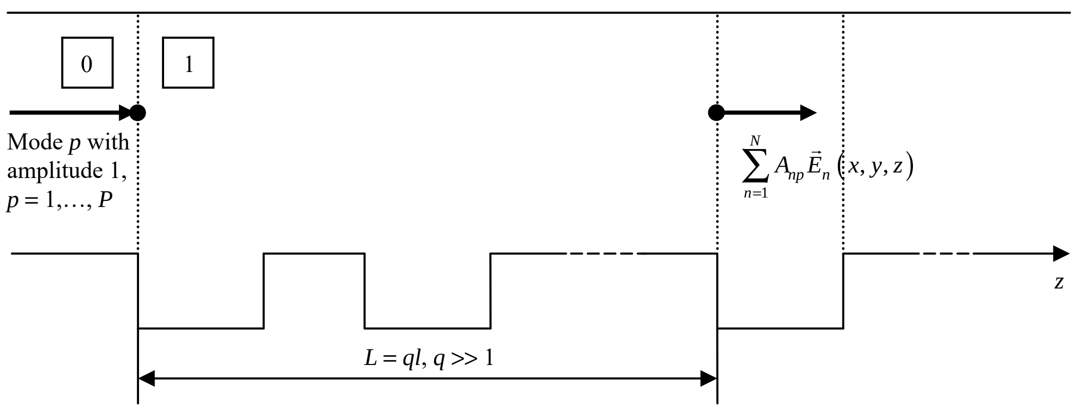

Fig. 1. Geometry of the problem

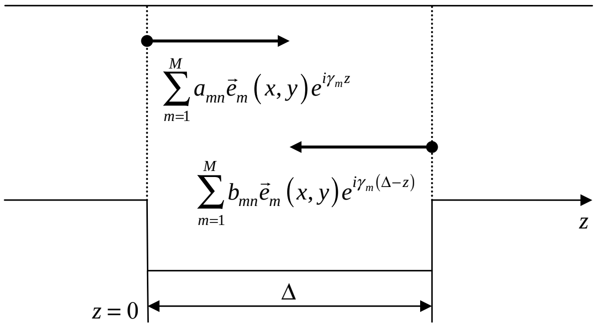

Fig. 2. Expansion of the *n*th eigenmode field in waveguide modes of the groove

Then the coefficients A_{np} are related to the elements of the transmission matrix of the semi-infinite structure by the relation

$$A_{np} = S_{np}^{(1,0)} e^{iL\Gamma_n}, \tag{2}$$

where Γ_n is the propagation constant of the *n*th eigenmode of the slow-wave structure.

In the process of finding the eigenmodes of the periodic slow-wave structure according to the approach described in [9], the representations of their fields in the grooves are obtained (Fig. 2)

$$\vec{E}_n(x,y,z) = \sum_{m=1}^{M} a_{mn}\vec{e}_m(x,y)e^{i\gamma_m z} + \sum_{m=1}^{M} b_{mn}\vec{e}_m(x,y)e^{i\gamma_m(\Delta-z)}. \tag{3}$$

Here γ_m is the propagation constant of the groove mode of number *m*, $\vec{e}_m(x,y)$ is its transverse field distribution, Δ is the groove width, a_{mn} and b_{mn} are the amplitudes of the waveguide modes in the groove, propagating or evanescing in opposite directions.

Substituting (3) into (1), we obtain the representation of the total field in the groove $q+1$ through the waveguide modes of the groove

$$\vec{E}(x,y,z) = \sum_{m=1}^{M} \vec{e}_m(x,y)e^{i\gamma_m z} \underbrace{\sum_{n=1}^{N} a_{mn}A_{np}}_{\alpha_{mp}} + \sum_{m=1}^{M} \vec{e}_m(x,y)e^{i\gamma_m(\Delta-z)} \underbrace{\sum_{n=1}^{N} b_{mn}A_{np}}_{\beta_{mp}}. \tag{4}$$

Consider an auxiliary waveguide unit, which is *q* periods of the slow-wave structure, with input port 0 corresponding to the lamella and output port 1 corresponding to the end of the groove (Fig. 3). Let us denote its scattering matrix by *s*.

