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EMPIRICAL INVESTIGATION OF THE THEORY OF PRODUCTION FUNCTION, WITH THE DATA OF ALLOY PRODUCTION IN UKRAINE

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In this research, a mathematical form of production function is investigated, which is a concept of microeconomics theory, with the actual data from the factory in Dnepropetrovsk Region of Ukraine, which produces the alloys from several input materials. A linear form of the production function was selected as the model, which consists of the variables that represent input materials together with their weighting factors, then the Lagrangean multiplier technique was used to transform this model in order to find the conditions for maximizing the output of the production, under a given cost constraint. The obtained conditions present the mathematical relations between the prices and the quantities of the input materials, which include unknown weighting factors. In order to get the values of the weighting factors, statistical analysis is made with the actual data. The result shows statistical significance of the model, therefore it is concluded that the selected linear function can be the production function.

INTRODUCTION

Production function of the microeconomics theory [1] gives the information for decision-making in producing industrial materials. In the theory, the production function defines the optimal combination of input materials with their weighting factors. In order to specify the weighting factors, the Lagurangean multiplier technique [1] is used under the conditions for maximizing the production, which is given by cost constraint that is made of the prices of the materials together with their quantities.

The mathematical forms of production function are given in the literatures of microeconomics, and Cobb-Douglas function [1] is known as an example in non-linear form. The procedure, the Lagrangean multiplier technique, of finding the conditions for maximizing the production under cost constraintis obtained from those literatures. In this research, a linear form of production function is selected, and the appropriateness of this form is tested with the data taken from the production system of alloy at a factory in Dnepropetrovsk of Ukraine.

The data that are used in this analysis include quantities and the prices of the input materials, i.e., lime, bentonite, ore, gas, electricity as well as the quantity of the final product, iron ore and pellets.

The descriptive statistics of those input materials are shown in Table 1 and 2. Correlations between the variables selected are given in Table 3. Figures 1–3 show time histories for the quantity of final products, prices of gas, electricity, ore, bentonite, and lime for 36 months. Figures 4–5 illustrate quantities dynamics for gas, bentonite, lime, electricity, and iron, ore also for 36 months.

Table 1. Descriptive statistics of the prices of gas, electricity, ore, lime and bentonite

Statistics	Gas price (UAH/m^3)	Electricity price (UAH/kWh)	Ore price (UAH/ton)	Lime price (UAH/ton)	Bentonite price (UAH/ton)
Mean	1.9167	0.3667	9.0667	700.00	566.67
Median	1.9100	0.4000	7.9000	700.00	550.00
Max.	2.1600	0.4300	11.600	700.00	600.00
Min.	1.6800	0.2700	7.7000	700.00	550.00
Std.Dev	0.1988	0.0704	1.8186	0.0000	23.905
Skewness	0.0510	–0.6094	0.7005	NA	0.7071
Kurtosis	1.5000	1.5000	1.5000	NA	1.5000
Obs.	36	36	36	36	36

Note Max. — maximum value; Min. — minimum value; Std. Dev. — standard deviation; Obs. — number of observations; NA — not available, because the lime price doesn't change over 36 months in the obtained database. UAH — Ukrainian currency (hryvnya); kWh — kilo watt-hour.

Table 2. Descriptive statistics of quantities of gas, electricity, ore, lime, bentonite and the final product

Statistics	Gas quantity	Electricity quantity	Ore quantity	Lime quantity	Bentonite quantity	Final product quantity
Mean	719570.6	18107646	3028497	44140.3	50020.2	1007887
Median	720530.0	18344268	3066452	43507.5	49443.5	998094.0
Max.	826160.0	20469762	3441243	50690.0	56923.0	1146490
Min.	620210.0	15400996	2568431	38676.0	42735.0	862363.0
Std.Dev	64555.82	1382721	253399.9	3807.67	4284.58	84840.57
Skewness	–0.056677	–0.281579	–0.0740	0.2199	–0.0296	0.1906
Kurtosis	1.834927	2.2615	1.9633	1.8112	1.6549	2.1655
Obs.	36	36	36	36	36	36

