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THE INTERACTION OF THE PUNCH TOROIDAL SURFACE WITH THE EYELET DURING GROMMETING TEXTILE MATERIALS

Introduction. *For the effective use of equipment for setting metal fittings in light industry products, in particular in footwear, leather goods, textiles, corsetry, and curtain products, the regularities of changing the strain force acting on metal grommets during their interaction with the punch toroidal surface should be taken into account.*

Problems Statement. *For solving many practical problems, it is necessary to calculate the parameters of the strain force acting on metal accessories (in particular grommets) from the working surface of the tool. Knowing the force parameters allows us to calculate the strength of the punch, to predict its wear resistance, and to determine the necessary kinematic and energy characteristics of the equipment.*

Purpose. *Establishing the regularities of changes in the strain force acting on metal fittings during their interaction with the toroidal surface of the punch, given the conditions of contact interaction and the physical and mechanical properties of the fittings material.*

Materials and Methods. *Theoretical and experimental methods of research based on the classical laws of mechanics and the simulation of the process of deformation with the use of information technologies have been employed. Microsoft Excel has been used to create the computation program. The analytical calculations and the experiment have been made for a 08 kp steel eyelet.*

Results. *The dependence of the total strain force acting on the eyelet on the punch step has been analytically established; the process of grommet deformation by the toroidal surface of the punch has been experimentally studied. The strain forces are most significantly affected by: the grommet thickness, the radius of the punch toroidal part, the radius of the point of transition of the punch conical surface into the toroidal one.*

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Conclusions. *The shape of the working surfaces and the geometric parameters of the punch have a decisive influence on the process pressure values. The resulting dependencies can be used for developing new press equipment for grommeting light industry products, as well as for selecting the operating conditions of the existing equipment. This improves the quality of process operations, reduces the energy consumption of equipment, and increases the labor productivity.*

Keywords: grommet, insertion of accessories, deformation effort, punch.

For solving many practical problems, there is a need to calculate the force parameters of deformation of the metal fittings (in particular grommets) with the working surface of the tool. Knowledge of force parameters allows us to calculate the strength of the tool, to predict its durability, to determine the required power of the equipment. The analysis of research in this area has shown that the stress-strain state of such structures has been partially calculated [1, 2]. The solution of the problem of grommet deformation with a combination of punch shaping surfaces has not been studied so far. Analytical ratios of this dependence determine the influence of the geometric parameters of the punch on its interaction with the grommet and, as a consequence, makes it possible to control the deformation force. The use of the obtained dependencies contributes to the efficient use of metal accessories in light industry products, including footwear, leather goods, textile, corset, curtain products [3—5].

The grommet deformation process consists of the three stages determined by the geometry of the punch's shaping surface [6].

The schematic representation of the deformation process stages is shown in Fig. 1 [1, 6].

The interaction of the working tool with the grommet at the first two stages has been described in [6].

This research deals with the force interaction of the working tool with the grommet at the third stage, when there is the deformation of grommets by the punch toroidal part. The radius of curvature of this surface is denoted by r_t (Fig. 1, *c*). The surface is an extension of the conical part, but the angles of inclination of the adjacent surfaces are different.

The mathematical apparatus of the calculation depends on the nature of the eyelet and the tool. They may contact with each other in the three ways: no contact, the contact is concentrated along the eyelet edge, and complete contact between the surfaces. Identifying these stages of deformation is an important task for the further calculations.

So, let us consider the initial stage of the grommet transition from the conical part of the punch to the toroid surface. As mentioned before, the angles of inclination of the conical and the toroid surfaces of the punch at the point of their convergence are different. Therefore, at the first stage of the transition, there is no contact with the shaping surface. Under the action of tangential stresses that take positive values on the conical part of the punch, the diameter of the upper end of the grommet slightly decreases before contact with the toroid surface at a certain point in deformation. Let us denote the grommet part that has come into the punch toroidal zone as l. As a result of the contact between the edge of the eyelet and the toroidal surface of the punch, a distributed load appears on the edge, the intensity of which we denote as p. The contact stresses result in an increase in the diameter of the eyelet end face and a growth in the deformation load, at a certain stage of which, the eyelet takes a conical shape (Fig. 2) [9].

Figure 2 shows part of the grommets that has come into the toroidal cavity of the punch at the time when the eyelet takes a conical shape. In this case, φ_0 indicates the angle of inclination of the radius, which connects the eyelet edge with the center of the toroid cross section. Let us determine the length of the grommet part within the toroid surface at this point. It determines the punch step, where there is no contact with the toroidal part and for which we do not calculate the load by the proposed algorithm.

Let us consider triangle ΔO_tBA and draw the altitude O_tC to the side AB. The angle AO_tC is equal to the angle of inclination α_{k2} of the punch conical part to the vertical axis, since they are the angles with mutually perpendicular sides. From this, it follows that the angle of inclination of the radius is:

$$\varphi_0 = 2\alpha_{k2}. \quad (1)$$

The magnitude of the toroidal part of the grommet is equal to:

$$l_0 = 2r_t \cdot \sin\alpha_{k2}. \quad (2)$$

With the further sliding on the toroidal surface, in the grommet, there appears a bending strain. This leads to meridal stresses and, as a consequence, plastic bending deformations in a certain cross-section of the eyelet. The nature of the interaction and, therefore, the mathematical apparatus of the calculations depend on the location of the section where there are plastic deformations. If the plastic bending deformations are at the point of the transition of the conical surface into the toroidal one, the upper part of the eyelet revolves, remaining elastic, and the parts contact along the circular line. Let us assume that the grommet completely fits the toroidal surface when plastic bending deformations act in the middle part of the grommet, which has passed into the punch toroidal zone. Let us determine the moment of beginning of the complete fit of the surfaces of the punch and the grommet, considering the condition of equilibrium of the fragment of the toroidal part of the grommet. The scheme of forces and torques that act in this case is shown in Fig. 3 [9].

