



# RESEARCH AND ENGINEERING INNOVATION PROJECTS OF THE NATIONAL ACADEMY OF SCIENCES OF UKRAINE

<https://doi.org/10.15407/scine18.05.038>

KHOROSHYLOV, S. V. (<https://orcid.org/0000-0001-7648-4791>),

and LAPKHANOV, E. O. (<https://orcid.org/0000-0003-3821-9254>)

Institute of Technical Mechanics of the National Academy of Sciences of Ukraine and the State Space Agency of Ukraine, 15, Leshko-Popelya St., Dnipro, 49005, Ukraine, +380 56 372 06-40, office.itm@nas.gov.ua

## TIME-PERIODIC SPACECRAFT ATTITUDE CONTROL WITH THE USE OF SLEWING PERMANENT MAGNETS

**Introduction.** Electromagnetic actuators are widely used in spacecraft (SC) attitude control systems. These actuators can be modified by using slewing permanent magnets (ASPM) as sources of torque instead of electromagnets. These modified devices consume less onboard electricity for SC attitude control than the conventional ones. **Problem Statement.** An algorithm for attitude stabilization of a SC using ASPM was proposed in previous studies, where the pole placement technique and pulse-width modulation (PWM) were used to design the controller. However, this approach does not allow the designers to find optimal values of the required magnetic torques, which may result in frequent engagement of the stepper motors of the ASPMs and their significant energy consumption. This controller has such a drawback because its gains are selected without taking into account time-periodic properties of the Earth magnetic field.

**Purpose.** The purpose of the study is to design the algorithm for the SC angular stabilization by the ASPMs taking into account time-periodic properties of the Earth magnetic field.

**Materials and Methods.** The solution of the time-periodic Riccati equation was used for the controller design. Mathematical modeling and computer simulation of SC motion was applied to build the model of the plan and validate the results.

**Results.** A time-periodic based SC attitude control algorithm has been designed. Taking into consideration the time-periodic properties of the magnetic field of Earth allow us to optimize the required magnetic control torques. This algorithm minimizes the frequency of the actuation of the ASPM sashes, and thus reduces onboard energy consumption.

**Conclusions.** The designed algorithm increases the control efficiency of SC attitude control by using jointly the ASPMs, time-periodic linear-quadratic regulator and pulse-width modulator.

**Keywords:** spacecraft, time-periodic regulator, and actuators with slewing permanent magnets.

Citation: Khoroshylov, S. V., and Lapkhanov, E. O. Time-Periodic Spacecraft Attitude Control with the Use of Slewing Permanent Magnets. *Sci. innov.*, 18(5), 38–48. <https://doi.org/10.15407/scine18.05.038>

New SC control algorithms, which guarantee the efficient operation during current and future space missions, are among very important areas of research in modern space science and technology. Attitude control is a subtask of SC motion control [1]. Attitude control system can use different actuators, for example reaction wheels, electromagnetic torquers, thrusters, control moment gyros and other modifications of these systems [1–3]. Electromagnetic torquers are frequently used for SC because they allow the system to control the SC attitude motion using only onboard electrical energy, which can be renewed in orbit. Today, magnetorquers are mainly used for SC detumbling and the reaction wheel desaturation. At the same time, the application of these actuators for other tasks of attitude control is a promising direction in the SC design due to their principal simplicity and reliability.

The main approaches for applying “mobile control” methods for electromagnets are given in Refs. [2, 4, 5]. These approaches make it possible to perform uniaxial orientation and stabilization of the SC and, at certain limited time intervals, triaxial stabilization of the SC. Proportional-differential (PD) regulators was used for electromagnetic attitude control in Ref. [6]. It should be noted, these approaches can provide pretty rough stabilization of the SC, which is explained by the specifics of the magnetic actuation. When an electromagnet generates a control torque in one control channel, disturbances occur in other ones due to the specificity of the channel coupling of the magnetic torques [4]. Thus, the accuracy of stabilization is determined by the ability of the controller to generate the required control torque in each channel and compensate disturbances in adjacent channels [7, 8].

References [11, 12] propose to use actuators with slewing permanent magnets (ASPM), which based on technologies for shielding of permanent magnetic fields [9, 10]. These devices consume less the onboard energy compared to electromagnets, because permanent magnets do not require power to generate the required dipole moments.

For example, Refs. [7, 11, 12] demonstrate that the ASPMs consume three times less electric energy than electromagnets with “mobile control” [8] and an order of magnitude less than conventional electromagnetic stabilization [1–3].

However, the pole placement method used in Ref. [12] for selecting the gains of the controller does not allow the designers to find optimal of magnetic torques at a certain control interval, which leads to quite frequent actuation of the ASPM stepper motors. This controller has such a drawback because its gains are selected without taking into account time-periodic properties of the magnetic field of Earth.

To mitigate this drawback a time-periodic model of the plant can be used for the controller design. References [20, 21] apply robust control methods to control such plants, but the robustness of the system to the variation of the model parameters is provided at a cost of control performance. In Ref. [23], an algorithm to control the time-periodic plant is synthesized by solving the state dependant Riccati equation. However, this approach requires solving the Riccati equation onboard during SC operation. References [13, 24] numerically solve the time-periodic Riccati equation to design optimal periodic controller. It should be noted that numerical algorithms for solving such an equation are a key problem and, despite existing publications on this matter, for example [24, 25], software for such problems is not widespread. Given this, there is of interest to study the possibility of using the solutions of the time-periodic Riccati equation for the synthesis of the SC control algorithm using ASPM.

