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GENERALIZATION OF EXPERIMENTAL ELASTICITY OF CAVITATION BUBBLES IN LRE PUMPS THAT DIFFER SIGNIFICANTLY IN SIZE AND PERFORMANCE

Introduction. *Consideration of cavitation phenomena in liquid rocket engine (LRE) pumps is necessary for determining the frequency characteristics of the engine, when calculating transient processes in propulsion systems during engine start-up and stop, and, especially, for addressing the problem of ensuring the stability of longitudinal oscillations of liquid rockets (POGO-oscillations).*

Problem Statement. *Currently, the theoretical determination of the characteristics of cavitation flows in LRE pumps has not been widespread because of extremely low accuracy. The disadvantage of the existing experimental and calculated dependences of elasticity, volume, and resistance of cavitation bubbles on the mode parameters is the limited range of cavitation numbers for which these dependences are reliable.*

Purpose. *The purpose of this research is to determine the elasticity, volume, and resistance of cavitation bubbles in LRE pumps in the whole range of existence of cavitation bubbles, based on the results of dynamic tests of 26 pumps that differ significantly in purpose, size, and performance.*

Materials and Methods. *The information and analytical method, the methods of the theory of oscillations, the impedance method, and the method of least squares have been used.*

Results. *It has been shown that the experimental values of the elasticity of cavitation bubbles for different pumps generally agree satisfactorily with each other. The dependence of the relative elasticity of cavitation bubbles on the number of cavitation and the flow coefficient has been approximated with the use of the formula that allows describing the cavitation phenomena in pumps in the entire range of existence of cavitation bubbles. Three types of deviations of experimental frequencies of oscillations from the natural frequencies of oscillations of fluid in a hydraulic system with a cavitating pump have been described. The first and second types of deviations are caused by the interaction of the fluid and the structure of the feed pipeline, while the third one is associated with developed self-oscillations of cavitation bubbles.*

Conclusions. *Semi-empirical dependences of elasticity, volume, and resistance of cavitation bubbles in LRE pumps on the mode parameters in the entire range of existence of cavitation bubbles have been built.*

Keywords: liquid rocket engine, POGO-oscillations, cavitating pump, frequency of oscillations of cavitation bubbles, experimental and computational method, elasticity, volume and resistance of cavitation bubbles, and number of initial cavitation.

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The consideration of cavitation phenomena in the liquid rocket engine pumps is necessary for determining the frequency characteristics of the engine and its fuel feeding system [1], as well as for calculating the transient processes in the liquid rocket propulsion system (LRPS) during engine start and stop [2]. However, it becomes most important for solving the problem of ensuring the stability of longitudinal oscillations of liquid rockets (POGO-oscillations) [3]. Cavitation bubbles located at the pump inlet, despite their small size, may lead to a reduction in the natural frequencies of fluid oscillations in the engine fuel feeding system to dangerous values when the rocket structure and the rocket fuel feeding system are involved in resonant interaction.

Currently, theoretical determination of the characteristics of cavitation flows in LRE pumps has not been widespread because of unsatisfactory accuracy [4, 5]. Therefore, approaches that involve the experimental data are employed, as a rule. There are two main such approaches. One approach is based on the definition and generalization of experimental dynamic transfer matrices of cavitating pumps of LRE [6, 7]. The other one is based on the use of the dependence of the experimental frequencies of cavitation bubble oscillations on the pressure at the pump inlet and the flow rate through the pump. This approach has been implemented in [8], where V. V. Pylypenko has proposed a method for the determination, and then found the elasticity and volume of cavitation bubbles, based on the results of dynamic tests of several pumps. The further development of this approach focuses on taking into account the inertia of backflows at the pump inlet. This has made it possible to agree with each other the experimental elasticities of cavitation bubbles, which are obtained from the frequencies of cavitation bubble oscillations in the modes with backflows of nine axial inducer front pumps [9]. In [10], the number of studied pumps increases to 18, among them there are the pumps of the oxidizer of the marching LRE stages of launch vehicles (LV). In this research, the experimental elasticities of cavitation bubbles have been agreed with each other for the modes of operation both with backflows at the pump inlet and without them. The dependence of the elasticity of cavitation bubbles on the mode parameters as part of the mathematical model of the dynamics of cavitating rocket pumps has been widely demanded in the problems of the dynamics of LRPS and liquid rockets, and its improvement is urgent to increase the accuracy and reliability of calculations and predictions. With the help of semi-empirical coefficients obtained in [10], some important problems of low-frequency dynamics of LRPS and liquid rockets have been solved. They include the analysis of stability of longitudinal oscillations and the selection of parameters of dampers of longitudinal oscillations [5], the prediction of amplitudes of longitudinal oscillations while upgrading LVs [11, 12], the study of the stability of the propulsion system [13], the determination of transient processes during the start and stop of the LRPS [4], the study of the influence of staggered start on the parameters of propulsion system launch [14], the influence of external and internal factors on the thrust misalignment at the LRE start [15], the simulation of engine start during fire tests in the throttle mode [16], etc.

The disadvantage of the dependences of the elasticity, volume, and resistance of cavitation bubbles on the mode parameters obtained in [8—10] is the limited range of cavitation numbers for which these dependences are valid. However, the practical problems of low-frequency dynamics of the LRPS put forward requirements for reliable determination of the characteristics of cavitation flows in pumps in wide ranges of pressure at the pump inlet (for example, for simulating the start of the LRPS): from the bursting pressures of the pumps to the pressures at which there is no cavitation in the pumps.

In addition, the dynamic tests of another 8 LRE pumps, which are known to the author and described in the literature, have not been properly considered. Among these pumps there are the pumps with new specific properties, which have not been considered in previous studies [8—10]. These are the pumps with a variable geometry of inducer front pump at the inlet and low-pressure booster pumps.

The purpose of this research is to determine the elasticity, volume, and resistance of cavitation bubbles in LRE pumps in the entire range of existence of cavitation bubbles, based on the results of dynamic tests of pumps that differ significantly in terms of purpose, size, and performance.

Some geometric and operational parameters of the studied pumps are presented in Tables 1 and 2. As compared with [10], the number of considered pumps increases to 26. These pumps include both the small-sized high-speed pumps of steering engines and the full-scale (for the problem of POGO-oscillations of liquid rockets) pumps of the oxidizer of marching engines of the 1st stage of LVs, as well as booster low-speed oxidizer and fuel pumps, pumps with constant and variable geometry of the inducer front pump at the entrance. The pumps are designed for different fuel compo-