Figure 4 shows a grommet fragment limited by two meridional planes turning one with respect to other by the angle γ, by the upper end of the eyelet and the horizontal plane located at the bottom of the punch toroidal surface. The load p distributed along the arc, the tangential stresses σ_θ, the friction forces acting on the line of contact $t_m = f \cdot p$, and the plastic torque M, the place of which application is unknown, but on the scheme shown in the lower part of the grommet, act on

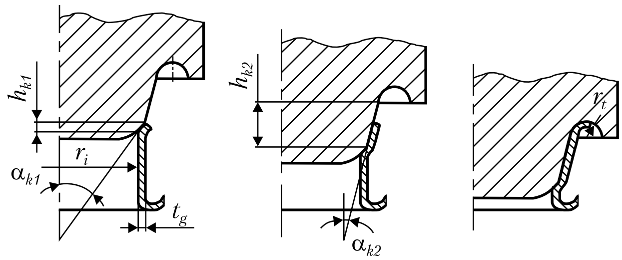

Fig. 1. Schemes of characteristic stages of grommet deformation: *a* — the first stage; *b* — the second stage; *c* — the third stage; r_i is the grommet internal radius; t_g is the grommets thickness; r_t is the radius of the punch toroidal part; h_{k1} is the height of the part of the punch, which is deformed at the first stage; α_{k1} is the angle of inclination to the axis at the first stage; h_{k2} is the height of the part of the punch, which is deformed at the second stage; α_{k2} is the angle of inclination to the axis in the second stage

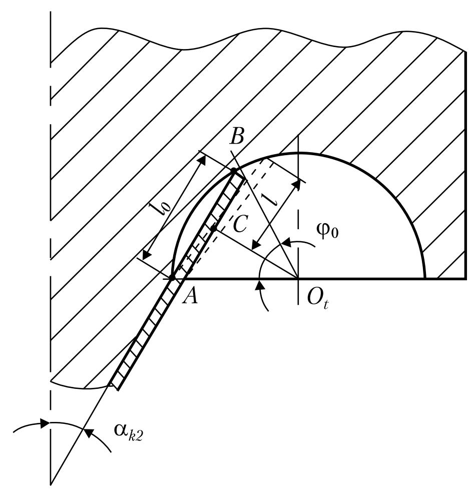

Fig. 2. The initial stage of the grommet transition into toroidal part of the punch: l is the intermediate value; α_{k2} is the angle of inclination to the axis; φ_0 is the angle of inclination of the radius; l_0 is the magnitude of the toroidal part of the grommet

the fittings. It is assumed that the friction forces act between the fittings and the punch and are calculated by the Coulomb law. The logical considerations indicate that the torques caused by the load p, tangential stresses and friction forces turn the fragment in opposite directions. The torques they create depend on the height of the grommet fragment and change during deformation. There-

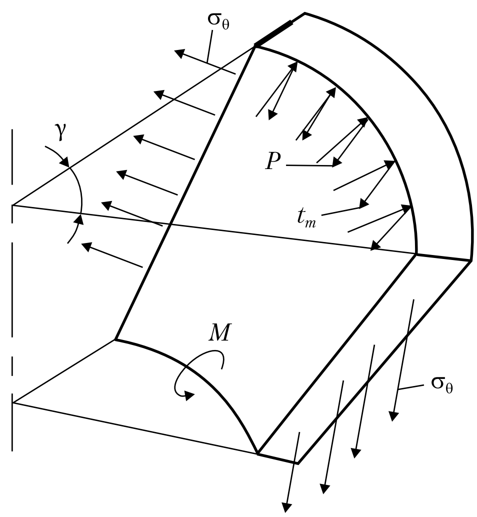

Fig. 3. Scheme of forces and torques acting on a fragment of the grommet toroidal part: γ is the angle of rotation of the meridional planes; σ_θ are the tangential stresses; p is the load; t_m are the friction forces; M is the plastic torque

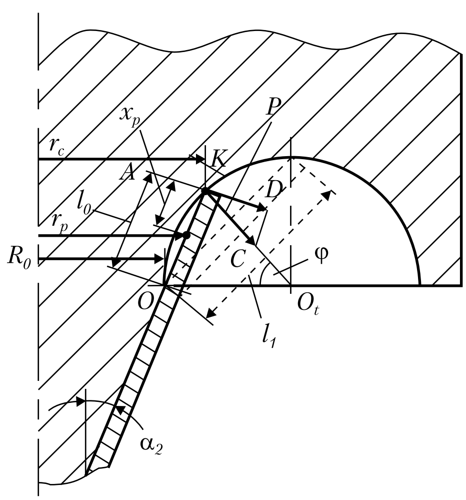

Fig. 4. The axial section of the punch toroidal part and the grommet fragment: α_2 is the angle of inclination to the axis; R_0 is the radius of the point of transition of the conical surface into the toroidal one; r_p is the radius of cross-section of the toroidal surface where plastic bend occurs; r_c is the radius of the punch and grommet contact point; x_p is the coordinate of the section where plastic bend acts; l_0 is the size of the initial contact area of the toroidal part; l_1 is the size of the toroidal part, where the contact becomes solid; φ is the angle of rotation of the radius

fore, it is possible to determine the location of the plastic torque in advance. To detect the cross-section where it is applied, let us consider the conditions of equilibrium of the fitting fragment.