The first series in (4) can be considered as the result of scattering of the *p*th wave incident on this element from port 0 and the combination of waves determined by the second series in (4) from port 1. Thus, we get

$$\alpha_{mp} = \sum_{k=1}^{M} s_{mk}^{(1,1)} e^{i\gamma_k\Delta}\beta_{kp} + s_{mp}^{(1,0)},$$
$$m = 1,...,M, \quad p = 1,...,P. \tag{5}$$

Substituting the representations of the amplitudes α_{mp} and β_{mp} from (4) into (5), we obtain

$$\sum_{n=1}^{N} a_{mn}A_{np} = \sum_{k=1}^{M} s_{mk}^{(1,1)} e^{i\gamma_k\Delta} \sum_{n=1}^{N} b_{kn}A_{np} + s_{mp}^{(1,0)},$$
$$m = 1,...,M, \quad p = 1,...,P.$$

or in the matrix form

$$\underbrace{(a - s^{(1,1)}\hat{\varphi}b)}_{M\times N} \underbrace{A}_{N\times P} = \underbrace{s^{(1,0)}}_{M\times P}, \tag{6}$$

where $\hat{\varphi} = diag\left\{e^{i\gamma_k\Delta}\right\}_{k=1}^{M}$.

We have $M \times P$ equations for $N \times P$ unknowns. Since $M > N$ due to the need to take into account evanescent modes in the grooves, the number of equations is larger than the number of unknowns.

There are two options for solving such a matrix equation.

Option 1.

We keep the first *N* rows of the matrices $D = a - s^{(1,1)}\hat{\varphi}b$ and $s^{(1,0)}$.

Option 2.

By left-multiplying of (6) by D^* we obtain

$$\underbrace{D^*D}_{N\times N} \underbrace{A}_{N\times P} = \underbrace{D^*s^{(1,0)}}_{N\times P}.$$

Hence it appears

$$A = (D^*D)^{-1}D^*s^{(1,0)}.$$

Using (2), we obtain

$$S^{(1,0)} = \hat{\Phi}^{-1}(D^*D)^{-1}D^*s^{(1,0)},$$

where $\hat{\Phi} = diag\left\{e^{i\Gamma_n L}\right\}_{n=1}^{N}$.

In this work, the second option is implemented.

We can elementarily obtain

$$S^{(0,0)} = s^{(0,0)} + s^{(0,1)}\hat{\varphi}b(D^*D)^{-1}D^*s^{(1,0)}.]$$

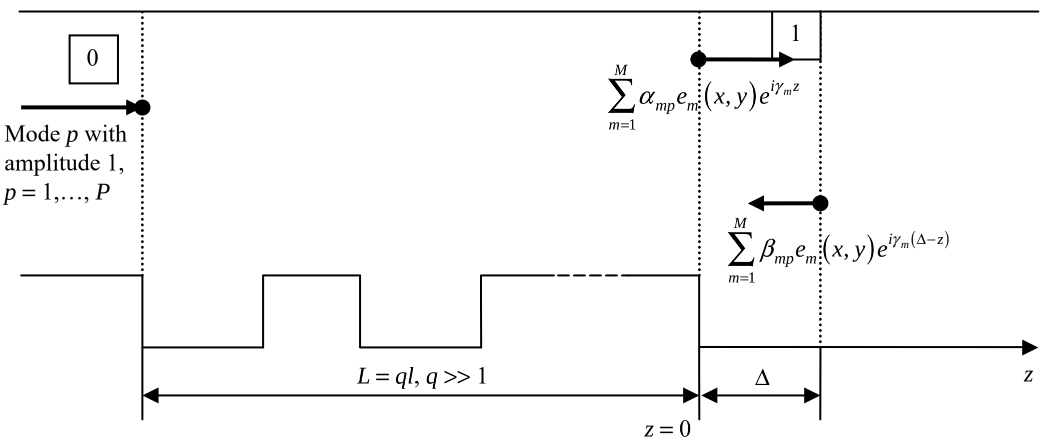

Fig. 3. The auxiliary waveguide unit

By analogous constructions, we obtain the matrix expressions for the remaining blocks of the scattering matrix of the semi-infinite structure:

$$S^{(1,1)} = \hat{\Phi}^{-1}(D^{*}D)^{-1}D^{*}(s^{(1,1)}\hat{\varphi}a - b)\hat{\Phi}^{-1},$$

$$S^{(0,1)} =$$

$$= s^{(0,1)}\hat{\varphi}(a + b(D^{*}D)^{-1}D^{*}(s^{(1,1)}\hat{\varphi}a - b))\hat{\Phi}^{-1}.$$