Table 1. **Some Geometrical Parameters of LRE Pump Inducers**

No.	Pump	D_E, cm	d_S, cm	S, cm	l_M, cm	Z	β_{1B}, degree	Engine	Pump	Propellant	References
1	1.1	12	6.3	5.4	9.85	2	8°9′	LRE-863	FP	UDMH	[1], [8], [9]
2	1.2	12	6.3	4.95	9.85	2	7°29′				[1]
3	1.3	12	6.3	4.55	9.85	2	6°53′				[1]
4	1.4	11	6.3	5.4	10.05	2	8°53′				[1]
5	2.1	5.6	2.6	2.52	3.7	2	8°9′	LRE-862	FP	UDMH	[1], [8], [9]
6	2.2	5.6	2.6	2.88	5.77	2	9°18′				[1]
7	2.3	5.6	2.6	3.15	5.77	2	10°9′				[1]
8	2.4	5.6	2.6	4.4	4.3	2	14°2′				[1]
9	2.5	5.6	2.6	5.76	4.3	2	18°8′				[1]
10	2.6	6.1	2.6	2.52	5.67	2	7°29′				[1]
11	2.7	6.1	2.6	2.16	5.67	2	6°26′				[1]
12	3	14.11	7	8.9	5.8	3	11°21′	LRE-218	OP	NT	[17
13	4.1	15.62	7.6	9.53	7.8	3	11°	LRE-264	OP	NT	[18]
14	4.2	15.62	7.6	12	7.8	3	13°40′	LRE-264	OP	NT	[18]
15	5.1	5.2	2	2.8	3.54	2	9°44′	LRE-852	OP	UDMH	[19], [20]
16	5.2	5	1.4	2.4	4.43	2	8°41′				[19], [20]
17	6	5.04	1.93	2.35	3.4	2	8°26′	LRE-861	OP	NT	–
18	7	15.62	7.6	8.65	7.8	3	10°	LRE-120	OP	LO	[21]
19	8	5.8	3.1	2.4	6	2	7°30′	LRE-8	FP	kerosene	[22]
20	9	12	4.5	5.4	7.1	2	8°9′	LRE-863	OP	NT	[23]
21	10	4.7*	1.3*	2.76*	4.8	2	10°35′				[24], [25]
22	11	10.6*	3.16*	4.9*	10	2	8°22′	LRE-8	OBP	LO	[26]
23	12	7.46	3.6	3.51	6	2	8°31′	LRE-119	FP	UDMH	[27]
24	13	29.3	13	16	10	3	9°52′	НК-15	OBP	LO	[28]
25	14	21.5	10.5	14	6.2	3	11°43′	НК-15	FBP	kerosene	[28]
26	15	12.11	5.4	7.85	5.5	3	11°40′	LRE-218	FP	UDMH	Data on pumps 17 and 26 have not been published separately

nents (in particular, cryogenic ones), operating conditions, and differ significantly in terms of structure. Among them, there are pumps with axial, lateral, annular and two-way fluid feed. The ranges of the main geometrical and operating parameters of the considered pumps overlap with those studied earlier [10] (see Table 3).

The symbols and abbreviations in Table 1 shall mean as follows: D_E is the outer diameter of the inducer; d_S is the inducer sleeve diameter; s is the inducer pitch; l_M the inducer length in the average diameter; Z is the number of blades; β_{1B} is the angle of blade mounting at the outer diameter; FP is fuel pump; OP is oxidizer pump; FBP is fuel boost pump; OBP is oxidizer boost pump; UDMH is unsymmetrical dimethylhydrazine; NT is nitrogen tetroxide; PK is lox; * marks the equivalent values at the inlet and outlet of the pumps with variable geometry.

Table 2. **Some Mode Parameters of LRE Pumps**

No.	Pump	$\overline{C}_{1M}$	ψ	$\rho \cdot W_{1M}^2 / 2$, bar	n_S	Δh_M, m	C_B	G_O, kg/s
1	1.1	0.0832	8.65	14	36.6	2.74	5424	80.4
2	1.2	0.0832	8.65	14	36.6	2.74	5424	73.7
3	1.3	0.0832	8.65	14	36.6	2.74	5424	67.7
4	1.4	0.1129	9.62	12.58	36.6	4.04	4055	62.6
5	2.1	0.1046	15.97	20.04	28.0	5.69	4978	23.6
6	2.2	0.1046	15.97	20.04	28.0	5.69	4978	26.9
7	2.3	0.1046	15.97	20.04	28.0	5.69	4978	29.5
8	2.4	0.1046	15.97	20.04	28.0	5.69	4978	41.2
9	2.5	0.1046	15.97	20.04	28.0	5.69	4978	53.9
10	2.6	0.0796	14.25	22.46	28.0	4.10	6364	29.2
11	2.7	0.0796	14.25	22.46	28.0	4.10	6364	25.0
12	3	0.1132	4.14	10.14	76.7	3.27	4506	280.5
13	4.1	0.1076	3.65	25.2	83.3	7.49	4737	267.2
14	4.2	0.1076	3.65	25.2	83.3	7.49	4737	336.4
15	5.1	0.1759	4.29	4.2	105.7	2.77	3727	12.7
16	5.2	0.0660	3.87	0.52	80.2	0.07	8623	4.3
17	6	0.1283	2.03	11.3	159.8	4.46	4731	17.4
18	7	0.0967	4.25	24.71	70.6	6.17	5121	240.4
19	8	0.0627	10.40	23.36	27.4	2.90	6566	21.9
20	9	0.0925	5.17	11.61	68.4	2.70	6070	96.5
21	10	0.0993	5.51	5.08	75.4	1.33	6432	14.8
22	11	0.1167	1.48	2.03	213.6	0.69	5585	36.2
23	12	0.0907	8.55	15.20	48.2	4.28	5392	30.3
24	13	0.1539	2.16	9.27	141.2	4.35	3839	585.6
25	14	0.1553	1.87	7.47	193.0	5.03	3602	261.2
26	15	0.1228	11.56	8.65	46.8	4.00	4527	192.7

Note: $\overline{C}_{1M}$ is the relative axial velocity at the inlet (feed ratio); ψ is the pressure coefficient; $\rho \cdot W_{1M}^2 / 2$ is the high-speed fluid pressure on the average diameter of the inducer; n_S is the pump rate coefficient; Δh_M is the disruptive cavitation reserve; C_B is the Rudnev cavitation coefficient; C_O is fluid consumption at zero attack angle.

Pump 1.1 (see Tables 1 and 2) is a fuel pump (unsymmetrical dimethylhydrazine (UDMH)) steering engine RD-863 designed by *Pivdenne* Design Office for the 1st stage of 15A15 and 15A16 (SS-17) rockets. Pump 2.1 is a fuel pump (UDMG) of RD-862 steering engine designed by *Pivdenne* Design Office for the 2nd stage of 15A15 and 15A16 (SS-17) rockets. Pumps 1.2—1.4 and 2.2—2.7 are test versions of pumps 1.1 and 2.1, respectively, differing from the prototypes in the pitch of the inducer line or the outer diameter of the inducer.

Pumps 1.1—1.4 and 2.1—2.7 were tested on the pumping stands of *Pivdenne* Design Office, with a model fluid in the 1970s as part of comprehensive study of the dynamics of LRE cavitating pumps. The results of these studies have been described in details in scholarly research literature [1, 8, 9]. Among them, we should single out those that made a breakthrough in research into the dynamics of LRE cavitating pumps:

- the causes and conditions of occurrence of cavitation bubble self-oscillations in the LRPS fuel feeding systems have been clarified;
- the effect of the mode and design parameters on the frequency, range and the area of existence of cavitation bubble self-oscillations has been established;
- the influence of the operating parameters of the inducer front pump on the experimental impedance and the pump amplification factor and so on has been studied.

This research uses the results of determining the experimental frequencies of cavitation bubble self-oscillations in autonomous fuel feeding systems with these pumps.

Pump 3 is an oxidizer pump (nitrogen tetroxide (NT)) of RD-218 propulsion engine developed by *Energomash* Design Bureau for the 1st stage of the 8K67 rocket and the *Cyclone* launch vehicle. The frequencies of cavitation bubble oscillations in the fuel feeding system with the pump used in this research were obtained on the fire stand of *Pivdenne* design office with the natural fuel component (in the 1960s) and on the stand for autonomous tests of water pumps (1984). During the autonomous pump tests, the flow rates at the pump inlet and outlet were measured in dynamic conditions, and the existence of cavitation bubble self-oscillations in the absence of backflows at the pump inlet was experimentally confirmed.

Pumps 4.1 and 4.2 are oxidizer pumps (NT) of four-chamber RD-264 cruise engine developed by *Energomash* Design Bureau for the 1st stage of 15A18, 15A18M, and *Dnipro* rockets (pump 4.1) and 15A14 rocket (pump 4.2). Pumps 4.1 and 4.2 were experimentally studied on the pump test stand of *Pivdenne* design office with water in 1987—1988. To expand the area of cavitation bubble self-oscillations, a flow receiver was installed in the fuel feeding pipeline of the stand, which made it possible to acoustically cut off a part of the pipeline from the tank to the flow receiver. When the receiver was “on” (the presence of an air cushion), the active and inertial resistance of the fuel feeding line decreased 2.4 and 6 times, respectively. This made it possible to increase the amplitudes and frequencies of cavitation bubble oscillations.