The scheme of forces and geometric parameters of the grommet and punch is convenient to consider in their axial section, as shown in Fig. 4 [9].

The net contact load P is determined as:

$$P = p \cdot r_c \cdot \gamma \cdot 6.28, \tag{3}$$

where r_c is the radius of the punch and grommet contact point.

To analyze the equilibrium condition, let us determine some geometric parameters. The AD line is directed perpendicular to the cross-section of the grommet. Based on the fact that the force of contact interaction is directed under the radius of the toroidal part, we determine the angle of inclination of CKD between the direction of AD and this radius. From the triangle ΔOO_tK it follows that the angle OKO_t is equal to $(180° - \varphi)/2$. Given the fact that the angle OKD is equal to 90, we get that $CKD = \varphi/2$.

Let us write the equation of equilibrium of torques in the cross-section of the grommet fragment with coordinate x, the count of which begins from the upper end:

$$M = -P \cdot \cos\left(\frac{\varphi}{2}\right) \cdot x + P \cdot f \sin\left(\frac{\varphi}{2}\right) \cdot x + q\frac{x^2}{2}. \tag{4}$$

In the above equation q indicates the distributed load that is caused by the action of tangential stresses:

$$q = 2\sigma_\theta \cdot \sin\left(\frac{\gamma}{2}\right) \cdot \cos\left(\frac{\varphi}{2}\right) \cdot t_g. \tag{5}$$

Based on the condition that the effect of contact load and tangential stresses ensures the emergence of a plastic torque in the grommet, the specific value of the torque can be determined by the known ratio:

$$M = \frac{\sigma_{s\rho} \cdot t_g^2}{4}, \tag{6}$$

where $\sigma_{s\rho}$ is the meridional stresses that act in the cross-section during plastic deformation.

It should be noted that their value is lower than the tension of fluidity as a result of mutual influence of tangential and meridional stresses. The same is true for the tangential stresses $\sigma_{s\theta}$. Given that the torque acts in an arc within the angle γ, its full value is equal to:

$$M = \frac{6.28\sigma_{s\rho} \cdot t_g^2 \cdot r_p \cdot \gamma}{4 \cdot 360}, \qquad (7)$$

where r_p is the radius of cross-section of the toroidal surface where plastic bend occurs.

For cross-section with coordinate $x = l$, the radius is $r_p = R_0$. Based on simple geometric considerations, it is possible to determine it in the average section of the toroidal part of the grommet:

$$r_p = R_0 + r_t \cdot \left(1 - \cos\frac{\varphi}{2}\right), \qquad (8)$$

where R_0 is the radius of the point of transition of the conical surface of the punch into the toroidal one.

With a certain approximation, ratio (8) can be written as function of the punch $\Delta H_l = H - (h_{k1} + h_{k2})$, which leads to the transition of the grommet to the toroidal zone:

$$r_p = R_0 + r_t\left(1 - \cos\left(\frac{H - (h_{k1} + h_{k2})}{2 \cdot r_t}\right)\right). \qquad (9)$$

Let is determine the stresses $\sigma_{s\rho}$ in this case. Neglecting the stresses that are normal to the grommet surface, the equation of relationship between stresses and strains in this case we write as follows:

$$\frac{\xi_\rho}{\xi_\theta} = \frac{\sigma_{s\rho}}{\sigma_{s\theta}},$$

where ξ_ρ, ξ_θ are the meridional and tangential deformations.

From the above, it follows that

$$\sigma_\rho = \frac{\xi_\rho \cdot \sigma_\theta}{\xi_\theta}. \qquad (10)$$

Given the insignificant difference between the radius of the toroidal surface and the thickness of the eyelet ($t_g / r_t = 0.4$), it can be assumed that the meridional tensile stresses act on the entire cross-section, and the neutral surface is located on its inner surface. Then the meridional deformations can be determined as follows:

$$\xi_\rho = \frac{t_g}{r_t - t_g}. \qquad (11)$$

Let us write the plasticity condition for this case as

$$\sigma_{s\rho} - \sigma_{s\theta} = \sigma_s, \qquad (12)$$

where σ_s is the stress under uniaxial load.