2. Dispersion analysis of a slow-wave structure. *2.1. Calculation of the group velocity of an eigenmode of a slow-wave structure.* An important characteristic of an eigenmode of a slow-wave structure is its group velocity v_g. In particular, its sign makes it possible to determine the direction of the energy transfer by the mode and, therefore, the sign of the propagation constants in representation (1). It can be calculated in the two ways:

1) through the angle α of the tangent to the dispersion curve

$$v_g = \frac{d\omega}{d\Gamma} = 2\pi l \frac{df}{d\varphi} = \frac{2\pi l}{\mathrm{tg}\,\alpha}, \qquad (7)$$

where φ is the phase incursion of the mode per period, and

2) through the average power flow P and the energy W stored per period according to the formula

$$v_g = Pl / W.$$

The second approach is preferable since the calculation of the derivative of the dispersion dependence in (7) reduces the calculation accuracy.

The average power flow of the eigenmodes of the slow-wave structure through the cross section can be calculated from their mode expansions in the series of regular waveguide modes

$$P = \frac{1}{2}\mathrm{Re}\int_S \left[\vec{E} \times \vec{H}^{*}\right] \cdot d\vec{s} =$$

$$= \frac{1}{2}\sum_{n=1}^{N}\left[|a_n|^2 - |b_n|^2\right] -$$

$$- \sum_{n=N+1}^{\infty} (\pm 1)\,\mathrm{Im}\,(a_n^{*}b_n)\exp(-|\gamma_n|\Delta).$$

Here the index $n = 1, 2, ..., N$ corresponds to the propagating modes of the waveguide ($\mathrm{Im}\,\gamma_n = 0$). In the second series, the + sign corresponds to the E modes, the – sign corresponds to the H and T modes.

The power flow of the eigenmode of the slow-wave structure is contributed not only by the propagating but also by the evanescent modes of the waveguide if the expansion contains the same modes evanescing in the opposite direction.

This formula is also used to normalize the eigenmodes of the slow-wave structure. The normalization is chosen in the same way as for the modes of regular waveguides so that

$$P = \frac{1}{2}\mathrm{Re}\int_S \left[\vec{E} \times \vec{H}^{*}\right] \cdot d\vec{s} = \frac{1}{2}.$$

To calculate the average energy stored per period, it is necessary to sum up its values for each section of the period formed by the regular waveguide.

Let us derive the formula for the average energy W stored in the volume V of the groove

$$W = \frac{1}{4}\int_V (\varepsilon_0 \varepsilon \vec{E} \cdot \vec{E}^{*} + \mu_0 \mu \vec{H} \cdot \vec{H}^{*})\,dv.$$

By substituting the analytic integrals, we obtain

$$W=\frac{\varepsilon_0\varepsilon}{4}\left\{\begin{aligned}&\sum_{n=1}^{N}|W_n|\left[\left(|a_n|^2+|b_n|^2\right)\Delta+2\,\mathrm{Re}(a_nb_n^*)\frac{\sin(\gamma_n\Delta)}{\gamma_n}\right]+\\&+\sum_{n=N+1}^{\infty}|W_n|\left[\left(|a_n|^2+|b_n|^2\right)\frac{1-\exp(-2|\gamma_n|\Delta)}{2|\gamma_n|}+2\Delta\,\mathrm{Re}(a_nb_n^*)\exp(-|\gamma_n|\Delta)\right]+\\&+\sum_{n=1}^{N}\delta_{ne}\frac{\chi_n^2|W_n|}{\gamma_n^2}\left[\left(|a_n|^2+|b_n|^2\right)\Delta-2\,\mathrm{Re}(a_nb_n^*)\frac{\sin(\gamma_n\Delta)}{\gamma_n}\right]+\\&+\sum_{n=N+1}^{\infty}\delta_{ne}\frac{\chi_n^2|W_n|}{|\gamma_n|^2}\left[\left(|a_n|^2+|b_n|^2\right)\frac{1-\exp(-2|\gamma_n|\Delta)}{2|\gamma_n|}-2\Delta\,\mathrm{Re}(a_nb_n^*)\exp(-|\gamma_n|\Delta)\right]\end{aligned}\right\}+$$