Pump 5.1 is a fuel (NDMG) pump of RD-852 steering engine developed by *Pivdenne* Design Office for the 1st stage of 8K64 (SS-7) rocket. Pump 5.2 is a redesigned ECSN-23 aviation pump that has a transparent inducer case and an inlet nozzle. The test results of pumps 5.1 and 5.2 are borrowed from monograph [19] and thesis [20]. They are obtained on a model fluid.

Table 3. **Range of Some Geometrical and Mode Parameters of the Inducers**

References	D_E, cm	d_S, cm	S, cm	ψ	n_S	G_O, kg/s
Previously [10]	5.0—15.62	2.0—7.6	2.16—12.0	2.03—15.97	28.0—159.8	4.3—336.4
In this research	4.7—29.3	1.3—13.0	2.16—16.0	1.48—15.97	27.4—213.6	4.3—585.6

Pump 6 is an oxidizer (NT) pump of RD-861 engine designed by *Pivdenne* Design Office for thrusting and controlling the 3rd stage of *Cyclone* LV. Pump 6 was experimentally studied on a test bench of the hydrodynamic laboratory of the ITM of the National Academy of Sciences of Ukraine and SSA of Ukraine, with water, in 1991. As a result of the dynamic tests of two types (in the mode of cavitation bubble self-oscillations and in the mode of pulsed excitation in the fuel feeding pipeline), the frequencies of cavitation bubble self-oscillations have been determined in a wide range of changes in the flow rate through the pump, with three fuel feeding pipelines with a length ratio of 1 : 2.3 : 4.1.

Pump 7 is an oxidizer (liquid oxygen) pump of RD-120 propulsion engine designed by *Energomash* Design Bureau for the 2nd stage of *Zenith* LV. Pump 7 was experimentally studied on the water pump stand of *Pivdenne* Design Office, in 1991, within the program of measuring pressure and cavitation characteristics. Low-frequency (up to 40 Hz) cavitation bubble self-oscillations in the pump system were excited by installing a flow receiver in the main line. The location of the receiver was chosen based on the results of previous calculations of the stability of the pump system. During the experimental studies of this pump, two samples of the pump were tested. Both collapsed during operation in the mode of cavitation bubble self-oscillations. In both cases, the collapse was caused by an unjustified structural modification that was introduced by the designer into the bearing relief mechanism.

Pump 8 is a fuel (kerosene) pump of four-chamber steering engine RD-8 designed by *Pivdenne* Design Office, for thrusting and controlling the flight of the 2nd stage of *Zenith* LV. The specific feature of the pump is the implementation of the standard design of the inducer front pump with three slots in the blades in the inlet section of the auger to increase the resistance of the fuel feeding system to cavitation bubble oscillations. Pump 8 was experimentally studied by employees of *Pivdenne* Design Office, with water, in 1982, both with a regular inducer front pump and with an auger without slots. The tests confirmed a high efficiency of the augers with slits to increase the stability of LREpower supply systems in relation to cavitation bubble oscillations.

Pump 9 is an oxidizer pump (AT) of steering engine RD-863 developed by *Pivdenne* Design Office for the 1st stage of 15A15 and 15A16 (SS-17) rockets. As a result of autonomous tests of pump 9 with water, in 1982, an unconventional area of the existence of cavitation bubble self-oscillations was discovered. The oscillations are caused by the interaction of the fluid in the feed pipeline with the structure of the pipeline.

Pump 10 is a model inducer centrifugal pump with a variable inducer pitch. The dynamic tests of the pump, with water, were made for two significantly different (9.5 times) lengths of the fuel feeding pipeline and for four numbers of shaft revolutions.

Pump 11 is a booster inducer front pump of the oxidizer (liquid oxygen) of RD-8 engine. Pump 11 was experimentally tested on a test bench of the hydrodynamic laboratory of the ITM of the National Academy of Sciences of Ukraine and the SSA of Ukraine, with water, in 1986. A distinctive feature of the dynamic tests of the booster front pump is that the booster shaft is driven by a hydro turbine on which the fluid pressure drops. After the hydro turbine, the fluid enters the outlet of the front pump, where it joins the main flow. The inducer front pump is made with a profiled sleeve, the pitch of the inducer line along the axis is variable. The blades of the inducer front pump are made perpendicular to the outer surface of the sleeve instead of the axis of rotation of the rotor.

Pump 12 is a fuel pump of RD-119 propulsion engine designed by *Energomash* Design Bureau for the 2nd stage of *Cosmos-2* (11K63) launch vehicle. The dynamic tests of pump 12 were made with a model fluid, they were among the first to confirm the distinctive features of cavitation bubble oscillations: the linear dependence of frequency and the nonlinear dependence of oscillation amplitude on the pressure at the pump inlet. During the pump tests, the stiff mode of excitation of cavitation bubble self-oscillation was observed for the first time.

Pumps 13 and 14 are booster pumps of the oxidizer (liquid oxygen) and fuel (kerosene) of NK-15 (NK-33) marching engine designed by the Kuznetsov Samara R&D Center. The dynamic tests of the pumps were carried out on a fire stand as part of the engine, with natural fuel components. These engines were installed on several different launch vehicles: the first stage of the H1-L3 rocket, the first stage of *Antares* (*Taurus* II) launch vehicle of *Orbital Sciences* Corporation, and the first stage of the *Soyuz*-2.1 V launch vehicle.

Pump 15 is a fuel pump of RD-218 propulsion engine designed by *Energomash* Design Bureau for the 1st stage of 8K67 rocket and *Cyclone* launch vehicle. The experimental data for this pump (as well as part of the data for pump 3) used in this research were obtained on the fire stand of *Pivdenne* Design Office, with the natural fuel component. Pumps 3 and 15 are the two ones with two-way fluid supply.

The results of experimental studies of the pumps were obtained by various authors in the course of fire and autonomous tests on various stands, during several decades. The natural frequencies of fluid oscillations, which are used further, in hydraulic systems with the studied cavitating pumps of LRE, were obtained in different ways. In addition to the conventional method [8] (based on the results of self-exciting cavitation bubble self-oscillations), we have used the methods of pulse excitations at the pump inlet (pumps 5.1, 5.2, 6, 10, 11), harmonic analysis (pumps 13, 14), and resonance frequencies of oscillations and frequency parameters of LRE fuel feeding systems (pumps 13, 14).

GENERALIZATION OF THE ELASTICITY OF CAVITATION BUBBLES

The method for determining the elasticity and volume of cavitation bubbles, based on the use of the experimental dependence of the frequency of cavitation bubble oscillations on the input pressure and the mode of pump operation, was proposed by V. V. Pylypenko in [8]. This method provides a well-known mathematical relationship between the parameters of the hydraulic system with a cavitating inducer centrifugal pump and the natural frequency of oscillations of this system. According to the hydrodynamic model of the cavitating pump [1], this relationship, as obtained from the theoretical analysis of cavitation bubble oscillations in pumping systems and expressed in relation to the elasticity of cavitation bubbles, in the simplest case (pure active load) is written as [10]:

$$B_1 \approx -\gamma\,(J_1 + J_{BF})\,(2\pi f_O)^2, \qquad (1)$$

where γ is the specific weight of the fluid; J_1 is the coefficient of inertial resistance of the fuel feeding pipeline; J_{BF} is the coefficient of inertial resistance of the fluid, which is caused by backflows at the pump inlet; f_O is the he natural frequency of fluid oscillations in a hydraulic system with a cavitating pump.