From the above, it follows that

$$\sigma_{s\theta} = \sigma_s - \sigma_{s\rho}. \qquad (13)$$

Substituting relationships (9), (10) into (13), we get:

$$\sigma_{s\rho} = \frac{\xi_\rho \cdot \sigma_s}{\xi_\theta\left(1 + \frac{\xi_\rho}{\xi_\theta}\right)}. \qquad (14)$$

The tangential deformities is found as follows (Fig. 4):

$$\xi_\theta = \frac{r_p - r_i}{r_i}. \qquad (15)$$

Then for the lower section of the toroidal part:

$$\xi_\theta = \frac{R_0 - r_i}{r_i}, \qquad (16)$$

in the middle of the toroidal part:

$$\xi_\theta = \frac{R_0 + r_t\left(1 - \cos\left(\frac{\varphi}{2}\right)\right) - r_i}{r_i} = \frac{R_0 + r_t\left(1 - \cos\frac{H - (h_{k1} + h_{k2})}{2r_t}\right) - r_i}{r_i}. \qquad (17)$$

Given this, equation (4) can be written as follows:

$$-P \cdot \cos\left(\frac{\varphi}{2}\right) \cdot x_p + P \cdot f \cdot \sin\left(\frac{\varphi}{2}\right) \cdot x_p + \frac{q \cdot x_p^2}{2} = \frac{6.28\sigma_{s\rho} \cdot s^2 \cdot r_p \cdot \gamma}{4 \cdot 360}, \qquad (18)$$

where x_p is the coordinate of the section under the action of plastic bending strain.

The equation has the three unknowns: the part of the punch that determines the angle of incli-

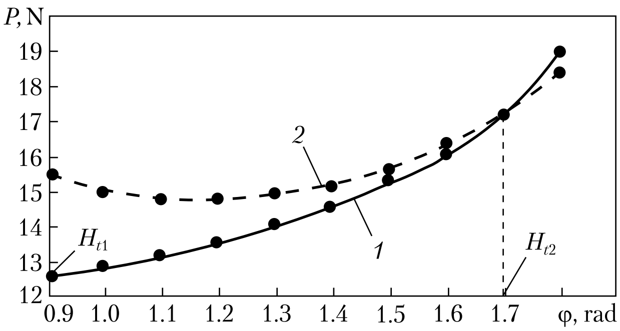

Fig. 5. The dependence of the contact force P on the angle of rotation of the toroidal part of the grommet φ: *1* — underr plastic deformation in the lower part; *2* — inside the toroidal part of the grommet

nation φ, $\left(\varphi = \dfrac{H - (h_{k1} + h_{k2})}{r_i}\right)$, the coordinate x_p and the equilibrium contact load P. Of course, to obtain the solution of this equation analytically is impossible, so we offer the following algorithm [6].

In a certain range, there are set several values of rotation of the radius of the toroidal part of the punch φ. For each, the length l of the toroidal part of the grommet is determined by the ratio $l = 2r_t \cdot \sin\varphi$. Using the equilibrium equation, we find the contact load P. The calculations are made for the bottom of the grommet, where $x = l$, and in the middle, where $x = l/2$, provided in these sections, there act plastic deformations. At this angle of rotation, the plastic bending strain acts in the section where the load is smaller. If plastic deformation acts in the lower plane, the toroidal part of the grommet remains almost straight and contact is realized along the contour. If the plastic torque acts in the middle part of the grommet, there is a complete fit of the surfaces of the grommet and the punch. To perform the calculations, we assume that the angle φ ranges within $0.9 \leqslant \varphi \leqslant 2.5$ rad.

The lower boundary of this range is equal to $2\alpha_{k2}$ at which the toroidal part of the grommet becomes straightforward, while the upper one is taken approximately, which corresponds to the grommet contact after the transition through the upper point of the punch toroidal part.

The angle γ is taken equal to γ = 0.26 rad. (γ = = 15°). It is known that the tangential stress is equal to the stress of approximately $\sigma_s = 200$ MPa; the radius of the beginning of the toroidal surface is $R_0 = 0.0055$ m; the thickness of the grommet wall is $t_g = 0.0003$ m.

Let us determine the contact load, assuming that $\sigma_\theta = \sigma_s$, from equation (18):

$$P = \frac{2\sigma_s \cdot \sin\left(\dfrac{\gamma}{2}\right) \cdot \cos\left(\dfrac{\varphi}{2}\right) \cdot t_g \left(\dfrac{x_p}{2}\right)^2 + \dfrac{6.28\sigma_{s\rho} \cdot t_g^2 \cdot r_p \cdot \gamma}{4 \cdot 360}}{x_p\left(\cos\left(\dfrac{\varphi}{2}\right) - f \cdot \sin\left(\dfrac{\varphi}{2}\right)\right)}. \quad (19)$$

The x_p takes two values: $x_p = l$ and $x_p = l/2$; $r_p = R_0$ or determined by ratio (9) [6].

The results of the calculations are shown in Fig. 4 that shows the dependence of the equating contact load distributed across the grommet section between the angle of rotation of the radius that connects this end with the center and the center of the toroidal surface. The H_{t1}, H_{t2} mark the boundaries of the punch where the contact occurs. For convenience of analysis, Fig. 5 shows the dependence of the punch step l on the angle of rotation of the radius φ.

At this stage of deformation, the increase in the slider step is equal to the length of the toroidal part of the grommet, while the step is determined by the ratio:

$$H = h_{k1} + h_{k2} + 2r_t \cdot \sin\varphi. \quad (20)$$

As it comes from the above dependencies, within the range $51° \leqslant \varphi \leqslant 97°$, the contact is concentrated and takes place along the circle. At higher angles, the grommet is fully fitted to the punch surface. More precisely, the transition to complete fit occurs at the upper point of the toroidal surface at angle φ = 90°. The specified range can be written in the magnitudes of the toroidal part of the grommet:

$$l_0 \leqslant \Delta H_l \leqslant l_1, \quad (21)$$

where $l_1 = 2r_t \cdot \sin 90°$ is the value of the toroidal part, in which the contact becomes continuous.