In previous studies [11], [12] the ASPMs were proposed to use for the SC attitude stabilization. A characteristic feature of this new class of magnetic actuators is the possibility to provide controlled shielding of permanent magnetic fields. Structurally an ASPM consist of three main elements: capsules (shields) with sashes, slewing permanent magnets and stepper motors. These actuators can apply discrete control torques generated by the interaction of the permanent magne-

tic field with the EMF as a result of opening and closing of the capsule sashes.

It should be noted that the application of this approach has quite significant limitations in terms of bandwidth. Given this, this approach should be used in long-term missions with very limited energy consumption where ensuring the speed of reorientation is not the main goal. Such missions may include:

- ◆ deorbiting of space debris using an aeromagnetic system [8], [11, 12];
- ◆ the SC attitude stabilization in passive flight modes, Sun Acquisition mode for battery charging, where accurate stabilization and settling time are optional, but minimum energy consumption is a priority;
- ◆ maintaining the SC operation in emergency situations caused by a partial failure of the power system.

In Ref. [12] the capsule sashes are controlled by a discrete controller and PWM. This approach allowed the authors to reduce the onboard electrical energy consumption for rough stabilization of an aerodynamically unstable SC with a flat sail in comparison with conventional control approaches [1, 2] with the electromagnets. However, as mentioned above, the pole placement technique used to design the discrete time-invariant controller does not allow minimizing the actuation of the capsule sashes, because in this case the optimized output is the cross product of Earth magnetic field (EMF) induction vector and magnetic dipole moment vector of permanent magnet instead of required magnetic torque. This article mitigates this drawback by directly optimizing the magnetic control torques of the ASPMs using a time-periodic regulator in conjunction with the pulse-width modulation (PWM) for SC attitude stabilization.

The purpose of the study is to design an algorithm for the SC angular stabilization by the ASPMs taking into account time-periodic properties of the plant.

To achieve this purpose, the following tasks are set:

1. To design the SC attitude controller using a time-periodic LQR and a PWM.

2. To simulate the orbital and attitude motion of the SC controlled by the ASPMs.

3. To compare the performance and the onboard electric energy consumption for the designed controller with the results obtained in previous studies.

## DYNAMIC MODEL

The following reference frames were used: the inertial reference frame (IRF), orbital reference frame (ORF), body frame reference frame (BFRF) [1], magnetic dipole reference frame (MDRF) [3, 14].

The origin  $O$  of the IRF is placed in the Earth center of mass, the axis  $OZ$  is directed along Earth's axis of rotation, the axis  $OX$ ,  $OY$  lie in the plane of the Earth equator so that the axis  $OX$  is pointed to the vernal equinox, and the axis  $OY$  complements the system to the right-handed one. The origin  $A$  of the ORF coincides with the SC center of mass. The axis  $AS$  is directed along the SC radius-vector  $r$  at the current point of the orbit, the axis  $AT$  is located in the plane of the orbit and forms an acute angle with the vector of the SC orbital velocity. The axis  $AW$  complements the system to the right-handed one. The axes of the BFRF coincide with the principal axes of the SC inertia. The axes of the BFRF are parallel and coincident in direction with the corresponding axes of the ORF when the SC orientation is nominal. When the SC deviates from the nominal orientation, the position of the BFRF relative to the ORF is determined by the following sequence of rotation angles: pitch  $\theta$ , roll  $\varphi$ , and yaw  $\psi$ . MDRF is the oblique dipole type reference frame [14].

To model the orbital motion of the SC with the ASPMs, a system of the differential equations of perturbed motion in orbital elements is used, which is integrated in the ORF [15]:

$$\dot{a} = 2a^2 h^{-1} \left( e \cdot \sin \vartheta \cdot S + \frac{p}{r} T \right),$$

$$\dot{e} = h^{-1} \left( p \cdot \sin \vartheta \cdot S + [(p+r) \cos \vartheta + r \cdot e] T \right),$$

$$\dot{i}' = r \cdot h^{-1} \cos(\vartheta + \omega) \cdot W,$$

$$\begin{aligned}\dot{\Omega} &= \frac{r \cdot \sin(\vartheta + \omega)}{h \cdot \sin i} W, \\ \dot{\omega} &= \frac{h^{-1}}{e} \left[ -p \cdot \cos \vartheta \cdot S + (p+r) \cdot \sin \vartheta T \right] - \\ &\quad - \cos i \frac{r \sin(\vartheta + \omega)}{h \cdot \sin i} W, \\ \dot{\vartheta} &= \frac{h}{r^2} + \frac{h^{-1}}{e} \left( p \cdot \cos \vartheta \cdot S - (p+r) \cdot \sin \vartheta \cdot T \right), \quad (1)\end{aligned}$$

where  $a$  is the semi-major axis of the orbit;  $e$  is the eccentricity of the orbit;  $h = \sqrt{\mu a(1-e^2)}$ ;  $\Omega$  is the argument of the ascending node;  $\omega$  is the argument of perigee;  $i$  is the inclination of the orbit;  $r$  is the SC radius-vector,  $r = \frac{a(1-e^2)}{1+e \cos \vartheta}$ ;  $p$  is the focal parameter of the orbit,  $p = a(1-e^2)$ ;  $\mu$  is the gravitational constant;  $\vartheta$  is the truth anomaly;  $t$  is the time of SC orbital motion;  $S$ ,  $T$ ,  $W$  are the projections of radial, transverse and normal perturbing accelerations on the ORF axes, respectively.