$$+\frac{\mu_0\mu}{4}\left\{\begin{aligned}&\sum_{n=1}^{N}\frac{1}{|W_n|}\left[\left(|a_n|^2+|b_n|^2\right)\Delta-2\,\mathrm{Re}(a_nb_n^*)\frac{\sin(\gamma_n\Delta)}{\gamma_n}\right]+\\&+\sum_{n=N+1}^{\infty}\frac{1}{|W_n|}\left[\left(|a_n|^2+|b_n|^2\right)\frac{1-\exp(-2|\gamma_n|\Delta)}{2|\gamma_n|}-2\Delta\,\mathrm{Re}(a_nb_n^*)\exp(-|\gamma_n|\Delta)\right]+\\&+\sum_{n=1}^{N}\delta_{nh}\frac{\chi_n^2}{\gamma_n^2|W_n|}\left[\left(|a_n|^2+|b_n|^2\right)\Delta+2\,\mathrm{Re}(a_nb_n^*)\frac{\sin(\gamma_n\Delta)}{\gamma_n}\right]+\\&+\sum_{n=N+1}^{\infty}\delta_{nh}\frac{\chi_n^2}{|\gamma_n|^2|W_n|}\left[\left(|a_n|^2+|b_n|^2\right)\frac{1-\exp(-2|\gamma_n|\Delta)}{2|\gamma_n|}+2\Delta\,\mathrm{Re}(a_nb_n^*)\exp(-|\gamma_n|\Delta)\right]\end{aligned}\right\}.$$

Here χ_n is the transverse wavenumber of the waveguide mode, W_n is its waveguide impedance,

$$\delta_{ne}=\begin{cases}1, & \text{for } E\text{-modes},\\ 0, & \text{for other modes},\end{cases}$$

$$\delta_{nh}=\begin{cases}1, & \text{for } H\text{-modes},\\ 0, & \text{for other modes}.\end{cases}$$

The formula for the average energy stored in the volume above the lamella has the same form, with the only difference that in it Δ must be replaced by $l-\Delta$.

2.2. An example of the dispersion analysis of a slow-wave structure. As an example of a slow-wave structure, consider a lamellar grating in a 7.2×0.8 mm^2 rectangular waveguide. Grooves 0.14 mm deep are cut in the wide wall of the waveguide. The period of the slow-wave structure is $l=0.1$ mm. The lamella and the groove have a width of 0.05 mm.

Using the method described in [9], we first find the eigenmodes of the periodic lamellar grating with a fixed index of x-dependences equal to 1, which corresponds to the fundamental H_{10} mode of the input rectangular waveguide. The convergence was investigated depending on the f_{cut} parameter, which controls the calculation accuracy. The projection bases include all the modes that propagate for the given f_{cut} parameter. An acceptable (graphical) accuracy is achieved at $f_{cut}=3\,600$ GHz.

In Fig. 4, the solid lines show the frequency dependences of the phase incursion per period for the

