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ON RANDOM ATTRACTOR OF SEMILINEAR STOCHASTICALLY PERTURBED WAVE EQUATION WITHOUT UNIQUENESS

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In this paper we investigate the dynamics of solutions of the semilinear wave equation, perturbed by additive white noise, in sense of the random attractor theory. The conditions on the parameters of the problem do not guarantee uniqueness of solution of the corresponding Cauchy problem. We prove theorem on the existence of random attractor for abstract noncompact multi-valued random dynamical system, which is applied to the wave equation with non-smooth nonlinear term. A priory estimate for weak solution of randomly perturbed problem is deduced, which allows to obtain the existence at least one weak solution. The multi-valued stochastic flow is generated by the weak solutions of investigated problem. We prove the existence of random attractor for generated multi-valued stochastic flow.

INTRODUCTION

In [1], [2], [3] as an adequate mathematical apparatus for describing the dynamics of a stochastically perturbed evolution systems has been proposed concept of random attractor, which has been applied to the stochastically perturbed reaction-diffusion system and 2D Navier-Stokes system, white noise-driven Burgers equation and to nonlinear wave equation with smooth nonlinear term. Recently many results concerning various properties of random attractors have appeared (see [4], [5] and the references therein). In particular, random attractors for stochastic damped nonlinear wave equation were investigated in [6], [7], [8]. Beginning with pioneering work [9], the ideas and methods of classical theory of global attractors systematically applied in the case of non-uniqueness of Cauchy problem. Modern research in this field with many applications contained in the monograph [10]. Theory of random attractors has been generalized for multi-valued case in [11], [12], [13] for the systems with attracting random compact set, and in [14], [15], [16] for the compact systems, which are dissipative in probability. In this paper we obtain a result on the existence and properties of random attractor for abstract asymptotically compact multi-valued random dynamical system (MRDS), which made it possible to prove the existence of random attractor for the semilinear wave equation with non-smooth nonlinear term, perturbed by additive white noise.

SETTING OF THE PROBLEM

In bounded domain $Q \subset R^n$, $n \geq 3$ we consider the problem:

$$\begin{cases} du_t + (\beta u_t - \Delta u + f(u))dt = \phi dw, \\ u|_{\partial Q} = 0, \end{cases} \tag{1}$$

where w is Wiener process, $\beta > 0$ and $\phi \in H^2(Q) \cap H_0^1(Q)$ are given, nonlinear term f is continuous (not necessary smooth) function, which satisfies conditions:

$$| f(u) | \leq C_1 (1 + | u |^{\frac{n}{n-2}}), \tag{2}$$

$$f(u)u \geq C_2 (F(u) - 1), \ F(u) \geq C_3 (| u |^{\frac{2n-2}{n-2}} - 1), \tag{3}$$

where $F(u) = \int_0^u f(s)ds.$

In [2], [6], [7] under additional conditions smoothness of f and $| f'(u) | \leq C_4 (1 + | u |^{p-1})$, $p < \dfrac{n}{n-2}$ authors proved the existence and investigated dimension of random attractor for stochastic flow, generated by the problem (1). Without restrictions on derivative of f two difficulties appear: absence of uniqueness of weak solution of Cauchy problem and impossibility of decomposition of the flow on compact and decaying parts. In deterministic case $(\phi = 0)$ in [17], using the apparatus of energy equations, the existence of global attractor for corresponding multi-valued semigroup was proved only under condition (2) and sign condition $\liminf\limits_{|u| \to \infty} \dfrac{f(u)}{u} > -\lambda_1$. Using energy equations approach, in [8] was proved existence of random attractor for stochastic wave equation with critical exponents on R^3. For non-autonomous case, when $f = f(t,u)$, and function $f(t,u)$ is smooth only on the first variable, theorems about existence of global attractor for multi-valued dynamical processes were obtained in [18].

The aim of this paper is to prove the existence of random attractor for multi-valued stochastic flow, generated by the problem (1), under conditions (2), (3). For this purpose we prove result about existence of random attractor for abstract non-compact MRDS system, which made it possible to investigate dynamics of solutions of the problem (1).