To generalize the experimental values of elasticity of cavitation bubbles obtained for different pumps, V. V. Pylypenko proposed relative elasticity of cavitation bubbles $\tilde{B}_1$:

$$\tilde{B}_1 = B_1 \cdot \frac{V_{SM}}{\rho \cdot W_{1M}^2 / 2},$$

where V_{SM} is the volume of the flow part of the inducer, where the bubbles are located before cavitation collapse. According to experimental studies [29], cavitation bubbles spread in a jump-like manner over the entire length of the inducer grid, when the length of the bubble is equal to 2.3 of the inducer pitch. In this case,

$$V_{SM} \approx 2{,}3 \cdot s \cdot \frac{D_E^2 - d_S^2}{4}.$$

Cavitation number k^* and flow rate q are used as arguments for the relative elasticity of cavitation bubbles, which allow for the agreement of experimental data for different pumps:

$$k^* = \frac{p_1 - p_{BD\,III}}{\rho \cdot W_{1M}^2 / 2},\ q = \frac{G}{G_O},$$

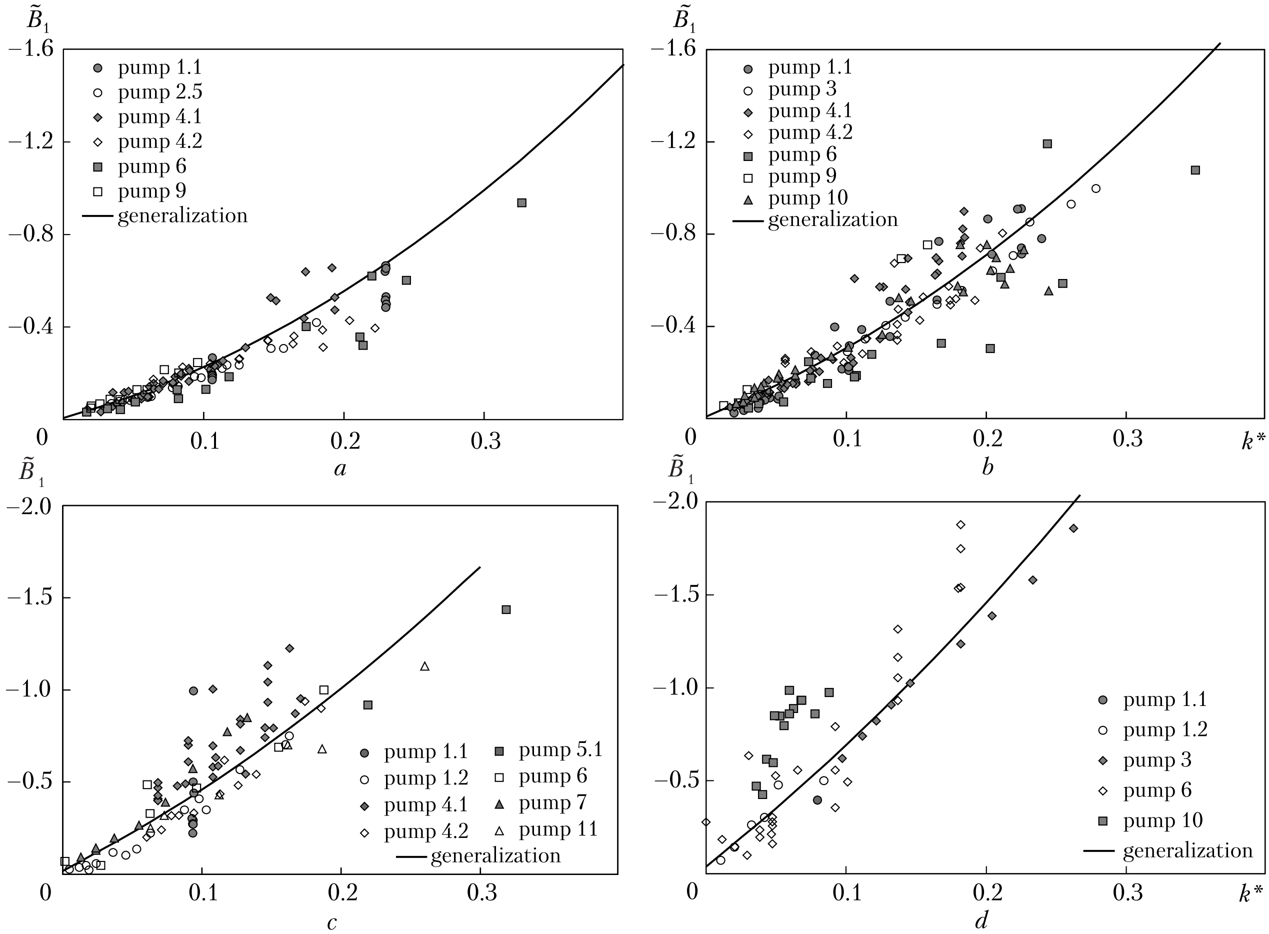

Fig. 1. Dependence of relative elasticity of cavitation bubble on the cavitation number and the flow rate: $a - q = 0.19$; $b - q = 0.27$; $c - q = 0.40$; $d - q = 0.56$

where p_1 is the fluid pressure at the pump inlet; $p_{BD\ III}$ is the pressure of pump breakdown under the 3rd critical mode; G is the fluid flow rate through the pump.

When determining the number of cavitation k^*, we have used the values of breakdown pressures $p_{BD\ III}$ obtained by the empirical formula that summarizes the experimental breakdown pressures of 11 inducer centrifugal pumps of LRE [30]. The expression for pump breakdown pressure from [30] includes the pressure of saturated vapors of the fuel component at the current temperature. Therefore, with the help of this type of cavitation number, the results of dynamic tests of the pumps on both high-boiling and cryogenic fuel components may be correctly summarized. The inertia of back flows at the pump inlet J_{BF} may be of the same order as that of the fuel feeding pipeline J_1 and, sometimes, even exceed it. The values of J_{BF} used in dependence (1) have been determined by the formula obtained in the two ways: by experimental frequencies of cavitation bubble oscillations at J_1, which are significantly different [31], and by experimental time dependences of the pressure and the fluid flow rate at the pump inlet [32].

In previous studies [8—10], the results of dynamic tests of inducer centrifugal pumps have been summarized. In these studies, the inducer front pumps have a constant pitch, constant bushing diameter, and constant outer diameter. Among the pumps studied in this paper, there are pumps with variable length geometry (pumps 10 and 11). The results of dynamic tests of these pumps have been generalized given the following circumstances. Usually, the fluid flow enters the auger at a certain attack angle. As soon as the flow enters the interblade channel, the difference between the angle of the blade and the angle of the flow decreases rapidly. The speed of rotation of the fluid flow is affected by the fluid inertia. In [33], based on the analysis of the two-dimensional potential flow, it has been found that the flow turns at an angle equal to the attack angle at a distance from the inlet, which is approximately equal to the grid step. At this distance, the main part of the inducer pressure is created, and the bubble is localized during partial cavitation in the pump. For variable geometry inducers, it is very important how these geometric characteristics change over this distance. Further, to generalize the experimental data of inducers with variable geometry, we have used equivalent D_E, d_S, S, which are equal to their average values at a density that is equal to unity.

Out of the pool of experimental data for the pumps discussed in this research (see Tables 1 and 2), we have chosen 16 groups with close values of q within the range of q from 0.14 to 0.67. Figure 1 shows the experimental values of relative elasticity of cavitation bubbles $\tilde{B}_1$ for four values of flow rate coefficient q = 0.19, 0.27, 0.40, and 0.56.