The model (1) takes into account the gravitational and aerodynamic perturbations. The aerodynamic perturbations are calculated using the atmosphere model of the European Space Agency (ECSS atmosphere standard) [16]. Gravitational perturbations are presented taking into account the influence of the first six zonal harmonics of the Earth's gravitational potential decomposition by spherical functions in the form of Legendre polynomials.

Dynamic Euler equations and kinematic equations in quaternion form are used to model the SC attitude motion. The dynamic equations are presented in vector form as follows:

$$J\dot{\omega} + \omega \times (J \cdot \omega) = \mathbf{M}^c + \mathbf{M}^p, \quad (2)$$

where  $J$  is the SC inertia tensor;  $\omega = [\omega_x \ \omega_y \ \omega_z]^T$  is the vector of the SC attitude velocity in the BFRF;  $\mathbf{M}^c = [M_x^c \ M_y^c \ M_z^c]^T$  is the vector of the SC control torque, which is generated by the ASPMs (in BFRF);  $\mathbf{M}^p = [M_x^p \ M_y^p \ M_z^p]^T$  is the vector of the total perturbation torque in the BFRF.

The control torque is determined as follows:

$$\mathbf{M}^c = \mathbf{m} \times (\mu_0 \mathbf{H}), \quad (3)$$

where  $\mathbf{m} = [m_x \ m_y \ m_z]^T$  is the vector of magnetic dipole moments of magnetic actuators that are used for the SC attitude control;  $\mathbf{H} = [H_x \ H_y \ H_z]^T$  is the vector of the EMF strength in BFRF;  $\mu_0 = 4\pi \cdot 10^{-7}$  H/m is the magnetic constant.

The model of the SC attitude motion takes into account the aerodynamic drag and gravitational perturbation torques which are calculated according to the method described in Ref. [17]. To determine the control magnetic torque, the NOAA Earth magnetic field model in dipole representation is used (first three harmonics) [18]. Kinematic equations are given as follows [19]:

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = 0.5 \cdot \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}, \quad (4)$$

where  $q_0, q_1, q_2, q_3$  are the components of the quaternion.

The yaw, roll and pitch angles can be calculated using the quaternion components and equations from Ref. [19].

## LINEARIZED MODEL

To facilitate control design, the mathematical model of the attitude motion is linearized using the approach from Ref. [13]. The linearized system (2), (4) can be given in the state space form as follows:

$$\dot{\mathbf{X}}(\mathbf{t}) = \mathbf{A}\mathbf{X}(\mathbf{t}) + \mathbf{B}(\mathbf{t})\mathbf{m}(\mathbf{t}), \quad (5)$$

where  $\mathbf{X}(\mathbf{t}) = [q_1 \ q_2 \ q_3 \ \omega_1 \ \omega_2 \ \omega_3]^T$  is the state vector;  $\mathbf{m}(\mathbf{t}) = [m_x \ m_y \ m_z]^T$  is the control magnetic dipole torque;  $\mathbf{A}$  is the system matrix, where  $\text{Dim}[\mathbf{A}] = 6 \times 6$ ; matrix  $\mathbf{A}$  has the following non-zero elements:

$$\begin{aligned}a_{14} &= a_{25} = a_{36} = 0,5; \quad a_{41} = 8 \cdot (J_{zz} - J_{yy}) \cdot \omega_0^2 / J_{xx}; \\ a_{46} &= (J_{xx} - J_{yy} + J_{zz}) \cdot \omega_0 / J_{xx}; \quad a_{52} = 6 \cdot (J_{zz} - J_{xx}) \cdot \omega_0^2 / J_{yy}; \\ a_{63} &= 2 \cdot (J_{xx} - J_{yy}) \cdot \omega_0^2 / J_{zz}; \quad a_{64} = (-J_{xx} + J_{yy} - J_{zz}) \cdot \omega_0 / J_{zz};\end{aligned}$$