MULTI-VALUED RANDOM DYNAMICAL SYSTEMS

Let $(X, \| \cdot \|)$ be separable Banach space with Boreal σ-algebra $\sigma(X)$, $C(X)$ is the set of all non-empty closed subsets of X, for $A, B \subset X$ we denote:

$\overline{A}$ is closure of A in X;

$$\operatorname{dist}(A,B)=\sup_{x\in A}\inf_{y\in B}\|x-y\|;$$

$$O_\delta(A)=\{y\in X \mid \operatorname{dist}(y,A)<\delta\};$$

$$B_r=\{x\in X \mid \|x\|\le r\},\ \|A\|_+=\sup_{a\in A}\|a\|.$$

Let (Ω,Φ,P) be a probability space, $\overline{\Phi}$ is P-completion of Φ, $\{\theta_t:\Omega\mapsto\Omega\}_{t\in R}$ is metric dynamical system [4], that is a measure preserving group of transformations in Ω such that the map $(t,\omega)\mapsto\theta_t\omega$ is measurable, where parameter t takes valued in R endowed with Boreal σ-algebra $\sigma(R)$.

The map $F:\Omega\mapsto C(X)$ is called random set $F(\omega)$, if F is measurable, that is the function $\omega\mapsto\operatorname{dist}(y,F(\omega))$ is measurable.

Definition 1. The map $G:R_+\times\Omega\times X\mapsto C(X)$ is called MRDS, if

1) $\forall\, x\in X$ the map $(t,\omega)\mapsto G(t,\omega)x$ is measurable;

2) $\forall\,\omega\in\Omega,\ \forall\, t,s\ge0,\ \forall\, x\in X$

$$G(0,\omega)x=x,\ \ G(t+s,\omega)x\subseteq G(t,\theta_s\omega)G(s,\omega)x.$$

Note, that it will be enough to assume that condition 2) takes place on θ_t-invariant set of full measure.

Definition 2. The random set $A(\omega)$ is called random attractor of MRDS G, if for P-almost all (P-a.a.) $\omega\in\Omega$:

1) $A(\omega)$ is compact;

2) $A(\theta_t\omega)\subset G(t,\omega)A(\omega)\ \ \forall\, t\ge0$;

3) $\forall\, r>0\ \ \operatorname{dist}(G(t,\theta_{-t}\omega)B_r,A(\omega))\to0,\ t\to+\infty$.

Theorem 3. Let assume that MRDS G satisfies the following conditions:

1) there exists P-almost everywhere (P-a.e.) bounded random set $B(\omega)$ such, that for P-a.a. $\omega\in\Omega$, $\forall\, r>0$

$$\operatorname{dist}(G(t,\theta_{-t}\omega)B_r,B(\omega))\to0,\ t\to+\infty; \tag{4}$$

2) $\forall\, t\ge0,\ \forall\,\omega\in\Omega$ the map $G(t,\omega):X\mapsto C(X)$ has compact valued and is upper semicontinuous;

3) $\forall\, r>0$ the map $(t,\omega)\mapsto\overline{G(t,\omega)B_r}$ is measurable;

4) for P-a.a. $\omega\in\Omega$, $\forall\, r>0$, $\forall\, t_n\uparrow+\infty$ arbitrary sequence $\xi_n\in$ $\in G(t_n,\theta_{-t_n}\omega)B_r$ is precompact in X.

Then the set

$$A(\omega)=\overline{\bigcup_{r>0}\Lambda_{B_r}(\omega)},\ \text{ where }\ \Lambda_B(\omega)=\bigcap_{T>0}\overline{\bigcup_{t\ge T}G(t,\theta_{-t}\omega)B} \tag{5}$$

is random attractor of MRDS G. It is P-a.e. unique and is a minimal among closed sets, satisfying (4).