Figure 1 shows that the values of $\tilde{B}_1$ for pumps of different purposes, sizes, and performance generally agree satisfactorily with each other for different values of the flow rate. There has been observed that the experimental data of eight pumps involved in this research agree well with those of 18 pumps summarized previously [10]. Separately, it should be noted that the experimental data of low-pressure booster pumps agree well with those of the high-pressure inducer centrifugal pumps, the pumps with constant and variable geometry at the pump inlet, the pumps with two and three blades, the pumps with axial, lateral, annular, and bilateral fluid feed, and the pumps in autonomous and fire tests. The difference between these parameters of the pumps and the test conditions, which have a lower level of influence, determine the spread of relative elasticity of the cavitation bubbles $\tilde{B}_1$.

To approximate the experimental data on the dependence of relative elasticity of cavitation bubbles $\tilde{B}_1(k^*, q)$ on the cavitation number k^* and the flow rate q we have used the formula:

$$\tilde{B}_1(k^*, q) = \frac{a(q) \cdot k^{*2} + b(q) \cdot k^* + B_{1O}(q)}{1 - \left(\frac{k^*}{k_O^*}\right)^2}, \qquad (2)$$

where $a(q)$, $b(q)$, and $B_{1O}(q)$ are some functions of flow rate q; k_O^* is the number of initial cavitation in the pump.

According to this formula, when the fluid pressure reaches the pressure of initial cavitation, the elasticity of the cavitation bubbles becomes equal to infinity (respectively, the reciprocal of the elasticity that is the flexibility of the cavitation bubbles is equal to zero). This reflects the physical meaning of initial cavitation in pumps. In addition, at $k^* = 0$ the relative elasticity of cavitation bubbles $\tilde{B}_1$ is not equal to zero, ass in the previous generalizations [8—10]. At $k^* = 0$, $\tilde{B}_1$ is equal to some function $B_{1O}(q)$. This reflects the experimental fact observed in the tests of pumps 1.1 and 2.1 that when approaching the pump breakdown pressure at different flow rates, the frequency of cavitation bubble oscillations tends to close (non-zero) values. The final value of the frequency of cavitation bubble oscillations in the case of pump breakdown indicates the final value of the volume of cavitation bubble under pump breakdown. It is possible that the cavitation-caused breakdown of the pump occurs at a certain volume of cavitation bubbles, regardless of the mode of operation of the pump in terms of flow rates. Thus, formula (2) correctly, in accordance with the known experimental data, describes cavitation phenomena in LRE pumps in the entire range of existence of cavitation bubbles.

Function $B_{1O}(q)$ has been determined based on the experimental data for $\tilde{B}_1$ of the studied pumps through their extrapolation to the zero cavitation

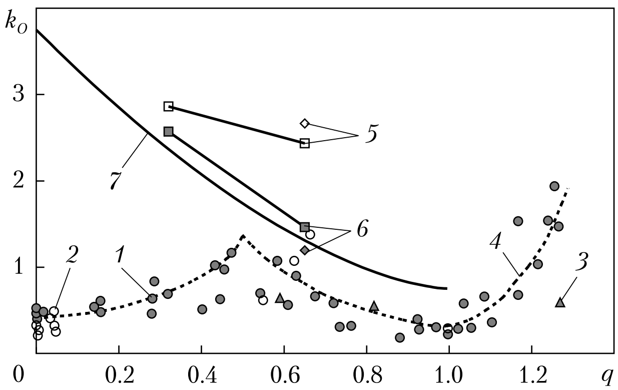

Fig. 2. Dependence of the number of initial cavitation in pumps on the flow rate: *1* — centrifugal pumps; *2* — diagonal pumps; *3* — axial pumps; *4* — envelope of experimental data [19]; *5* — inducer with a pointed leading edge that is perpendicular to the axis of rotation [19]; *6* — inducer with an arrow-shaped (beveled) leading edge [19]; *7* — approximation by points of dependence *6*

numbers. This function weakly depends on the flow rate q and may be represented by the empiric formula:

$$B_{1O}(q) = -0.004 - 0.147 \cdot q^{2.5}. \qquad (3)$$

Figure 2 shows the experimental initial cavitation numbers k_O in diagonal, centrifugal, and axial pumps depending on the flow rate q taken from [19]:

$$k_O = \frac{p_1 - p_S}{\rho \cdot W_{1E}^2 / 2},$$

where p_S is the pressure of fluid saturated vapor; $\rho \cdot W_{1E}^2 / 2$ is the high-speed fluid pressure on the outer diameter of the inducer.

In the absence of backflows at the pump inlet, within the range of variation of flow rate q from 0.5 to 1, initial cavitation number k_O monotonously decreases. In the presence of backflows at the pump inlet, there are different opinions about the appearance of the first cavitation bubbles in inducer centrifugal pumps. The authors of [29, 34] believe that when there are backflows in the pump, if the suction pressure is reduced, cavitation occurs near the periphery of the active flow at the entrance to the impeller, since there the relative velocities are the highest.

Research by Yu. M. Vasylieva and S. M. Kurochkin [19] has shown that “the place of the first cavitation bubbles are the central parts of the microvortices formed in the zone of separation of the boundary layer. In the throttling mode of the pump operation, observation of initial cavitation is difficult because of backflows that periodically “blow” the bubbles formed towards the main flow. Figure 2 presents some results of the experimental determination of the beginning of cavitation in inducer centrifugal pumps with high anti-cavitation qualities. The design of inducer front pumps with pointed leading edges significantly improves the cavitation safety margin at the breakdown pressure, but at the same time materially worsens it at the beginning of cavitation [19] (see positions 5 and 6 in Fig. 2). In addition, these studies have shown that in the presence of backflows at the pump inlet in the range of q from 0 to 0.5, initial cavitation number k_O monotonously decreases.

To approximate the dependence of initial cavitation number k_O for an inducer with a pointed arrow-shaped (beveled) leading edge (position 6 in Fig. 2) on flow rate q we propose the empirical formula:

$$k_O = 0.75 + 3(1 - q)^{1.6}. \qquad (4)$$

There is the following relationship between k_O and k_O^*:

$$k_O^* = \frac{p_1 - p_{BD\,III}}{p_1 - p_S} \; \frac{4}{\left(1 + \dfrac{d_S}{D_E}\right)^2} k_O.$$

According to formula (2), with the use of ratios (3) and (4), the obtained experimental values of $\tilde{B}_1$ for the studied pumps have been approximated, and unknown functions $b(q)$ and $a(q)$ have been determined:

$$a(q) = -5.30,$$
$$b(q) = -0.29 - 5.84 \cdot q - 7.77\, q^2. \qquad (5)$$

The values of $\tilde{B}_1$ calculated according to formulas (2)—(5) are presented in Fig. 1, They agree satisfactorily with the presented experimental data. However, part of the experimental data for some pumps is not consistent with the data of other pumps