The calculations have shown that the boundaries of the concentrated and distributed on the surface contacts take place at other friction coefficient in the range $0.1 \leqslant f \leqslant 0.2$, and the radius of the toroid surface significantly affects the strain force and the boundaries of the concentrated and distributed contacts. Figures 6, 7 feature the corresponding graphs calculated for $r_t = 0.001$ m and $r_t = 0.0015$ m.

Analyzing these graphs, we see a significant effect of the radius of the toroidal part at the time of transition to continuous contact. At $r_t = 0.001$ m and $r_t = 0.0015$ m, the punch step and the deformation force decrease [6].

Therefore, when the radius of the punch toroidal part is equal to $r_t = 0.00075$ m, at a punch step within $h_{k1} + h_{k2} \leqslant H \leqslant h_{k1} + h_{k2} + 2 \cdot r_t \cdot \sin 51°$, there is no contact on the toroid surface, and the load is not determined by this algorithm.

Within $h_{k1} + h_{k2} + 2 \cdot r_t \cdot \sin 51° \leqslant H \leqslant h_{k1} + h_{k2} + 2 \cdot r_t \cdot \sin 90°$, the contact is concentrated and takes place along the circle. The strain force can be determined by ratio (20).

To explain the geometric considerations, let us consider Fig. 8 that features the scheme of interaction of the grommet and the punch. This scheme shows an increase in the strain force acting on the toroidal part of the grommet ΔN_t and the contact force P. From the triangle ABO_t it follows that the angle ABO_t between the vectors $\Delta\bar{N}_t$ and $\bar{P}$ is equal to $90 - \varphi$.

Given the fact that the force P is the normal projection of ΔN_t on the contact surface, we write:

$$\Delta N_t = \frac{P}{\cos(90° - \varphi)} = \frac{P}{\sin\varphi}. \qquad (22)$$

To determine the total deformation, it should assumed that the contact load acts along the circle, not along the arc. Therefore, the value obtained from (19) shall be multiplied by $6.28 / \gamma$. It can be assumed with a certain approximation that, at this stage, the bending strain at $x_p = l_1$ is determined by ratio (20). In this case, the strain force is equal to:

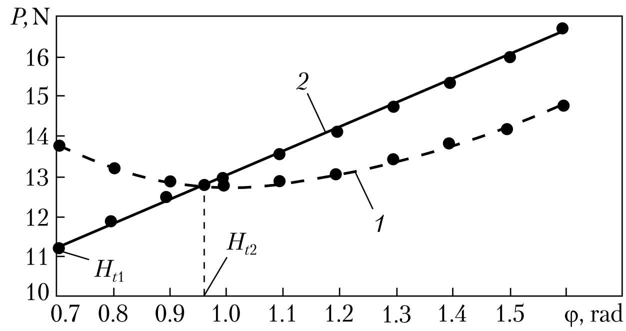

Fig. 6. Dependence of the contact force P on the angle of rotation of the grommet toroidal part φ at $r_t = 0.001$ m: *1* — under plastic deformation in the lower part; *2* — inside the toroidal part of the grommet

$$\Delta N_t = \frac{360}{\gamma} \times$$

$$\times \frac{2\sigma_s \cdot \sin\left(\frac{\gamma}{2}\right) \cdot \cos\left(\frac{\varphi}{2}\right) \cdot t_g \left(\frac{x_p}{2}\right)^2 + \frac{6.28\sigma_{s\rho} \cdot t_g^2 \cdot r_p \cdot \gamma}{4 \cdot 360}}{x_p \left(\cos\left(\frac{\varphi}{2}\right) - f \cdot \sin\left(\frac{\varphi}{2}\right) \cdot \sin\varphi\right)}. (23)$$

As a function of the punch step, with the toroidal part of the slider denoted as $H_t = H - (h_{k1} + h_{k2})$, ratio (23) is written as follows:

$$\Delta N_t = \frac{360}{\gamma} \times$$

$$\times \frac{2\sigma_s \cdot \sin\left(\frac{\gamma}{2}\right) \cdot \cos\left(\frac{H_t}{2r_t}\right) \cdot t_g \left(\frac{H_t}{2}\right)^2 + \frac{6.28\sigma_{s\rho} \cdot t_g^2 \cdot r_p \cdot \gamma}{4 \cdot 360}}{H_t \left(\cos\left(\frac{H_t}{2r_t}\right) - f \cdot \sin\left(\frac{H_t}{2r_t}\right) \cdot \sin\left(\frac{H_t}{r_t}\right)\right)}. \quad (24)$$

The previous studies have found that in the case of deformation of the toroidal surface of the punch, when the slider moves with a step within $H \geqslant h_{k1} + h_{k2} + 2 \cdot r_t \cdot \sin 90°$, the contact is continuous. The strain force at this stage is determined based on the condition of the work balance (Fig. 9) [6].