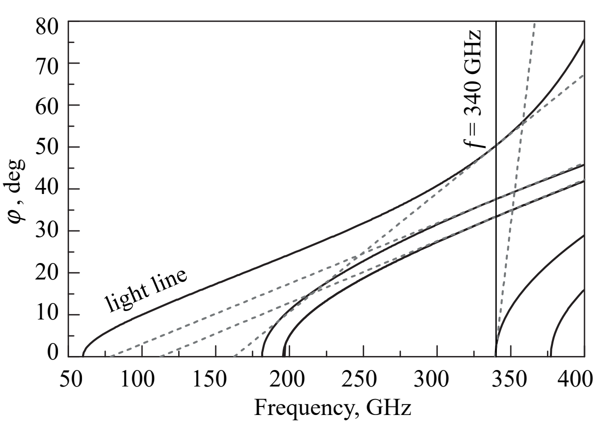

Fig. 4. Dispersion curves and tangents to them for $n=1$

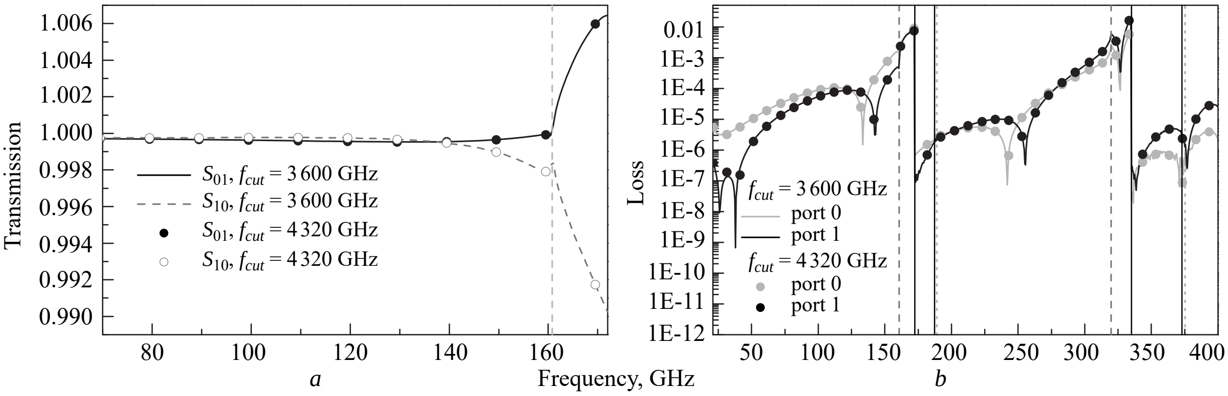

Fig. 5. Frequency dependences of the transmission coefficients (*a*) and the residual in the energy balance equation (*b*) for the semi-infinite slow-wave structure calculated for q = 10 and different accuracy parameters f_{cut} (ports of incidence are indicated in the legend)

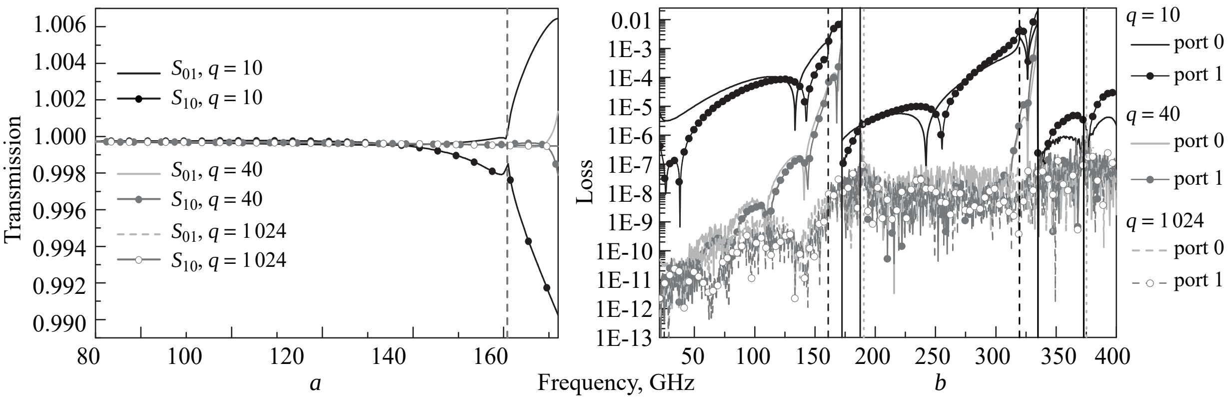

Fig. 6. Frequency dependences of the transmission coefficients (*a*) and the residual in the energy balance equation (*b*) for the semi-infinite slow-wave structure calculated for f_{cut} = 3 600 GHz and different lengths of the auxiliary finite fragment ql (ports of incidence are indicated in the legend)

eigenmodes of the slow-wave structure with the index of x-dependence n = 1 and the dashed lines are tangents to them at frequency f = 340 GHz. The angles of inclination of the tangents were calculated through the group velocity, which, in turn, was calculated through the average power flow and the energy stored per period. As can be seen from the figures, the group velocity is in good agreement with the tangent.