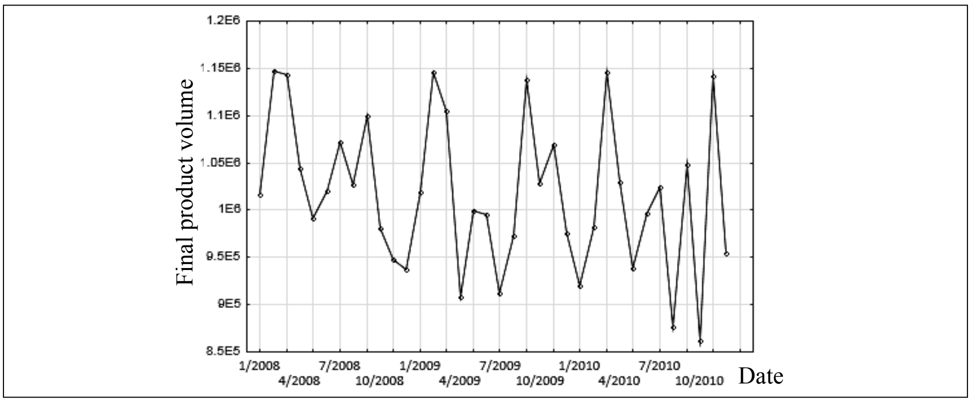

Fig. 1. Quantity of final products for 36 months from January 2008 (tons)

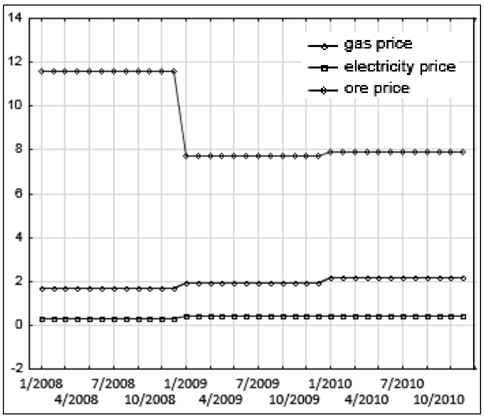

Fig. 2. Prices of gas, electricity, and ore for 36 months

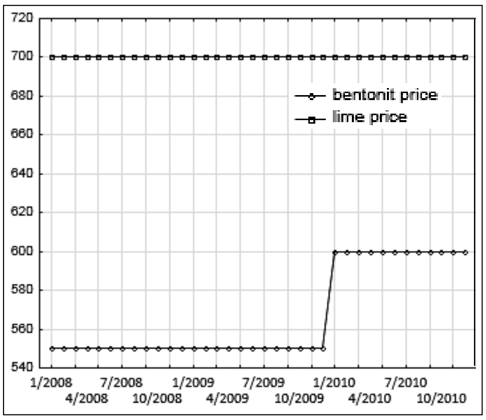

Fig. 3. Prices of bentonite and lime for 36 months

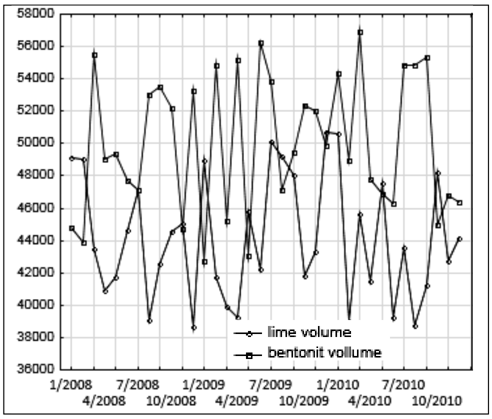

Fig. 4. Quantities of gas, bentonite and lime for 36 months

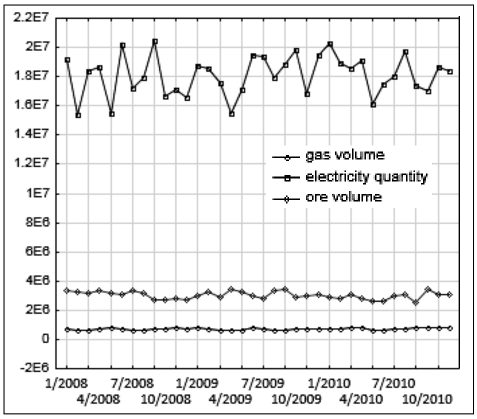

Fig. 5. Quantities of electricity and ore for 36 months

Table 3. Correlations of quantities and/or prices of the final product and input materials

Variables	Final product	Gas price	Electricity price	Ore price	Bentonite price	Gas quantity	Electricity quantity	Ore quantity	Lime quantity	Bentonite quantity
Final product	1									
Gas price	−0.2193	1								
Electricity price	−0.1896	0.9322	1							
Ore price	0.1589	−0.8292	−0.9752	−1						
Bentonite price	−0.2121	0.8778	0.6449	−0.4600	1					
Gas quantity	−0.2175	0.2983	0.2213	−0.1596	0.3370	1				
Electricity quantity	0.0719	0.1548	0.1773	−0.1794	0.0920	0.2842	1			
Ore quantity	0.1454	−0.2165	−0.1238	0.0590	−0.2933	−0.1523	−0.0819	1		
Lime quantity	−0.0558	−0.0401	0.0268	−0.0659	−0.1202	−0.0736	0.1100	0.2609	1	
Bentonite quantity	0.0325	0.0825	0.0868	−0.0837	0.0593	0.0632	0.2326	−0.1484	−0.3286	1

Note: Lime price is omitted from this table because the lime price doesn't change over the given 36 months as shown in Fig. 3, therefore it doesn't have any correlation with other variables.