This condition can be written as follows:

$$A_\partial = N_t \cdot H_t. \qquad (25)$$

That is, the work performed by the punch at the step H_t and the load N_t is equal to the work of grommet deformation A_∂ consisting of the two

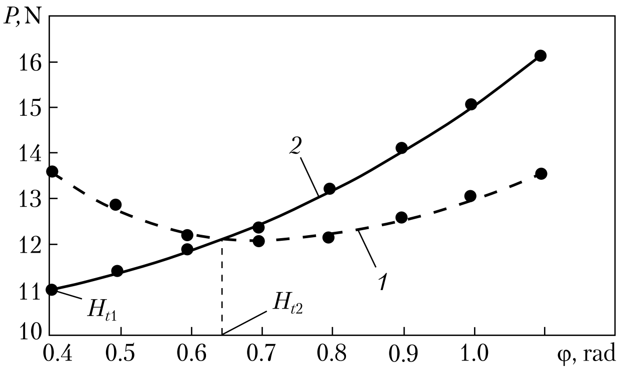

Fig. 7. Dependence of the contact force P on the angle of rotation of the grommet toroidal part φ at $r_t = 0.0015$ m: *1* — under plastic deformation in the lower part; *2* — inside the toroidal part of the grommet

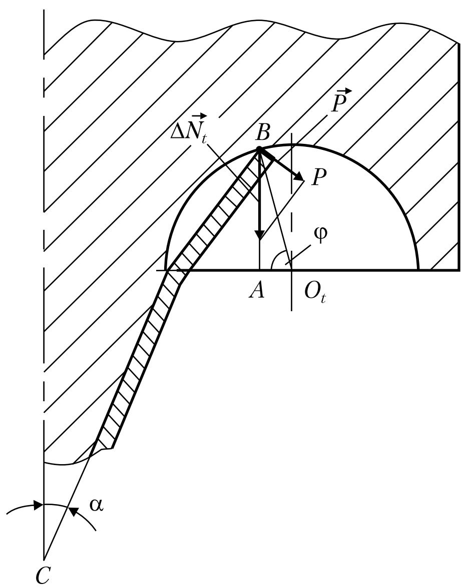

Fig. 8. Scheme of interaction of the grommet and the toroidal surface of the punch: α is the angle of inclination to the axis; ΔN_t is the strain force acting on the toroidal part of the grommet; φ is the angle of rotation of the radius; P is the contact force

parts: the work of the meridional A_ρ stresses and the work of the tangential stresses A_θ:

$$A_\partial = A_\rho + A_\theta. \quad (26)$$

Let us determine these values depending on the punch step. The angle of rotation of the radius of the toroid surface is connected with the punch step by the ratio $\varphi = H_t / r_t$. The specific work of meridional stresses is equal to the product of the torque acting in the cross-section of the grommet and the angle of rotation of this section φ:

$$A_\rho = M \cdot \varphi, \quad (27)$$

its increase at this stage of deformation is

$$\Delta A_\rho = M \cdot \Delta\varphi = \frac{M \cdot \Delta H_t}{r_t}. \quad (28)$$

The specific torque can be determined by previously mentioned ratio (4).

Given that the torque acts along the circle, its total value is equal to:

$$M = \frac{3.14\sigma_{s\rho} \cdot t_g^2 \cdot R}{2}, \quad (29)$$

where R is the radius of the cross-section of the toroidal surface where the torque acts.

Based on the geometric considerations, the radius is determined by the formula:

$$R = R_0 + r_t \cdot (1 - \cos\varphi) = R_0 + r_t\left(1 - \cos\left(\frac{H_t}{r_t}\right)\right). \quad (30)$$

The meridional stresses are determined by ratio (14):

$$\sigma_{s\rho} = \frac{\xi_\rho \cdot \sigma_s}{\xi_\theta\left(1 + \frac{\xi_\rho}{\xi_\theta}\right)}, \quad (31)$$

while the tangential stresses are found from the plasticity conditions (13): $\sigma_{s\theta} = \sigma_s - \sigma_{s\rho}$.

Using (30), we determine the tangential strain as follows:

$$\xi_\theta = \frac{R_0 + r_t\left(1 - \cos\frac{H - (h_{k1} + h_{k2})}{r}\right) - r_i}{r_i}. \quad (32)$$

The specific increase in the tangential stresses is equal to:

$$\Delta A_\theta = 6.28\sigma_\theta \cdot \xi_\theta \cdot R, \quad (33)$$

and the total increase in the tangential stresses is

$$\Delta A_\theta = 6.28\sigma_\theta \cdot \xi_\theta \cdot R \cdot t_g \cdot \Delta H_t, \quad (34)$$

Where the product of the grommet thickness t_g and the increase in the punch step at this stage of deformation ΔH_t is the increase in the cross-section area of the toroidal part of the grommet.