Remark 4. From condition (4) we have existence of P-a.a. bounded random set $D(\omega)\supset B(\omega)$ such, that for P-a.a. $\omega\in\Omega$, $\forall\, r>0$

$$\exists\, T = T(\omega, r)\ \ \forall\, t \ge T\ G(t, \theta_{-t}\omega)B_r \subset D(\omega). \tag{6}$$

Remark 5. In the paper [11] theorem 3 was proved under condition of compactness of $B(\omega)$ (which is stronger than condition 4)), in paper [14] it was proved under condition of precompactness of $G(t,\omega)B_r$, which is also stronger than condition 4), because

$$\xi_n \in G(t_n, \theta_{-t_n}\omega)B_r \subset G(1, \theta_{-1}\omega)G(t_n - 1, \theta_{-t_n}\omega)B_r \subset G(1, \theta_{-1}\omega)D(\theta_{-1}\omega).$$

Proof. It is well known [5], that

$$y \in \Lambda_B(\omega) \Leftrightarrow \exists\, t_n \uparrow +\infty\ \ \exists\, y_n \in G(t_n, \theta_{-t_n}\omega)B : y_n \to y.$$

So from 4) $\exists\, \Omega_0$, $P(\Omega_0) = 1$ such that $\forall\, \omega \in \Omega_0$, $\forall\, r > 0$ $\Lambda_{B_r}(\omega) \ne \varnothing$. Assuming that $\forall\, \omega \in \Omega_0$ condition 4) takes place, we have that $\forall\, \omega \in \Omega_0$, $\forall\, r > 0$ $\Lambda_{B_r}(\omega) \subset B(\omega)$. Further $\forall\, \omega \in \Omega_0$, $\forall\, \{z_n\} \subset \Lambda_{B_r}(\omega)$ $\exists\, t_n \uparrow +\infty$, $\exists\, \xi_n \in G(t_n, \theta_{-t_n}\omega)B_r$ such that $\| \xi_n - z_n \| \le \frac{1}{n}$. Then due to 4) the sequence $\{z_n\}$ is precompact, so $\forall\, \omega \in \Omega_0$, $\forall\, r > 0$ $\Lambda_{B_r}(\omega)$ is compact. Let us prove that $\forall\, \omega \in \Omega_0$, $\forall\, r > 0$ $\Lambda_{B_r}(\omega)$ attracts B_r in the sense of (4). If not, then $\exists\, \delta > 0$ $\exists\, t_n \uparrow +\infty$, $\exists\, y_n \in G(t_n, \theta_{-t_n}\omega)B_r$ such that $dist(y_n, \Lambda_{B_r}(\omega)) \ge \delta$. But on some subsequence $y_n \to y \in \Lambda_{B_r}(\omega)$, and we have contradiction. So $\forall\, \omega \in \Omega_0$ the set $A(\omega) = \overline{\bigcup_{r>0} \Lambda_{B_r}(\omega)}$ satisfies condition 3) of Definition 2. Moreover, $A(\omega) \subset B(\omega)$, so $A(\omega)$ is bounded P-a.e. Let us prove that $A(\omega)$ is compact P-a.e. Let us put $\Omega^0 = \bigcap_{n=1}^{\infty} \Omega_n$, where $\Omega_n = \{\omega \in \Omega_0 \mid \theta_{-n}\omega \in \Omega_0\}$. Then $P(\Omega^0) = 1$. For $\omega \in \Omega^0$ let us define the set $K(\omega) = \bigcup_{r>0} \Lambda_{B_r}(\omega)$. For every sequence $\{\xi_n\} \subset K(\omega)$ and for arbitrary $n \ge 1$ we can find $r_n > 0$ such that $\xi_n \in \Lambda_{B_{r_n}}(\omega)$. Then

$$\forall\, n \ge 1\ \forall\, \omega \in \Omega^0\ \ \exists T = T(\omega, n)\ \ \forall\, t \ge T\ \ G(t, \theta_{-t}\omega)B_{r_n} \subset D(\theta_{-n}\omega).$$

For $n \ge 1$ $\exists\, \eta_n \in \bigcup_{t \ge T+n} G(t, \theta_{-t}\omega)B_{r_n}$ such that $\| \eta_n - \xi_n \| \le \frac{1}{n}$. So $\forall\, n \ge 1$ $\exists\, m \ge T$ such that

$$\eta_n \in G(n+m, \theta_{-m-n}\omega)B_{r_n} \subset G(n, \theta_{-n}\omega)G(m, \theta_{-m}\theta_{-n}\omega)B_{r_n} \subset$$

$$\subset G(n, \theta_{-n}\omega)D(\theta_{-n}\omega).$$

Let us consider for arbitrary $R > 0$ sets:

$$\Omega(R) = \{\omega \,|\, \|D(\omega)\|_{+} \le R\},$$

$$\Omega_{\infty}(R) = \{\omega \,|\, \theta_{-n}\omega \in \Omega(R) \text{ for infinitely many } n\}.$$