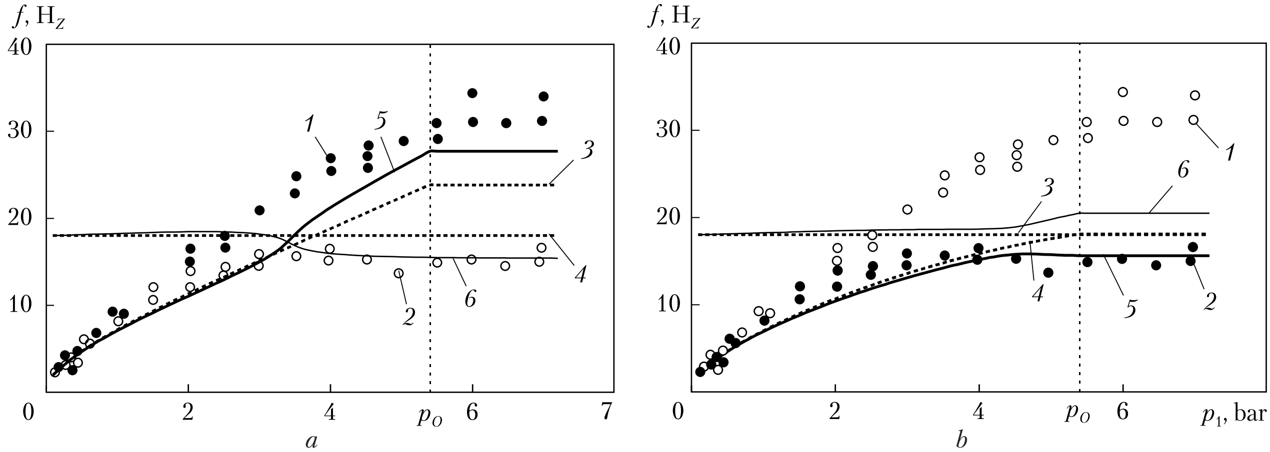

Fig. 3. Dependence of the frequencies of cavitation bubble oscillations in a hydraulic system with a cavitating pump with the short (*a*) and the long (*b*) pressure pipeline: *1*, *2* — experimental frequencies of oscillations with the short and the long pressure pipeline, respectively [35]; *3*, *4* — theoretical oscillation frequencies of the two lower vibration tones in the absence of interaction between the pipeline structure and the fluid [36]; *5*, *6* — theoretical frequencies of oscillations given this interaction (*5* — dominant roots of the characteristic equation); p_0 is the calculated pressure at the beginning of cavitation in the pump

and dependence (2) that summarizes the experimental values of the relative elasticity of cavitation bubbles $\tilde{B}_1$. These deviations have different causes and may be conditionally divided into three types.

We consider two types of deviations by the example of dynamic tests of pump 5.1, as described in [35]. Here, there are presented (see Fig. 3) the results of experimental studies on the determination of the dependence of the natural frequencies of oscillations in a hydraulic system with a cavitating pump on the pressure at the pump inlet for two significantly different pressure pipelines (0.4 and 10.5 m long). The natural frequencies of oscillations have been determined by transient processes in the hydraulic system, which are caused by impulse excitations at the pump inlet. The peculiarity of the test results is that in the range of pump inlet pressures from the breakdown one to 2 bar, the dependences of the natural frequencies on the inlet pressure, which are obtained at different pump loads, are close. In the pressure range from 2 bar to 7 bar, these dependences diverge sharply (see Fig. 3). In the case of the long pressure pipeline, this dependence is close to a horizontal straight line, i.e. the frequency of cavitation bubble oscillations becomes independent of the pressure at the pump inlet.

On the basis of mathematical modeling in [36], it has been shown that the observed discrepancy in the dependence of the frequency of cavitation bubble oscillations on the pressure at the pump inlet is caused by the interaction of the fluid and the structure of the pipeline, which in this case is sensitive to the length (impedance) of the pressure pipeline. This interaction leads to qualitatively different dependences of the oscillation frequencies on the pump inlet pressure.

Figure 3, *a* shows that in the case of the short pressure pipeline, the frequency of oscillations built on the dominant root of the characteristic equation (the root with the smallest modulo real part), after the intersection with the oscillation frequency of the structure, continues to increase with increasing pressure at the pump inlet (Fig. 3, *a*, curve *5*). With the long pressure pipeline (Fig. 3, *b*), the frequency of oscillations corresponding to the dominant root of the characteristic equation does not cross the frequency of oscillations of the structure and becomes constant as the pressure at the pump inlet increases.

Figure 3 shows that the calculated frequency of oscillations of the dominant root (curve *5*) practically coincides with the frequency of fluid oscillations (curve *3*) up to pressure at the pump

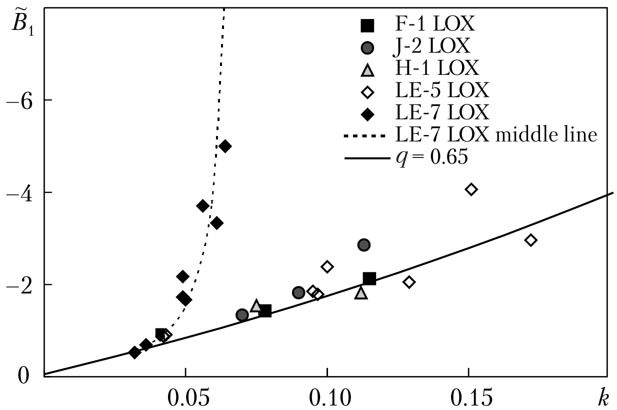

Fig. 4. Experimental relative elasticity of cavitation bubbles of some liquid oxygen pumps [37] and calculated dependence of relative elasticity of cavitation bubbles on the cavitation number for a flow rate of 0.65

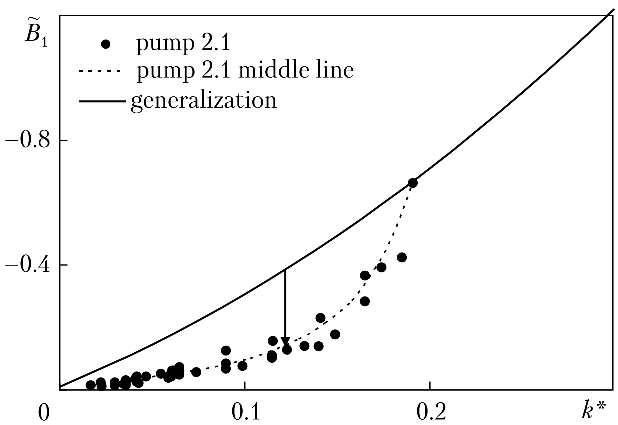

Fig. 5. Experimental relative elasticities of cavitation bubbles for pump 2.1 and calculated dependence of relative elasticity of cavitation bubbles on the cavitation number for the flow rate $q = 0.27$

inlet $p_1 = 3.2$ bar. As p_1 further increases, the frequency sharply deviates upward. In Fig. 3, *b* the calculated frequency of the dominant root (curve *5*) is close to the frequency of fluid oscillations (curve *4*) up to pressure $p_1 = 4.0$ bar. As p_1 further grows, the frequency becomes constant. So, Fig. 3, *a* and *b*, features two types of deviations of the frequency of oscillations of the related system from the frequency of oscillations of the fluid in this system. These deviations are caused by the interaction of the fluid and the structure of the pipeline system. Obviously, the experimental oscillation frequencies with such deviations cannot be used to determine elasticity of cavitation caverns $\tilde{B}_1$, since they are not natural frequencies of fluid oscillations.

The first type of deviations of the experimental frequencies of oscillations from the natural frequencies of fluid oscillations (see Fig. 3, *a*) has been observed in the results of dynamic tests of pumps 7, 9, 10, 11, and 12, with various intensities, mainly at high flows (and, therefore, at higher oscillation frequencies). The deviation of this type can be seen in Fig. 1, *d*, for pump 10.

This kind of deviations of experimental oscillation frequencies is apparently quite common. Thus, in addition to the dynamic tests of the six pumps under consideration, there have been known the studies of the liquid oxygen pump of Japanese LE-7 engine for which this type of deviation of the experimental oscillation frequencies have been reported [37] (see Fig. 4). Here, a characteristic feature of this type of deviations fully manifests itself: as the number of cavitation increases, firstly the experimental frequency of oscillations coincides with the frequency of fluid oscillations, but further deviates sharply upwards.

The second type of deviations of the experimental frequencies of oscillations from the natural frequencies of oscillations of the fluid (see Fig. 3, *b*) has been reported in the results of dynamic tests of pumps 13 and 14. During the fire tests of oxidizer 13 and fuel 14 booster pumps, their output loads are rather complex and included the main pumps as well as the engine piping all the way to the gas generator and combustion chamber. Such loads could very likely have an output impedance from these booster pumps with a large negative phase value. This makes the test conditions of pumps 5.1, 13, and 14 similar. It is likely that the second type of deviation of the experimental oscillation frequencies are found under complex loads that involve concentrated or distributed flexibility in the low range of oscillation frequencies.