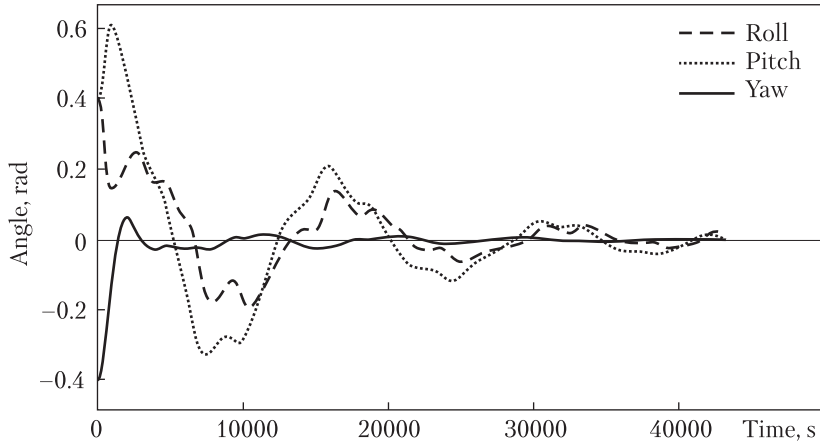


Fig. 1. Variations of the attitude angles

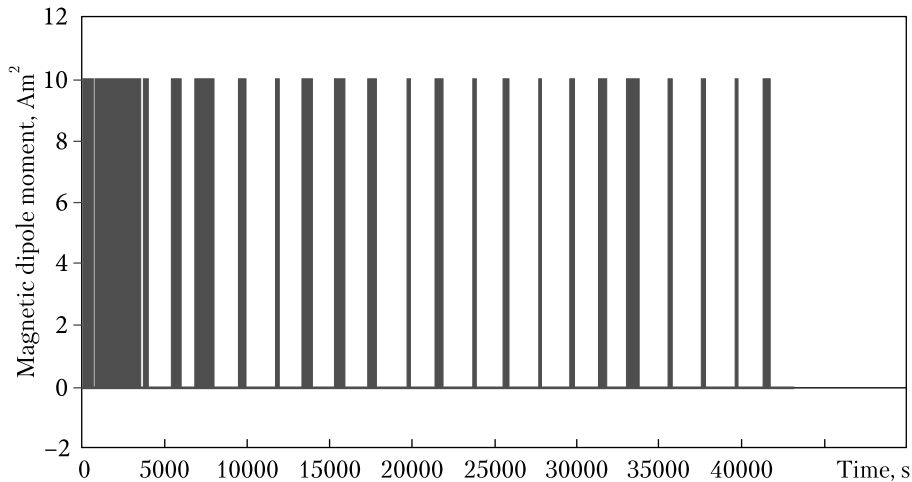


Fig. 2. Modulated magnetic dipole moment in the roll channel for case 1

$\omega_0$  is the absolute value of the mean SC orbital velocity;  $J_{xx}, J_{yy}, J_{zz}$  are the principle moments of the SC inertia;  $B$  is the input matrix,  $\text{Dim}[B] = 6 \times 3$ , matrix  $B$  has the following non-zero elements:

$$b_{42} = \frac{\mu_0 H_z}{J_{xx}}; b_{43} = -\frac{\mu_0 H_y}{J_{xx}}; b_{51} = -\frac{\mu_0 H_z}{J_{yy}}; b_{53} = \frac{\mu_0 H_x}{J_{yy}};$$

$$b_{61} = \frac{\mu_0 H_y}{J_{zz}}; b_{62} = -\frac{\mu_0 H_x}{J_{zz}}.$$

Since the modern SC control system is a discrete computer system, the model (4) is represented in the following discrete form:

$$\mathbf{X}_{k+1} = A_k \mathbf{X}_k + B_k \mathbf{m}_k, \quad (6)$$

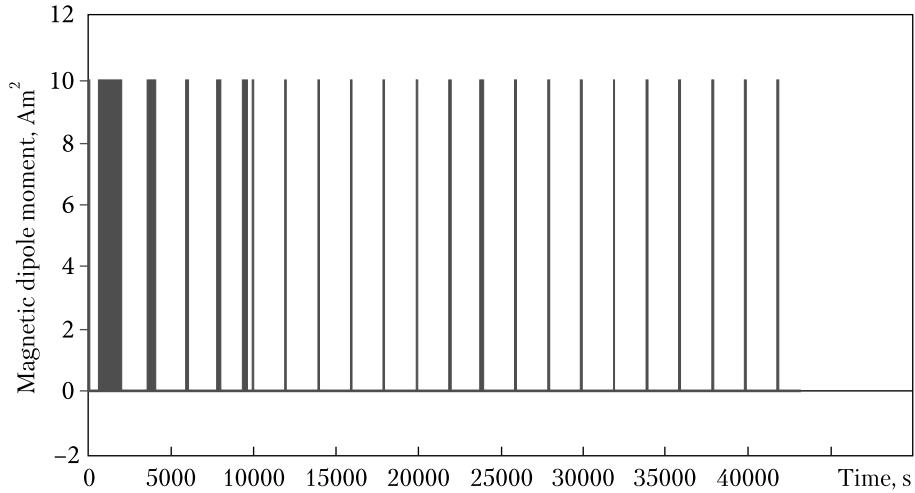
where  $A_k = (I + At_s)$ ;  $B_k = B(t)t_s$ ;  $t_s$  is the sample time;  $k$  is the sample number;  $\mathbf{m}_k = [m_x^k \ m_y^k \ m_z^k]^T$  is the discretized control vector.

### CONTROL DESIGN

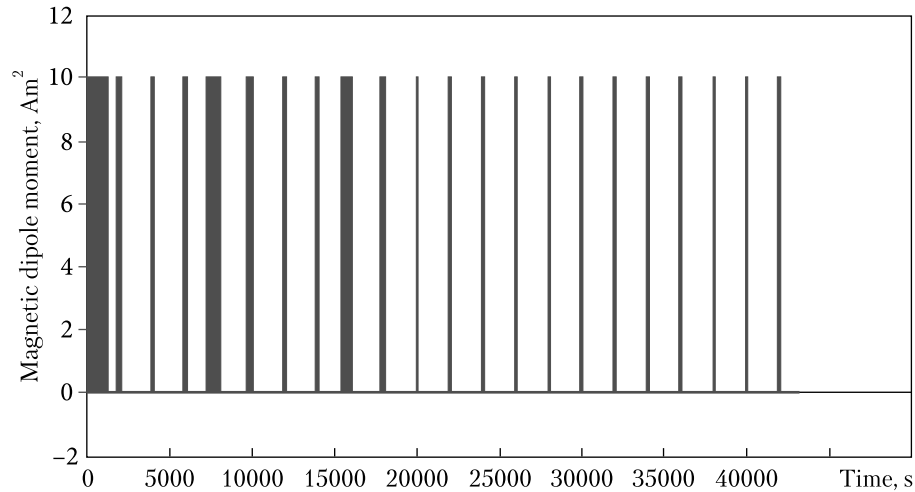
The discrete-time linear-quadratic regulator (DLQR) problem [24] is a widely used methodology to design controllers. The goal of the DLQR design is to find a static gain  $K$  for the full-state feedback law that minimizes the quadratic cost function:

$$J = \min \sum_{k=0}^{\infty} (Q^T \mathbf{X}_k Q + R^T u_k R), \quad (7)$$

where  $Q, R$  are the cost matrices.



**Fig. 3.** Modulated magnetic dipole moment in the pitch channel



**Fig. 4.** Modulated magnetic dipole moment in the yaw channel

Among various positive properties, such a controller demonstrates impressive robustness, which allows designers to use it for systems whose actual parameters differ significantly from the nominal ones [23]. According to this methodology the control is given in the following form:

$$\mathbf{m}_k = K(\mathbf{X}^r - \mathbf{X}_k), \quad (8)$$

where  $\mathbf{X}^r$  is the reference input vector.

Since the matrices  $B$  and  $B_k$  depend on the truth anomaly, their values vary with the orbital

period  $T$ , therefore they satisfy the following conditions:

$$B(t) = B(t+T), \quad (9)$$

$$B_k = B_{k+p},$$

where  $p = T/t_s$ ,  $T = 2\pi \left( \sqrt{\frac{\mu}{a^3}} \right)^{-1}$ .

To control such a time-periodic plant, it is natural to use a periodic discrete linear-quadratic regulator (PDLQR) [25]:

$$\mathbf{m}_k = K_k(\mathbf{X}^r - \mathbf{X}_k), \quad (10)$$

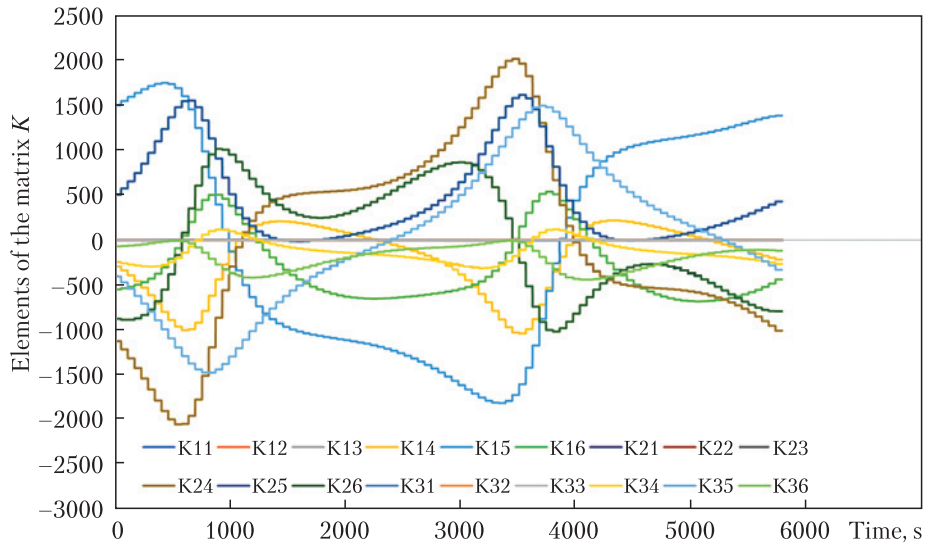


Fig. 5. Variation of the PDLQR gains

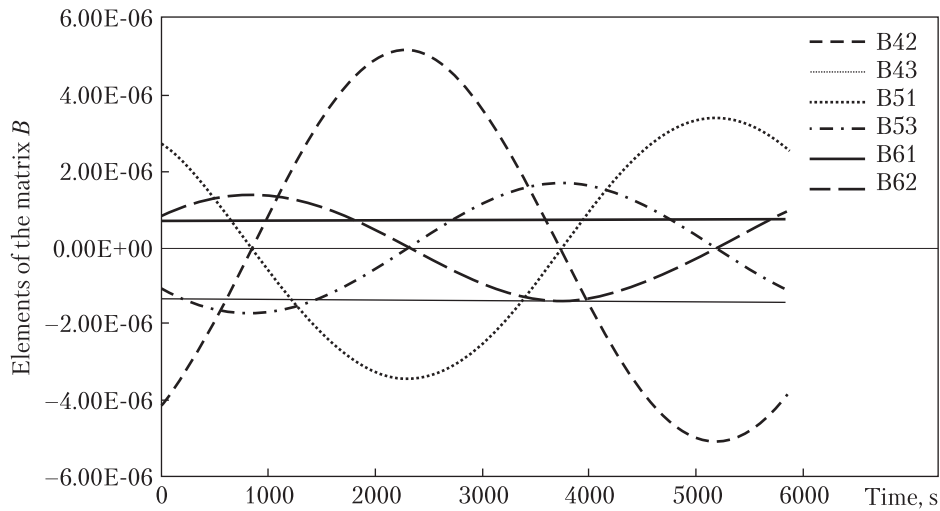


Fig. 6. Variation of the matrix B elements

The periodic gains matrix  $K_k = K_{k+p}$  is calculated as:

$$K_k = (R + B^T P_{k+1} B)^{-1} B^T P_{k+1} A_k, \quad (11)$$

where  $P_{k+1}$  is the solution of the discrete periodic Riccati equation (DPRE).

$$P_k = Q + A_k^T P_{k+1} A_k - A_k^T P_{k+1} B (R + B^T P_{k+1} B)^{-1} B^T P_{k+1} A_k, \quad (12)$$

The solution of the DPRE satisfies the following conditions  $P_k = P_{k+p}$ .