Due to Poincare's recurrence theorem $P(\Omega_{\infty}(R)) \ge P(\Omega(R))$. As the set $D(\omega)$ is bounded P-a.e., we have $P(\Omega_{\infty}(R)) \to 1$, $R \to \infty$. For $\omega \in \Omega^0 \cap$ $\cap\, \Omega_{\infty}(R)$ we can find subsequence $\{n_k(\omega)\}_{k=1}^{\infty}$ such that for every $k \ge 1$ $\eta_{n_k} \in G(n_k, \theta_{-n_k}\omega)B_R$. So $\forall\, R > 0$, $\forall\, \omega \in \Omega^0 \cap \Omega_{\infty}(R)$ $K(\omega)$ is precompact in X and we deduce that $A(\omega)$ is compact P-a.e.

From 3) one can easy obtain (see [11]) that the map

$$\omega \mapsto \bigcup_{n=1}^{\infty} \bigcap_{k=0}^{\infty} \overline{\bigcup_{t=k}^{\infty} G(t, \theta_{-t}\omega)B_n}$$

is $\overline{\Phi}$-measurable. So from [3] there exists compact random set $\widetilde{A}(\omega)$ such that $\widetilde{A}(\omega) = A(\omega)$ for P-a.a. $\omega \in \Omega$. Thus the set $\widetilde{A}(\omega)$ is compact random set, which satisfies condition 3) of definition 2. Then from results of [11] MRDS G has random attractor, which coincides with $A(\omega)$ P-a.e. and it is a minimal among closed sets, satisfying (4). Theorem is proved.

RANDOM ATTRACTOR FOR STOCHASTIC FLOW, GENERATED BY THE PROBLEM (1)

We consider problem (1) under conditions (2),(3), w is two-sided real Wiener process. Let us consider canonical Wiener probability space (Ω, Φ, P). Then we have $w(t,\omega) = \omega(t)$ $\forall\, t \in R$ and formula $\theta_s\omega(t) = \omega(t+s) - \omega(s)$ defines metric dynamical system $\{\theta_s : \Omega \mapsto \Omega\}_{s \in R}$ [4].

Following to [2], we define $W = W(t,\omega)$ as a solution of the problem

$$\begin{cases} dW_t + \beta W_t = dw, \\ W(0) = W_t(0) = 0. \end{cases} \tag{7}$$

We make the change of variable $v(t) = u(t) - \phi W(t)$. Equation (1) turns into

$$\begin{cases} v_{tt} + \beta v_t - \Delta v + f(v + \phi W(t)) = \Delta \phi W(t), \\ v|_{\partial Q} = 0. \end{cases} \tag{8}$$

Now we deduce a priory estimate for weak solution $\varphi = \begin{pmatrix} v \\ v_t \end{pmatrix}$ of randomly perturbed problem (8) in the phase space $X = H_0^1(Q) \times L^2(Q)$ with usual norm $\|\varphi\|_X$, $|\cdot|$ is a norm in $L^2(Q)$, $\|\cdot\|$ is a norm in $H_0^1(Q)$.

Multiplying (8) by $\widetilde{v} = v_t + \eta v$ for $\eta > 0$ small enough, we have [2]

$$\frac{1}{2}\frac{d}{dt}(|\widetilde{v}|^2+\|v\|^2)+\eta\|v\|^2+(\beta-\eta)|\widetilde{v}|^2-\eta(\beta-\eta)(\widetilde{v},v)+$$

$$+(f(u),\widetilde{v})=(\widetilde{v},\Delta\phi W),$$

where for sufficiently small $\eta>0$:

$$\eta\|v\|^2+(\beta-\eta)|\widetilde{v}|^2-\eta(\beta-\eta)(\widetilde{v},v)\geq\frac{\eta}{2}(\|v\|^2+|\widetilde{v}|^2),$$