Figure 5 shows the experimental relative elasticities of cavitation bubbles $\tilde{B}_1$ for pump 2.1 ($q = 0.27$), which significantly differ from the calculated dependency $\tilde{B}_1(k^*, q)$ (2)—(5). The deviations of the experimental points of pump 2.1 differ from the two types of deviations considered above. Near the region of existence of cavitation bubble self-oscillations ($k^* = 0.015$ i $k^* = 0.191$), where the observed self-oscillations are close to the harmonic one in terms of shape, these deviations are minimal. In the middle part of the region of instability, the amplitudes of oscillations increase and the very shape of oscillations that become discontinuous may change. Such self-oscillations are called developed self-oscillations of cavitation bubbles [1]. It has been shown in [38] that with large fluctuations of the pump parameters, there may be realized a blocking mode that is typical for the critical cavitation flow in the flow part of the pump. At the same time, the frequency of cavitation bubble self-oscillations may differ from the natural frequency of oscillations more than 2 times (see the arrow in Fig. 5).

This (third) type of deviations of the experimental frequencies of oscillations from the natural frequencies of fluid oscillations has been observed in some operating modes where there are developed cavitation bubble self-oscillations, during the dynamic tests of pumps 2.2—2.7, 1.1 (short feed pipeline), 1.3, and 1.4. The mathematical modeling of blocking modes in pumps [38] has shown that with developed cavitation bubble self-oscillations there is a delay in the time of pressure recovery at the pump inlet from cavitation hollow. This always leads to a decrease in the self-oscillation frequency as compared with the natural frequency of the fluid.

DETERMINATION OF THE VOLUME AND RESISTANCE OF CAVITATION BUBBLES

For solving the problems of low-frequency dynamics of LRPS and liquid rockets, the hydrodynamic model of LRE cavitating pumps [1, 4, 5, 11—16] includes elasticity of cavitation caverns $\tilde{B}_1(k^*, q)$, volume $\tilde{V}_C(k^*, q)$, and negative cavitation resistance $\tilde{B}_2(k^*, q)$. These parameters of cavitation flow in pumps may be determined based on the dependencies of the experimental frequencies of cavitation bubble oscillations on pressure at the pump inlet p_1 and flow rate through pump G. According to the definition of elasticity of cavitation bubbles:

$$\tilde{B}_1 = 1 \Big/ \frac{\partial \tilde{V}_C}{\partial k^*}.$$

According to combined (experimental and computational) method for determining the elasticity and the volume of cavitation bubbles [8], relative volume of cavitation bubble $\tilde{V}_C(k^*, q)$ may be found by integrating the expression:

$$\tilde{V}_C(k^*, q) = \int_{k^*}^{k_O^*} \frac{dk^*}{\tilde{B}_1(k^*, q)}. \qquad (6)$$

Having substituted formula (2) into (6), we obtain that

$$\tilde{V}_C(k^*, q) = \frac{1}{\sqrt{-\Delta}} \cdot \ln \frac{2ak^* + b - \sqrt{-\Delta}}{2ak^* + b + \sqrt{-\Delta}} \frac{2ak_O^* + b + \sqrt{-\Delta}}{2ak_O^* + b - \sqrt{-\Delta}}$$
$$\left(1 - \left(\frac{b}{2a \cdot k_O^*}\right)^2\right) - \frac{1}{k_O^{*2}} \cdot \left(\frac{k^* - k_O^*}{a} - \frac{b}{2a^2} \ln \frac{a \cdot k^{*2} + b \cdot k^*}{a \cdot k_O^{*2} + b \cdot k_O^*}\right),$$
$$\Delta = 4aB_{1O} - b^2. \qquad (7)$$

Using (7), we may determine negative cavitation resistance $\tilde{B}_2(k^*, q)$:

$$\tilde{B}_2(k^*, q) = -\frac{\partial \tilde{V}_C}{\partial q} \Big/ \frac{\partial \tilde{V}_C}{\partial k^*} = -\tilde{B}_1(k^*, q) \cdot \frac{\partial \tilde{V}_C}{\partial q}.$$

We do not present here the analytical expression for $\tilde{B}_2(k^*, q)$ because of its bulkiness In practical calculations, an approximate estimate that uses the same formula can be employed.

For transiting from dimensionless dependencies $\tilde{V}_C(k^*, q)$ and $\tilde{B}_2(k^*, q)$ to the dimensional ones $V_C(k^*, q)$ and $B_2(k^*, q)$ it is necessary to use the following formulas:

$$V_C(k^*, q) = \tilde{V}_C(k^*, q) \cdot V_{SM},$$

$$B_2(k^*, q) = \tilde{B}_2(k^*, q) \cdot \frac{\rho \cdot W_{1M}^2 / 2}{G_O}.$$

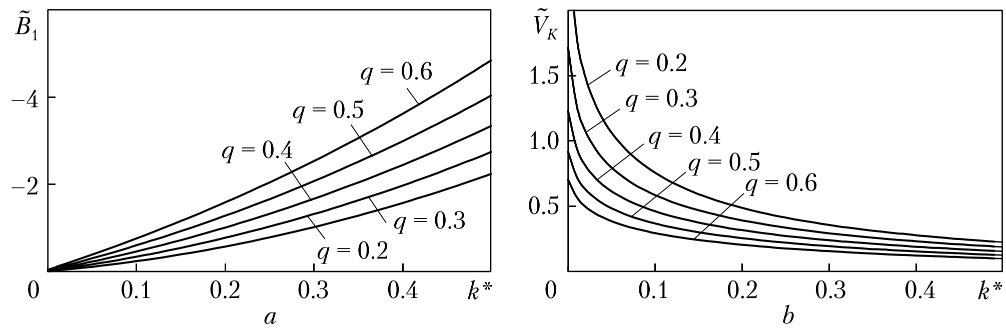

Fig. 6. Calculated dependences of the relative elasticity (*a*) and the volume (*b*) of cavitation bubbles on the number of cavitation and the flow rate in the region of probable existence of cavitation bubble self-oscillations

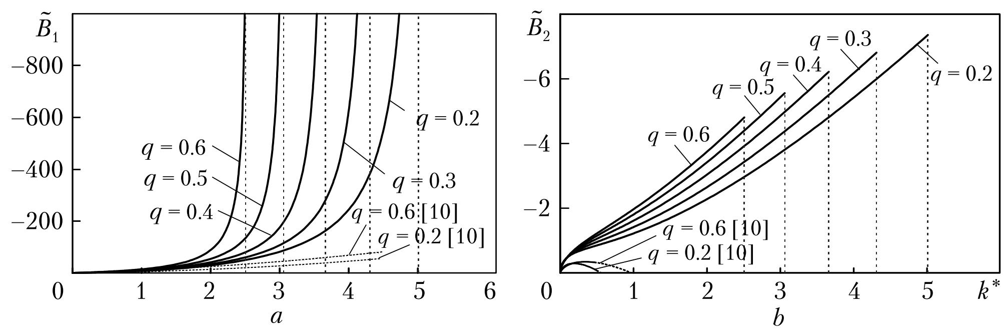

Fig. 7. Calculated dependences of the relative elasticity (*a*) and the relative resistance (*b*) of cavitation bubbles on the number of cavitation and the flow rate in the entire region of existence of cavitation bubbles, based on the data of this research and research [10]