Given ratio (32), we get:

$$\Delta A_\theta = \frac{\sigma_\theta \left(R_0 + r_t \cdot \left(1 - \cos\left(\frac{\Delta H_t}{r_t} \right) \right) - r_i \right) \cdot t_g \cdot 6.28 R \cdot \Delta H_t}{r_i}. \quad (35)$$

Given ratios (34), (35) the equation of the work balance is as follows:

$$N_t \cdot \Delta H_t = \frac{M \cdot \Delta H_t}{r_t} + \frac{\sigma_\theta \left(R_0 + r_t \cdot \left(1 - \cos\left(\frac{H_t}{r_t} \right) \right) - r_i \right) \cdot t_g \cdot 6.28 R \cdot \Delta H_t}{r_i}. \quad (36)$$

From the above, it follows that:

$$N_t = \frac{M}{r_t} + \frac{\sigma_\theta \left(R_0 + r_t \left(1 - \cos\left(\frac{H_t}{r_t} \right) \right) - r_i \right) \cdot t_g \cdot 6.28 R}{r_i}. \quad (37)$$

It should be noted that the resulting ratio allows us to calculate the increase in the strain force on the toroid surface of the punch, rather than the total strain force.

As a result of the research, we have developed the algorithm for calculating the strain force, depending on the punch step throughout the deformation period. The calculation begins from the moment of contact of the punch with the grommet for which the step is equal to zero $h = 0$. At the first and second stages, the deformation is realized by the lower and upper cones of the punch with forces that are determined by the following ratios [6].

For the former cone, at $0 \leqslant H \leqslant h_{k1}$:

$$N_{k1} = \frac{-\sigma_s \left(1 + \frac{tg\alpha_{k1}}{f} \right) \cdot \left[1 - \left(\frac{r_i}{r_i + H \cdot tg\alpha_{k1}} \right)^{\frac{f}{tg\alpha_{k1}}} \right] \cdot 6.28 r_i \cdot t_g}{r_i}, \quad (38)$$

its highest value at $H = h_{k1}$ is

$$N_{h1} = \frac{-\sigma_s \left(1 + \frac{tg\alpha_{k1}}{f} \right) \cdot \left[1 - \left(\frac{r_i}{r_i + h_{k1} \cdot tg\alpha_{k1}} \right)^{\frac{f}{tg\alpha_{k1}}} \right] \cdot 6.28 r_i \cdot t_g}{r_i}. \quad (39)$$

For the latter cone, at $h_{k1} \leqslant H \leqslant h_{k2}$:

$$N_{k2} = \frac{-\sigma_s \left(1 + \frac{tg\alpha_{k2}}{f} \right) \times \left[1 - \left(\frac{r_{0k2}}{r_{0k2} + (H - h_{k1}) \cdot tg\alpha_{k2}} \right)^{\frac{f}{tg\alpha_{k2}}} \right] \cdot 6.28 r_{0k2} \cdot t_g}{\cos\alpha_{k2}} + \frac{-\sigma_s \left(1 + \frac{tg\alpha_{k1}}{f} \right) \cdot \left[1 - \left(\frac{r_i}{r_i + h_{k1} \cdot tg\alpha_{k1}} \right)^{\frac{f}{tg\alpha_{k1}}} \right] \cdot 6.28 r_i \cdot t_g}{\cos\alpha_{k1}}, \quad (40)$$

its highest value at $H = h_{k1} + h_{k2}$ is equal to:

$$N_{h_{k2}} = \frac{-\sigma_s \left(1 + \frac{tg \quad _{k2}}{f} \right) \times \left[1 - \left(\frac{r_{0k2}}{r_{0k2} + h_{k2} \cdot tg\alpha_{k2}} \right)^{\frac{f}{tg \quad _{k2}}} \right] \cdot 6.28 r_{0k2} \cdot t_g}{\cos\alpha_2} + \frac{-\sigma_s \left(1 + \frac{tg\alpha_{k1}}{f} \right) \cdot \left[1 - \left(\frac{r_{0k1}}{r_i + h_{k1} \cdot tg\alpha_{k1}} \right)^{\frac{f}{tg \quad _{k1}}} \right] \cdot 6.28 r_i \cdot t_g}{\cos\alpha_1}. \quad (41)$$

Within $h_{k2} \leqslant H \leqslant h_{k2} + 2r_t \cdot \sin\alpha_{k2}$, practically, there is no contact on the toroidal surface, and according to this algorithm, the strain force is not calculated, its value is determined by interpolation of the strain forces in the adjacent intervals. The next stage of deformation, where the contact is realized by the end of the grommet with the toroidal surface of the punch, takes place within the limits $h_{k2} + 2r_t \cdot \sin\alpha_{k2} \leqslant H \leqslant h_{k2} + 2r_t \cdot \sin 90°$, and the force is determined by the ratio:

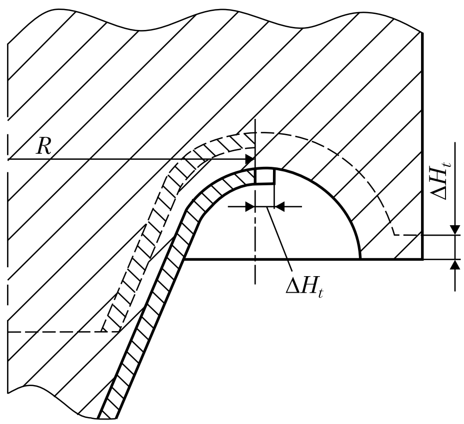

Fig. 9. Deformation by the toroidal surface of the punch at continuous contact: ΔN_t is the strain force acting on the toroidal part of the grommet; R is the radius of the cross-section of the toroidal surface where the torque works

$$N_t = \frac{360}{\gamma} \times \frac{2\sigma_s \cdot \sin\left(\frac{\gamma}{2}\right) \cdot \cos\left(\frac{H_t}{2r_t}\right) \cdot t_g \left(\frac{H_t}{2}\right)^2 + \frac{6.28\sigma_{s\rho} \cdot s_g^2 \cdot r_p \cdot \gamma}{4 \cdot 360}}{H_t \left(\cos\left(\frac{H_t}{2r_t}\right) - f \cdot \sin\left(\frac{H_t}{2r_t}\right)\right) \cdot \sin\left(\frac{H_t}{r_t}\right)} + N_{h_{k2}}. \quad (42)$$