$$|(\widetilde{v},\Delta\phi W)|\leq\frac{\eta}{4}|\widetilde{v}|^2+C_\eta|W(t)|^2,$$

$$(f(u),\widetilde{v})\geq\frac{d}{dt}(F(u),1)+\eta C_2(F(u),1)-\eta C_1|Q|-(f(u),\phi(W_t+\eta W)),$$

$$|(f(u),\phi)\|W_t+\eta W|\leq\frac{C_2\eta}{2}(F(u),1)+C_\eta(1+|W_t+\eta W|^{\frac{2n-2}{n-2}}).$$

From the above inequalities for some small $\delta>0$ we obtain

$$\frac{d}{dt}(|\widetilde{v}|^2+\|v\|^2+(F(u),1))+\delta(|\widetilde{v}|^2+\|v\|^2+(F(u),1))\leq g(t,\omega),\tag{9}$$

where

$$g(t,\omega)=C(1+|W(t,\omega)|^{\frac{2n-2}{n-2}}+|W_t(t,\omega)|^{\frac{2n-2}{n-2}}),$$

and $C>0$ is some constant, which does not depend on ω.

So from Gronwall's lemma $\forall\, t\geq s$

$$\begin{gathered}|\widetilde{v}(t)|^2+\|v(t)\|^2+(F(u(t)),1))\leq\\ \leq e^{-\delta(t-s)}(|\widetilde{v}(s)|^2+\|v(s)\|^2+(F(u(s)),1))+\int_s^t e^{-\delta(t-\tau)}g(\tau,\omega)d\tau.\end{gathered}\tag{10}$$

From (10) we deduce final estimate: $\exists C>0\ \ \forall\, t\geq s\ \ \forall\,\omega\in\Omega$

$$\|\varphi(t)\|_X^2\leq C(1+e^{-\delta(t-s)}\|\varphi(s)\|_X^{\frac{2n-2}{n-2}}+\int_s^t e^{-\delta(t-\tau)}g(\tau,\omega)d\tau).\tag{11}$$

From estimate (11) we can claim [18] that $\forall\,\omega\in\Omega\ \ \forall\, s\in R\ \ \forall\,\varphi_s\in X$ there exists at least one (weak) solution of (8) $\varphi(\cdot)=\begin{pmatrix}v(\cdot)\\ v_t(\cdot)\end{pmatrix}$ on $[s,+\infty)$, $\varphi(s)=\varphi_s$. In further arguments we denote it by $\varphi(t,\omega,s,\varphi_s)$. Moreover, every solution of (8) on $[s,+\infty)$ belongs to $C([s,+\infty);X)$, satisfies (11) and the following equality:

$$\frac{d}{dt}I_\omega(t,\varphi(t))+\beta I_\omega(t,\varphi(t))=H_\omega(t,\varphi(t)),\tag{12}$$

where

$$I_\omega(t,\varphi(t))=\frac{1}{2}|v_t|^2+\|v\|^2+(F(u),1)+\frac{\beta}{2}(v_t,v),$$

$$H_\omega(t,\varphi(t)) = (v_t, \Delta\phi W(t)) + \frac{\beta}{2}(v, \Delta\phi W(t)) - \frac{\beta}{2}(f(u), v) +$$

$$+ (f(u), \phi W_t) + \beta(F(u),1), \;\; u(t) = v(t) + \phi W(t).$$

Moreover, we have the following result.

Lemma 6. [18] Let $\omega_n \to \omega_0$ in Ω, $t_n \to t_0 \ge s$, $\varphi_n(\cdot)$ is solution of (8) on $[s,+\infty)$ with random parameter ω_n, $\varphi_n(s) \to \varphi_s$ weakly in X. Then there exists $\varphi(\cdot)$ — solution of (8) on $[s,+\infty)$ with random parameter ω_0, $\varphi(s) = \varphi_s$ such that on some subsequence

$$\varphi_n(t_n) \to \varphi(t_0) \text{ weakly in } X,$$

$$H_{\omega_n}(t_n, \varphi_n(t_n)) \to H_{\omega_0}(t_0, \varphi(t_0)).$$

If, moreover, $\varphi_n(s) \to \varphi_s$ strongly in X, then $\varphi_n(t_n) \to \varphi(t_0)$ strongly in X.