3. Calculation of the scattering matrix of a semi-infinite slow-wave structure. Below the results of calculating the scattering matrix of the junction of a 7.2 × 0.8 mm^2 rectangular waveguide with a semi-infinite slow-wave structure with the parameters described in Section 2 are represented.

The frequency dependences of the transmission and residual in the energy balance equation (loss) are shown in Fig. 5. For the calculation, the fields were expanded in groove 11 (q = 10). The transmission is presented only in single-mode regime to better see the violation of the reciprocity. The vertical dashed, dotted, and solid lines correspond to the cutoff frequencies of the groove, lamella, and slow-wave structure.

As can be seen from the figures, the curves for f_{cut} = 3 600 GHz and 4 320 GHz coincide with graphical accuracy. But the closer is the frequency to the upper limit of the single-mode band, the more the energy balance and the reciprocity are violated.

As the length of the finite fragment ql increases, the energy balance and reciprocity improve and eventually become acceptable, which is illustrated in Fig. 6. At first sight, it seems that the larger the q value, the more accurate the result. However, increasing q has two opposite effects. On the one hand, the accuracy increases due to the attenuation of the modes of the slow-wave structure that are not taken into account. On the other hand, it decreases due to the deterioration of the accuracy of the phase factor in (2) since its accuracy is proportional to q (not shown in Fig. 6). In practice, a compromise value for q must be found.

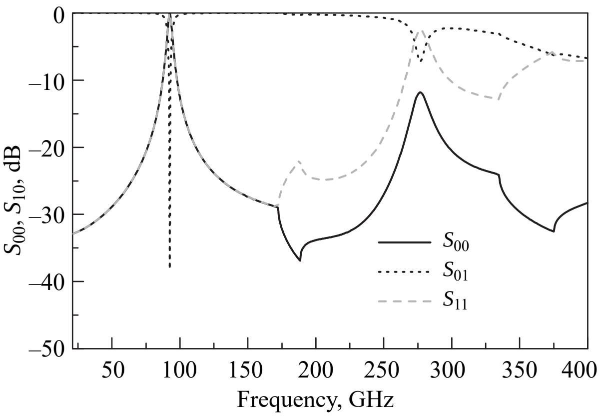

Fig. 7. Resonant frequency response of the semi-infinite slow-wave structure with deepened resonant first groove calculated for f_{cut} = 3 600 GHz, q = 1 024, h_0 = 0.78 mm

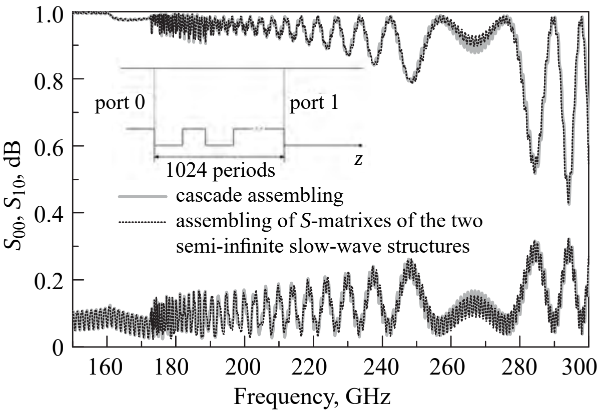

Fig. 8. Frequency response of a finite fragment of the slow-wave structure: comparison of the two calculation approaches. The fragment is 1 024 periods long

In addition, the conservation of the scattering matrix was tested once one period was added to the semi-infinite slow-wave structure. This was the key property in the derivation of the nonlinear equation of a semi-infinite structure in [5, 6]. The results coincide with a good accuracy.

The interaction of a semi-infinite structure with a resonant object is of interest. Let us deepen the first groove so that it becomes resonant. Let us choose its depth equal to h_0 = 0.78 mm, which corresponds to a quarter-wave groove at 96 GHz. The corresponding frequency response is shown in Fig. 7. The results are plausible: there is reciprocity, energy balance, and total reflection resonance. As expected, reflection resonances are observed near the frequencies that are multiples of 96 GHz.