Elasticity $\tilde{B}_1(k^*, q)$ and volume $\tilde{V}_C(k^*, q)$ of cavitation bubbles, which are built within the region of probable existence of cavitation bubble self-oscillations (see Fig. 6), have a conventional view. Within the range of cavitation number k^* up to 0.2, the values of $\tilde{B}_1(k^*, q)$ calculated by proposed formula (2) and those from [10] are numerically close to each other, as expected. However, they differ significantly in the entire region of existence of cavitation bubbles. Figure 7 shows elasticity $\tilde{B}_1(k^*, q)$ and resistance $\tilde{B}_2(k^*, q)$ of cavitation bubbles for various flow rates q, as calculated based on the data of this research and research [10]. One can see that according to the data of this research, at the numbers of initial cavitation (dashed lines), $\tilde{B}_1(k^*, q)$ tends to infinity (while the flexibility of cavitation bubbles tens to zero), whereas cavitation resistance $\tilde{B}_2(k^*, q)$ grows in modulus as k^* increases, but takes finite values. Elasticity $\tilde{B}_1(k^*, q)$ according to the data of research [10] increases slowly and takes finite values. Dependencies $\tilde{B}_2(k^*, q)$ obtained based on the data of this research qualitatively agree with the theoretical ones [1] and significantly differ from those determined previously by the combined experimental and computational method [8–10]. Figure 7, *b* shows that dependence $\tilde{B}_2(k^*, q)$ from research [10], and experimental and computational dependencies $\tilde{B}_2(k^*, q)$ from [8, 9] are similar. $\tilde{B}_2(k^*, q)$ has an extremum and crosses the ordinate axis much earlier than the cavitation begins. This limits the region of reliable definition of $\tilde{B}_1(k^*, q)$, $\tilde{V}_C(k^*, q)$, and $\tilde{B}_2(k^*, q)$ in [8–10].

This change of negative resistance of cavitation bubbles $\tilde{B}_2(k^*, q)$ is caused mainly by the chosen type of dependence of the initial cavitation number on the flow rate $k^*_O(q)$, which within the range from 0 to 0.5, according to the experimental studies of industrial pumps [19] (see Fig. 2), is taken increasing. Dependence of volume of cavitation bubbles $\tilde{V}_C(k^*, q)$ is stratified by flow rate q, with larger q corresponding to lesser $\tilde{V}_C$ (see Fig. 6 *b*). Initial cavitation numbers $k^*_O(q)$ are the boundary conditions for integrating dependence $\tilde{V}_C(k^*, q)$ (see Fig. 6). If the dependence of the initial cavitation number is an increasing function, near the beginning of cavitation, the stratification of $\tilde{V}_C(k^*, q)$ by q is broken, and curves $\tilde{V}_C(k^*, q)$ at various q cross each other. This inevitably leads to distortion of $\tilde{B}_2(k^*, q)$ (see Fig. 7, *b*) and limitation of the region of reliable definition of $\tilde{B}_1(k^*, q)$, $\tilde{V}_C(k^*, q)$, and $\tilde{B}_2(k^*, q)$.

CONCLUSIONS

According to the results of the dynamic tests of 26 LRE pumps, the experimental elasticities of cavitation bubbles in pumps with extended ranges of their main geometric and mode parameters have been determined. The results of experimental studies of the pumps have been obtained by various authors during fire and autonomous tests on various stands for several decades. The natural frequencies of fluid oscillations in hydraulic systems with the studied pumps have been obtained in various ways, by the following methods: self-excited cavitation bubble oscillations, pulsed excitations at the pump inlet, harmonic analysis of the working process in the mode, and resonant frequencies of oscillations in the frequency characteristics of the LRE fuel feeding systems.

It has been shown that the experimental values of the elasticity of cavitation bubbles for pumps of different purposes, sizes, and performance generally agree satisfactorily with each other for different values of the flow rate. It should be noted that the experimental data of booster low-pressure pumps with high-pressure inducer centrifugal pumps, pumps with constant and variable geometry at the pump inlet, pumps with two and three blades, pumps with axial, side, annular, and two-way fluid feeding systems agree satisfactorily as well.

The dependence of the relative elasticity of cavitation bubbles on the number of cavitation and the flow rate has been approximated by the formula that allows describing cavitation phenomena in pumps in the entire range of existence of cavitation bubbles. The dependence of the number of initial cavitation on the flow rate of LRE pumps with high anti-cavitation qualities has been explained.

Three types of the deviations of the experimental frequencies of oscillations on the natural frequencies of oscillations of fluid in a hydraulic system with a cavitating pump have been described. The first and second types of deviations are caused by the interaction of the fluid and the structure of the fluid feeding pipeline. In the case of the first type of deviations, the experimental frequencies of oscillations sharply deviate upward from the natural frequencies, while in the second case, they become a constant value. The third type of deviations is caused by developed cavitation bubble self-oscillations, when the frequency of the oscillatory process may decrease more than 2 times relative to the natural frequency of the fluid oscillations.

From the analytical expression of the relative elasticity of cavitation bubbles, the dependencies of the volume and the negative resistance of cavitation bubbles on the number of cavitation and the flow rate have been built. It has been shown that the dependence of the elasticity of cavitation bubbles at the numbers of initial cavitation tends to infinity, which significantly differs from the previously obtained dependences of the elasticity of cavitation bubbles. The dependence of the resistance of cavitation bubbles increases in modulus as the number of cavitation grows, which is consistent with the data of theoretical studies. The previously obtained dependences of the negative resistance of cavitation bubbles have an extremum and crossed the ordinate axis much earlier than the initial numbers of cavitation.

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УЗАГАЛЬНЕННЯ ЕКСПЕРИМЕНТАЛЬНОЇ ПРУЖНОСТІ КАВІТАЦІЙНИХ КАВЕРН У НАСОСАХ РРД, ЯКІ ІСТОТНО ВІДРІЗНЯЮТЬСЯ ЗА РОЗМІРАМИ ТА ПРОДУКТИВНІСТЮ

Вступ. Урахування кавітаційних явищ у насосах рідинних ракетних двигунів (РРД) є необхідним при визначенні частотних характеристик двигуна, при розрахунках перехідних процесів у двигунових установках при запуску й зупинці двигуна і, особливо, в задачі забезпечення стійкості поздовжніх коливань рідинних ракет (POGO-коливань).

Проблематика. Теоретичне визначення характеристик кавітаційних течій у насосах РРД нині не набуло поширення через вкрай низьку точність. Недоліком існуючих експериментально-розрахункових залежностей пружності, об'єму та опору кавітаційних каверн від режимних параметрів є обмежений діапазон чисел кавітації, в якому ці залежності достовірні.

Мета. Визначення пружності, об'єму та опору кавітаційних каверн у насосах РРД у всьому діапазоні існування кавітаційних каверн за результатами динамічних випробувань 26 насосів, що істотно відрізняються за призначенням, розмірами та продуктивністю.

Матеріали й методи. Використано інформаційно-аналітичний метод, методи теорії коливань, імпедансний метод та метод найменших квадратів.

Результати. Показано, що експериментальні значення пружності кавітаційних каверн для різних насосів загалом задовільно узгоджуються між собою. Проведено апроксимацію відносної пружності кавітаційних каверн від числа кавітації та коефіцієнта витрати за формулою, яка дозволяє описати кавітаційні явища в насосах у всьому діапазоні існування кавітаційних каверн. Описано три види відхилень експериментальних частот коливань від власних частот коливань рідини в гідравлічній системі з кавітуючим насосом. Перший і другий види відхилень обумовлені взаємодією рідини та конструкції живильного трубопроводу, третій — розвиненими кавітаційними автоколиваннями.

Висновки. Побудовано напівемпіричні залежності пружності, об'єму та опору кавітаційних каверн у насосах РРД від режимних параметрів у всьому діапазоні існування кавітаційних каверн.

Ключові слова: рідинний ракетний двигун, POGO-коливання, кавітуючий насос, частота кавітаційних коливань, експериментально-розрахунковий спосіб, пружність, об'єм і опір кавітаційних каверн, число початкової кавітації.