Let us put $\overline{W} = \begin{pmatrix} \phi W \\ \phi W_t \end{pmatrix}$ and define the maps:

$$S : R_d \times \Omega \times X \mapsto P(X),$$

$$S(t,s,\omega)\varphi_0 = \{\varphi(t,\omega,s,\varphi_0 - \overline{W}(s,\omega)) + \overline{W}(t,\omega)\}, \tag{13}$$

$$G : R_+ \times \Omega \times X \mapsto P(X),$$

$$G(t,\omega)\varphi_0 = \{\varphi(t,\omega,0,\varphi_0) + \overline{W}(t,\omega)\}. \tag{14}$$

It is easy to show [2], that for every $s \in R$

$$G(t,\omega)x = S(t+s, s, \theta_{-s}\omega)x, \tag{15}$$

and the map S for every $\omega \in \Omega$ generates multi-valued process [18], that is

$$\forall \tau \in R \;\; S(\tau,\tau,\omega)x = x, \;\; \forall t \ge r \ge s \;\; S(t,s,\omega)x \subset S(t,r,\omega)S(r,s,\omega)x.$$

The main result of the paper is the following.

Theorem 7. The formula (14) defines MRDS, which has random attractor in the phase space $X = H_0^1(Q) \times L^2(Q)$.

Proof. Let us prove condition 2) of definition 1. From (14) $\forall\, \omega \in \Omega$ $\forall\, x \in X$ $G(0,\omega)x = x$. For $t_1, t_2 \ge 0$ we have

$$G(t_1 + t_2, \omega)x = S(t_1 + t_2 + s, s, \theta_{-s}\omega)x \subset$$

$$\subset S(t_1 + t_2 + s, t_2 + s, \theta_{-s}\omega)S(t_2 + s, s, \theta_{-s}\omega)x =$$

$$= S(t_1 + t_2 + s, t_2 + s, \theta_{-t_2 - s}\theta_{t_2}\omega)S(t_2 + s, s, \theta_{-s}\omega)x = G(t_1, \theta_{t_2}\omega)G(t_2,\omega)x.$$

Now let us verify conditions of theorem 3 (from conditions 2), 3) we, in particular, obtain, that the map G has closed values and is measurable). Let

$\xi_n \in G(t,\omega)\eta_n$. Then $\xi_n = \varphi_n(t,\omega,0,\eta_n) + \overline{W}(t,\omega)$ and from lemma 6 we have condition 2). Let us consider for fixed $a \in R$, $r \geq 0$, $y \in X$ the set

$$C = \{(t,\omega) \mid \operatorname{dist}(y, \overline{G(t,\omega)B_r}) \leq a\}.$$

If $(t_n,\omega_n) \in C$, $(t_n,\omega_n) \to (t_0,\omega_0)$, then $\exists\, \eta_n \in B_r$, $\eta_n \to \eta_0$ weakly in X, $\exists\, y_n \in G(t_n,\omega_n)\eta_n$, $\| y_n - y \| \leq a$. As $y_n = \varphi_n(t_n,\omega_n,0,\eta_n) + \overline{W}(t,\omega_n)$, so due to lemma 6 $y_n \to z = \varphi(t_0,\omega_0,0,\eta_0) + \overline{W}(t,\omega_0)$ weakly in X. Thus $z \in G(t_0,\omega_0)B_r$ and $\| z - y \| \leq \liminf \| y_n - y \| \leq a$. Therefore $(t_0,\omega_0) \in C$ and condition 3) takes place.