Since the solution of equation (12) is not a trivial task, the algorithm presented in Ref. [25] and used in this study to solve the DPRE is described below. The extended state vector is introduced as follows:

$$Z_k = [X_k \ Y_k]^T, \quad (13)$$

where  $Y_k = P_k X_k$ .

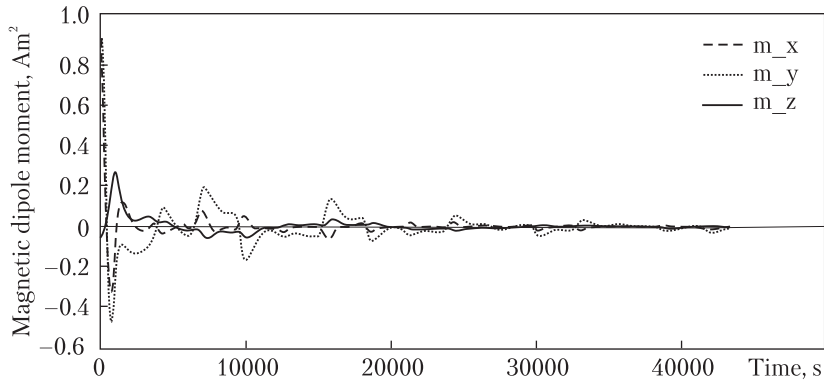


Fig. 7. Variation of the PDLQR output

For this vector, there is a matrix  $k$ , which provides the following transition:

$$\mathbf{Z}_{k+p} = \Pi_k \mathbf{Z}_k,$$

$$\Pi_k = E_{k+p-1}^{-1} F_{k+p-1} \cdots E_{k+1}^{-1} F_{k+1} E_k^{-1} F_k, \quad (14)$$

where  $E_k = \begin{bmatrix} I & BR^{-1}B^T \\ 0 & A_k^T \end{bmatrix}$ ,  $F_k = \begin{bmatrix} A_k^T & 0 \\ -Q & I \end{bmatrix}$ .

Since  $\Pi_k$  is a simplistic matrix, it can be represented using the ordered Schur decomposition [25] in the following form:

$$\begin{bmatrix} W_{11k} & W_{12k} \\ W_{21k} & W_{22k} \end{bmatrix}^T \Pi_k \begin{bmatrix} W_{11k} & W_{12k} \\ W_{21k} & W_{22k} \end{bmatrix} =$$

$$= \begin{bmatrix} S_{11k} & S_{12k} \\ 0 & S_{22k} \end{bmatrix} \quad (15)$$

where  $W_k$  is an orthogonal matrix;  $S_{11k}$  is an upper-triangular matrix, which has all of its eigenvalues outside the unit circle.

Using this decomposition, the DPRE solution can be found as follows:

$$P_k = W_{21k} W_{11k}^{-1}. \quad (16)$$

The magnitude of an ASPM output can have only two values: zero and the value of the magnetic dipole torque of the permanent magnet. Taking into account this peculiarity of the ASPM application, the control magnetic torque is modulated using the technique given in Ref. [12].

Thus, the required pulse duration of the PWM are calculated for each control sample after finding the discrete dipole moments as follows:

$$dt_x = \frac{|m_x^k|}{m_{IM}} t_s,$$

$$dt_y = \frac{|m_y^k|}{m_{IM}} t_s, \quad (17)$$

$$dt_z = \frac{|m_z^k|}{m_{IM}} t_s,$$

where  $m_{IM}$  is the magnetic dipole torque of the permanent magnets.

The number of the capsule openings and their closings can be calculated considering the number of the pulses during a given sample interval and, thus, the electricity consumption of stepper motors.

**Simulation of the SC control.** The efficiency of the designed PDLQR based algorithm is demonstrated by simulating the SC orbital and attitude motion using the parameters summarized in Tables 1–3. Table 1. SC parameters.

The weight matrixes  $Q$  ( $\text{Dim}(Q) = 6 \times 6$ ) and  $R$  ( $\text{Dim}(R) = 3 \times 3$ ) have the following nonzero elements:  $q_{11} = q_{22} = q_{33} = 1.0$ ;  $q_{44} = q_{55} = q_{66} = 0.9$ ;  $R_{11} = R_{22} = R_{33} = 0.6$ .

The discretization period  $t_s$  of 20 s is selected for simulations. The variations of the SC attitude angles for the time interval of 12 hours are plotted in Fig. 1. Figures 2–4 show modulated mag-



Table 1. Spacecraft Parameters

Parameter	Value
Mass, kg	100
Cross-section, m <sup>2</sup>	0.5
Center of pressure offset, m	0.1
Magnet torque, A · m <sup>2</sup>	10

Table 2. Elements of the Inertia Matrix

Element	$J_{xx}$	$J_{yy}$	$J_{zz}$	$J_{xy}$	$J_{xz}$	$J_{yz}$
Value, kg · m <sup>2</sup>	8	12	15	0.03	0.025	0.04

Table 3. Orbital Parameters

Parameter	Value
Semi-major axis, m	7006030.15
Eccentricity	0.001
Inclination, deg	72

netic torque in each control channel. One period variations of the optimized gains of the PDLQR, nonzero elements of the matrix  $B$ , and the PDLQR output are depicted in Fig. 5, Fig. 6, and Fig. 7, respectively.