4. The problem of homogenization of the slow-wave structure. The possibility of using the scattering matrix of a semi-infinite slow-wave structure to reconstruct the scattering matrix of its finite fragment was investigated. This is a kind of homogenization of the finite slow-wave structure fragment, when the calculation is performed using the scattering matrices of the slow-wave structure junctions with a regular waveguide, and the phase incursion between the boundaries of the fragment is calculated according to the phase velocity of the eigenmode. The study showed that such homogenization is possible only for a fragment whose length is an integer number of the periods. For this, the fragment must begin and end with waveguides of different sizes: one corresponds to the groove, and the other to the lamella (i.e., it must look like the auxiliary element in Fig. 3). To calculate it, one needs to know the matrices of two semi-infinite slow-wave structures with different input ports. Again, one port corresponds to the groove and the other to the lamella. The comparison of the transmission and reflection results for 1 024 periods is shown in Fig. 8. There is a good qualitative agreement between the results. It is expected that the calculation accuracy can be improved by taking into account a few evanescent eigenmodes of the slow-wave structure.

The scattering matrix of a finite number of grooves in a rectangular waveguide cannot be calculated using only the scattering matrix of a semi-infinite slow-wave structure, cannot be calculated, since the phase incursion of the eigenmode of the periodic structure is not determined for a non-integer number of the periods. This is a typical problem that arises during the homogenization of metamaterials when an attempt is made to replace a finite layer of the metamaterial with a dielectric with equivalent material parameters.

Naturally, a finite number of grooves in a rectangular waveguide can be calculated by the generalized scattering matrix method by connecting the finite slow-wave structure fragment starting with a lamella and ending with a groove with a step transition from the groove to the lamella.

Conclusion. An efficient method for calculating the scattering matrix of a semi-infinite slow-wave structure has been proposed. The convergence of the method has been investigated. A number of tests were carried out to check the reliability of its implementation. Also, a representation of the group velocity of the slow-wave structure eigenmode in terms of its expansion coefficients in series of waveguide modes in the partial regions of the period has been obtained. The obtained scattering matrix has been already used to build a hot model of resonators as part of vacuum electronics devices [8]. The method can be generalized to study the scattering by a half-space filled with a metamaterial.

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УЗАГАЛЬНЕННЯ МЕТОДУ ЧАСТКОВИХ ОБЛАСТЕЙ
НА ПРОБЛЕМИ РОЗСІЮВАННЯ НАПІВНЕСКІНЧЕННИМИ
СПОВІЛЬНЮВАЛЬНИМИ СИСТЕМАМИ

Предмет і мета роботи. Досліджено матрицю розсіювання напівнескінченної сповільнювальної системи, утвореної канавками в прямокутному хвилеводі. Метою дослідження було розроблення методу розрахунку напівнескінченної періодичної структури.

Методи та методологія. Побудовано узагальнення методу часткових областей на напівнескінченні періодичні структури. Поля періодичної частини структури розкладаються в ряди з власних мод періодичної структури з урахуванням умови на нескінченності, що дозволяє отримати лінійне матричне рівняння для знаходження матриці розсіювання. Розглядались лише моди періодичної структури, що поширюються. Щоб зробити ці представлення достовірними, поля узгоджувались на періоді, дещо віддаленому від зчленування регулярного хвилеводу з періодичним.

Результати роботи. Отримано матричні рівняння для визначення блоків матриці розсіювання напівнескінченної структури. Для перевірки достовірності отриманих рівнянь було проведено низку досліджень. До їх числа входили тест на збіжність, взаємність, енергетичний баланс та збереження матриці розсіювання при додаванні одного періоду до напівнескінченної структури. Основне підтвердження отримане шляхом порівняння матриці розсіяння скінченного фрагмента сповільнювальної системи, одержаної двома способами: через матриці розсіювання напівнескінченної сповільнювальної системи і через каскадну збірку матриць розсіювання хвилевідних елементів, що складають структуру.

Висновки. Отримано алгоритм розрахунку матриці розсіювання напівнескінченної структури. Він може бути використаний для побудови строгої «гарячої» моделі пристроїв вакуумної електроніки з використанням сповільнювальних систем.

Ключові слова: *напівнескінченна ґратка, сповільнювальна система, метод часткових областей.*