According to (15)

$$G(t,\theta_{-t}\omega)\varphi_0 = S(0,-t,\omega)\varphi_0 = \{\varphi(0,\omega,-t,\varphi_0 - \overline{W}(-t))\}.$$

So from (11) we deduce

$$\| G(t,\theta_{-t}\omega)B_r \|_+^2 \leq C(1 + e^{-\delta t}(r + \| \overline{W}(-t) \|_X)^{\frac{2n-2}{n-2}} + \int_{-t}^{0} e^{\delta\tau} g(\tau,\omega)d\tau). \quad (16)$$

It means that the conditions (6) and 1) take place with $D(\omega) = B_{C(2+r(\omega))}$, where

$$r(\omega) = \int_{-\infty}^{0} e^{\delta\tau} g(\tau,\omega)d\tau < \infty \quad P-a.e.$$

Let us verify condition 4). If $\xi_n \in G(t_n,\theta_{-t_n}\omega)\eta_n$, where $t_n \uparrow +\infty$, $\eta_n \to \eta$ weakly in X, then from (15) $\xi_n = \varphi_n(0,\omega,-t_n,\eta_n - \overline{W}(-t_n,\omega))$ and from estimate (16) $\xi_n \to \xi$ weakly in X. For $M > 0$ we consider

$$z_n(t) = \varphi_n(t-M,\omega,-t_n,\eta_n - \overline{W}(-t_n,\omega)) + \overline{W}(t-M,\omega) \in S(t-M,-t_n,\omega)\eta_n \subset$$
$$\subset S(t-M,-M,\omega)(\varphi_n(-2M,\omega,-t_n,\eta_n - \overline{W}(-t_n)) + \overline{W}(-2M)).$$

So $z_n(t) = \widetilde{\varphi}_n(t-M,\omega,-M,\gamma_M^n - \overline{W}(-M)) + \overline{W}(t-M)$, where

$$\gamma_M^n = \varphi_n(-2M,\omega,-t_n,\eta_n - \overline{W}(-t_n)) + \overline{W}(-2M) \to \gamma_M \quad \text{weakly in } X.$$

Then due to lemma 6 $\forall\, t \in [0,M]$

$$z_n(t) \to z(t) = \widetilde{\varphi}_n(t-M,\omega,-M,\gamma_M - \overline{W}(-M)) + \overline{W}(t-M) \quad \text{weakly in } X.$$

From equality (12), applying to function $\widetilde{\varphi}_n$, we obtain

$$I_\omega(0,\xi_n) = e^{-\beta M} I_\omega(-M,\gamma_M^n - \overline{W}(-M)) + \int_{-M}^{0} e^{\beta p} H_\omega(p,\widetilde{\varphi}_n(p)dp.$$

As

$$\lim_{n\to\infty} \int_{-M}^{0} e^{\beta p} H_\omega(p,\widetilde{\varphi}_n(p)dp = \int_{-M}^{0} e^{\beta p} H_\omega(p,\widetilde{\varphi}(p)dp =$$

$$= I_{\omega}(0, \widetilde{\varphi}(0)) - e^{-\beta M} I_{\omega}(-M, \gamma_M - \overline{W}(-M)),$$

so

$$\frac{1}{2}\liminf \| \xi_n \|_X^2 \le \frac{1}{2} \| \xi \|_X^2 + e^{-\beta M} \, | I_{\omega}(-M, \gamma_M - \overline{W}(-M)) | +$$

$$+ e^{-\beta M} \limsup | I_{\omega}(-M, \gamma_M^n - \overline{W}(-M)) | .$$

From the last inequality, passing to the limit when $M \to \infty$, we have inequality $\liminf \| \xi_n \|_X \le \| \xi \|_X$, which means, that the sequence $\{\xi_n\}$ is precompact in X. Theorem is proved.

CONCLUSIONS

For semilinear wave equation, perturbed by additive white noise, in sense of the random attractor theory the dynamics of solutions is investigated.

In particular, the existence of random attractor for abstract noncompact multi-valued random dynamical system is proved. The abstract theory allows to apply this result to the wave equation with non-smooth nonlinear term. A priory estimate for weak solution of randomly perturbed problem in the phase space is deduced, which contributes to obtain the existence of the weak solutions. The existence of random attractor for generated multi-valued stochastic flow is proved.

Thus for the class of mathematical models with non-smooth dependencies between determining parameters of the problem, controlled by nonlianerized piezoelectric and viscoelasticity theory with nonlinear stochastic perturbations, the opportunity of long-time forecasts for state functions is obtained. As a result, it became possible to direct the state functions to the desired asymptotic level.

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