Comparing the simulation results for the PDLQR with the PWM and the similar results obtained

in Ref. [12], here after just the base line controller, the following conclusions can be made:

1) the PDLQR-based attitude control algorithm allows the controller to significantly reduce the frequency of the sash engagement. For example, the number of the controlling pulses of the PDLQR-based controller (see Fig. 4) is significantly less in comparison with the case of the base line controller [12];

2) for the model of the stepper motors from Ref. [10] used for actuating the sashes, the onboard energy consumption during half a day interval (43200 s) is 5374.5 J and 15500.5 J for PDLQR-based and base line controllers, respectively.

The article synthesizes and investigates a new controller for controlling the SC attitude using the ASPMs. The controller is designed using the solution of the time-periodic Riccati equation, which allows us to take into account the peculiarities of the orbital variation of the Earth's magnetic field. This approach makes it possible to explicitly optimize the values of the required magnetic torques of the actuators and decrease the frequency of actuation of the ASPM sashes and goes down the energy consumption of such actuators in total. The application of these algorithms in practice makes the ASPMs especially attractive for SC attitude control, when strict requirements for minimizing energy consumption are imposed.

## REFERENCES

1. Alekseev, Yu. S., Belous, Ye. V., Belyaev, G. V. (2012). *Design of control systems for objects of rocket and space technology. T. 2. Design of control systems for spacecraft and modules of orbital stations [Text]: tutorial* (Eds. Yu. S. Alekseev, Yu. M. Zlatkin, V. S. Krivtsov, A. S. Kulik, V. I. Chumachenko). Kharkiv: National Aerospace University "KhAI", Scientific and Industrial Enterprise "Khartron-Arkos" [in Russian].
2. Alpatov, A. P. (2016). *Spacecraft dynamics*. Kyiv: Naukova Dumka [in Russian].
3. Ovchinnikov, M. Yu., Roldugin, D. S. (2019). Recent advances in the active magnetic control of satellites. *Spacecraft & Technologies*, 2(28), 73–86. <https://doi.org/10.26732/2618-7957-2019-2-73-86> [in Russian].
4. Alpatov, A. P., Dranovsky, V. I., Saltykov, Yu. D., Khoroshylov, V. S. (1978). *Dynamics of spacecraft with magnetic control systems: monograph* (Eds. A. P. Alpatov). Moscow: Mashinostroenie [in Russian].
5. Wang, P., Shtessel, Y. (1999). Satellite attitude control via magnetorques using switching control laws. *14th Triennial World Congress, (Beijing, P.R. China)*. Beijing, 8021–8026. URL: <https://www.sciencedirect.com/science/article/pii/S1474667017573681> (Last accessed: 03.05.2021)
6. Torcynski, M. T., Amini, R., Massioni, P. (2010, April). Magnetorque based attitude control for a nanosatellite testplatform. *AIAA Infotech@Aerospace (20–22 April 2010, Atlanta, Georgia)*. Atlanta, 1–9. URL: <https://core.ac.uk/download/pdf/207102607.pdf> (Last accessed: 03.05.2021)