At the last stage, at $h_{k2} + 2r_t \cdot \sin 90° \leqslant H \leqslant h_{k2} + 3.14r_t$, the force is determined by the ratio:

$$F_{fix} = \frac{M}{r_t} + \frac{\sigma_\theta \left(R_0 + r_t\left(1 - \cos\left(\frac{H_t}{r_t}\right)\right) - r_i\right) \cdot t_g \cdot 6.28R}{r_i} + N_t. \quad (43)$$

The plastic torque M and σ_θ are calculated by ratios (13), (14), (15), and (29).

According to formula (43), the dependence of the strain force acting on the grommet on the punch step has been calculated (Fig. 10). The calculations have been made for a 08 kp steel grommet in Microsoft Excel [8].

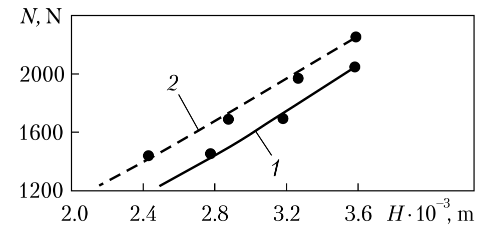

Fig. 10. Dependence of the force of grommet deformation by the punch toroidal part on the punch step: *1* — calculated values; *2* — experimental values

To confirm the adequacy of the obtained mathematical model (43) of the dependence of the total strain force acting on the grommet N on the step of the punch H, experimental studies of the process of grommet deformation by the toroidal surface of the punch have been conducted. With the help of the measuring system, we have obtained diagrams of the dependence of the strain force acting on the grommet N and the punch step H.

The strain force acting on the grommet from the toroidal part of the punch obtained analytically has been compared with the experimental results (Fig. 10).

Having analyzed the obtained results, we conclude that the shape of the working surfaces and the geometric parameters of the punch have a significant influence on the force N. The most influencing parameters are the grommet thickness t_k; the radius of the toroidal part of the punch r_t, the radius of the point of transition of the punch conical surface into the toroidal one R_0.

The difference between the calculated and the experimental values of the strain force N is about 16%. This is an acceptable error. Thus, formula (43) can be used for approximate calculations of the maximum strain forces acting on metal grommet from toroidal punch surface N [9].

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ВЗАЄМОДІЯ ТОРОЇДАЛЬНОЇ ПОВЕРХНІ ПУАНСОНА З ЛЮВЕРСОМ ПРИ ЙОГО ВСТАНОВЛЕННІ В ТЕКСТИЛЬНІ МАТЕРІАЛИ

Вступ. Для ефективного використання обладнання для встановлення металевої фурнітури у вироби легкої промисловості, зокрема у взуттєві, шкіргалантерейні, текстильні, корсетні, гардинні вироби, слід враховувати закономірності зміни зусилля деформування металевих люверсів при їх взаємодії з тороїдальною поверхнею пуансона.

Проблематика. При вирішенні практичних задач виникає потреба обчислення силових параметрів деформування металевої фурнітури (зокрема люверсів) робочою поверхнею інструмента. Знання силових параметрів дозволяє розрахувати міцність пуансона, прогнозувати зносостійкість, визначати необхідні кінематичні та енергетичні характеристики обладнання.

Мета. Встановлення закономірностей зміни зусилля деформування металевої фурнітури при її взаємодії з тороїдальною поверхнею пуансона з урахуванням контактної взаємодії та фізико-механічних властивостей матеріалу фурнітури.

Матеріали й методи. Використано методи досліджень, що базуються на класичних законах механіки та моделювання технологічного процесу деформації з використанням інформаційних технологій. Для програми розрахунку застосовано *Microsoft Excel*. При аналітичних обчисленнях та в експерименті було використано металевий люверс, виготовлений зі сталі 08 кп.

Результати. Встановлено аналітично закономірності зміни сумарного зусилля деформування люверса від ходу пуансона та проведено експериментальні дослідження процесу деформування люверса тороїдальною поверхнею пуансона. Найбільш суттєво на зусилля деформування впливають: товщина люверса, радіус тороїдальної частини пуансона, радіус точки переходу конічної поверхні пуансона в тороїдальну.

Висновки. Визначальний вплив на технологічне зусилля мають форма робочих поверхонь та геометричні параметри пуансона. Отримані залежності можна застосувати при розробці нового пресового обладнання для встановлення металевої фурнітури у вироби легкої промисловості та при виборі режимів роботи обладнання. Це дозволить підвищити якість виконання технологічних операцій, знизити енергоємність обладнання та підвищити продуктивність праці.

Ключові слова: люверс, встановлення фурнітури, зусилля деформування, пуансон.