7. Sofyali, A., Jafarov, E. M., Wisniewski, R. (2018). Robust and global attitude stabilization of magnetically actuated spacecraft through sliding mode. *Aerospace Science and Technology*, 76, 91–104. <https://doi.org/10.1016/j.ast.2018.01.022>
8. Alpatov, A., Lapkhanov, E. (2019). The use of mobile control methods for stabilization of a spacecraft with aeromagnetic deorbiting system. *System technologies*, 6, 41–54. <https://doi.org/10.34185/1562-9945-6-125-2019-04>
9. Dmitrenko, V. V., Nyunt, P. W., Vlasik, K. F., Grachev, V. M., Grabchikov, S. S., Muravyev-Smirnov, S. S., Novikov, A. S., Ulin, S. E., Uteshev, Z. M., Chernysheva, I. V., Shustov, A. Y. (2015). Electromagnetic Shields Based on Multilayer Film Structures. *Bulletin of the Lebedev Phys IRF Institute*, 2(42), 43–47.
10. Nyunt, P. W. (2015). A method for suppressing the influence of constant magnetic fields on the nuclear-physical and cosmophysical apparatus (PhD) (Engineer). Moscow [in Russian].
11. Lapkhanov, E., Khoroshylov, S. (2019). Development of the aeromagnetic space debris deorbiting system. *Eastern-European Journal of Enterprise Technologies*, 5(5 (101)), 30–37. <https://doi.org/10.15587/1729-4061.2019.179382>
12. Alpatov, A., Khoroshylov, S., Lapkhanov, E. (2020). Synthesizing an algorithm to control the attitude motion of SC equipped with an aeromagnetic deorbiting system. *Eastern-European Journal of Enterprise Technologies*, 1(5(103)), 37–46. <https://doi.org/10.15587/1729-4061.2020.192813>
13. Yang, Y. (2019). *SC modeling, attitude determination, and control: quaternion-based approach*. Taylor & Francis Group, LLC.
14. Ovchinnikov, M. Yu., Penkov, V. I., Roldugin, D. S., Pichuzhkina, A. V. (2018). Geomagnetic field models for satellite angular motion studies. *Acta Astronautica*, 144, 171–180. <https://doi.org/10.1016/j.actaastro.2017.12.026>
15. Fortescue, P., Stark, J., Swinerd, G. (2011). *SC systems engineering*. John Wiley & Sons Ltd. Chichester.
16. ECSS-E-ST-10-04C. Space engineering, Space environment (2008). Noordwijk: ECSS Secretariat, ESA-ESTEC, Requirements & Standards Division, 198.
17. Khramov, D. A. (2015). Visual modelling spacecraft motion. *Technical Mechanics*, 2, 49–58.
18. International Geomagnetic Reference Field. NOAA. URL: <https://www.ngdc.noaa.gov/IAGA/vmod/igrf.html> 32 (Last accessed 03.05.2021)
19. Alpatov, A. P., Khoroshylov, S. V., Maslova, A. I. (2019). *Contactless de-orbiting of space debris by the ion beam. Dynamics and control*. Kyiv: Akadempriodyka, 150. <https://doi.org/10.15407/akadempriodyka.383.170>
20. Khoroshylov, S. (2018). Relative motion control system of SC for contactless space debris removal. *Sci. innov.*, 14(4), 5–16. <https://doi.org/10.15407/scine14.04.005>
21. Khoroshilov, S. V. (2017). Synthesis of robust controller for ion beam shepherd control system. *Technical Mechanics*, 1, 26–39. <https://doi.org/10.15407/itm2017.01.026>
22. Dwidar, H. R., Owis, A. H. (2013). Relative Motion of Formation Flying with Elliptical Reference Orbit. *International Journal of Advanced Research in Artificial Intelligence*, 2(6), 79–86. <https://doi.org/10.14569/IJARAI.2013.020613>
23. Khoroshylov, S. (2020). Relative control of an ion beam shepherd satellite in eccentric orbits. *Acta Astronautica*, 176, 89–98. <https://doi.org/10.1016/j.actaastro.2020.06.027>
24. Peng, H., Zhao, J., Wu, Z., Zhong, W. (2011). Optimal periodic controller for formation flying on libration point orbits, *Acta Astronautica*, 69, 537–550. <https://doi.org/10.1016/j.actaastro.2011.04.020>
25. Varga, A. (2008). On solving periodic Riccati equations. *Numerical Linear Algebra with Applications*, 15(9), 809–835.

Received 18.08.2021

Revised 11.04.2022

Accepted 07.06.2022

С. В. Хорошилов (<https://orcid.org/0000-0001-7648-4791>),  
Е. О. Лапханов (<https://orcid.org/0000-0003-3821-9254>)

Інститут технічної механіки Національної академії наук України  
і Державного космічного агентства України,  
вул. Лешко-Попеля, 15, Дніпро, Україна, 49005,  
+380 56 372 06-40, office.itm@nas.gov

#### ЧАСОПЕРІОДИЧНЕ КЕРУВАННЯ КУТОВИМ РУХОМ КОСМІЧНИХ АПАРАТІВ З ВИКОРИСТАННЯМ ПОВОРТНИХ ПОСТІЙНИХ МАГНІТІВ

**Вступ.** Електромагнітні виконавчі органи широко використовуються в системах керування космічними апаратами (КА). Ці виконавчі органи можна модифікувати, використовуючи, замість електромагнітів, поворотні постійні магніти (ВППМ) для генерації керуючого моменту. Останні споживають менше бортової електроенергії для керування орієнтацією КА, ніж звичайні електромагніти.

**Проблематика.** В попередніх дослідженнях було запропоновано алгоритм орієнтації й стабілізації КА з використанням ВППМ. Для розробки алгоритму керування було застосовано підходи синтезу модальних регуляторів із широтно-імпульсною модуляцією (ШІМ). Однак такий підхід не дозволяє знайти оптимальні значення необхідних магнітних моментів, що може призвести до частого вмикання крокових двигунів ВППМ та суттєвого споживання енергії, оскільки його коефіцієнти підсилення підбрано без урахування часоперіодичних властивостей магнітного поля Землі.

**Мета.** Розробка алгоритму кутової стабілізації КА з ВППМ з урахуванням часово-періодичних властивостей установки магнітного поля Землі.

**Матеріали та методи.** Для розробки регулятора використано розв'язок часоперіодичного рівняння Ріккати. Для побудови моделі плану та перевірки результатів було застосовано математичне та комп'ютерне моделювання руху КА.

**Результати.** Розроблено часоперіодичний алгоритм керування орієнтацією КА. Врахування часово-періодичних властивостей магнітного поля Землі дозволяє оптимізувати значення необхідних магнітних керуючих моментів. Цей алгоритм мінімізує частоту спрацьовування ступок ВППМ і таким чином знижує бортове енергоспоживання.

**Висновки.** Розроблений алгоритм підвищує працездатність та ефективність керування орієнтацією КА шляхом використання ВППМ, часоперіодичного лінійно-квадратичного регулятора та широтно-імпульсного модулятора.

*Ключові слова:* космічний апарат, часоперіодичний регулятор, виконавчі органи з поворотними постійними магнітами.