METHODOLOGY

Production function is a theory to indicate the levels of production of industrial materials with various input materials, X_i, where $i = 1,2,....,n$, such as raw materials,electricity and gas. The producers and/or sellers wish higher level of production, $Q(X_1, X_2, X_3,..., X_n)$, but the constraints are given by the total cost or budget, C^o, together with the prices P_{x_i} for different kinds of input materials X_i respectively, where

$$C^o = \sum_{i=1}^{n} P_{x_i} X_i. \tag{1}$$

Under this constraint, the condition for obtaining the maximum production is to be found, using the Lagrangean multiplier technique, as shown below: at first, the Lagrangean is defined as the follows:

$$Z = Q(X_1, X_2, X_3,....X_n) + \lambda(C^o - \sum_{i=1}^{n} P_{x_i} X_i), \tag{2}$$

here, λ is an unknown variable, which is called the "Lagrangean multiplier".

The first order condition to get the maximum production, $Q(X_1, X_2, X_3,..., X_n)$, is that the partial derivatives of Z by each of $X_1, X_2, X_3,..., X_n$ and λ are equal to zero, i.e.,

$$\partial Z / \partial X_i = \partial Q / \partial X_i - \lambda P_{x_i} = 0, \tag{3}$$

$$\partial Z / \partial \lambda = C^o - \sum_{i=1}^{n} P_{x_i} X_i = 0. \tag{4}$$

For example, by dividing *i*-th equation by $(i+1)$-th equation of the above (1)–(4), we get the following:

$$\frac{\partial Q / \partial X_i}{\partial Q / \partial X_j} = \frac{P_{X_i}}{P_{X_j}}, \tag{5}$$

where, $i \neq j$.

The above equation (5) means that the ratio of marginal production of inputs (the ratio of these two partial derivatives of production function by X_i and X_j) should be equal to the ratio of the prices of these X_i and X_j in order to get the maximum production [1]. In other words, although producers and/or sellers wish to achieve the higher/larger production, the maximum production is always constrained by the total cost or total budget and the prices, and the maximum production is obtained only where and/or when the ratio of marginal productions, $\frac{\partial Q / \partial X_i}{\partial Q / \partial X_j}$, and the ratio of the corresponding two prices, $\frac{P_{X_i}}{P_{X_j}}$, are equal. This point is the equilibrium to achieve the maximum production, which is given under the total cost constraint (equation (1)). In other words, the production is at the maximum, and there is enough amount of budget when equation (5) is satisfied.

The mathematical model of the production function needs to be found. In this research, a linear model (equation (6)) is assumed, and then empirical analysis is made for testing the fitting of the model to the actual data:

$$Q = \sum_{i=1}^{n} a_i X_i, \tag{6}$$

where

$$\sum_{i=1}^{n} a_i = 1, \tag{7}$$

here a_i is a weighting factor to combine various input materials, X_i, to make up a production function Q.

In order to make the statistical test, the variables included in the equation (6) are not enough because the actual value of Q. is unknown, therefore this model needs to be transformed to the other linear equations, with the Lagrangean multiplier technique as shown below, with which each quantity of input material, X_i, can be mathematically indicated as the function of the total cost, C^o, and the prices of various input materials, $P_{x_1}, P_{x_s}, P_{x_3}, ..., P_{x_n}$, together with rest of the other input materials, X_j, where $i \neq j$, which are available in the actual database. Then, the linear regression analysis can be carried out for the statistical test.

For the linear model, $Q = \sum_{i=1}^{n} a_i X_i$, the Lagrangean is:

$$Z = \sum_{i=1}^{n} a_i X_i. + \lambda(C^o - \sum_{i=1}^{n} P_{X_i} X_i). \tag{8}$$

Given the cost constraint, the first order condition for maximizing the production, $\sum_{i=1}^{n} a_i X_i$, is that the partial derivatives of Z by each of $X_1, X_2, X_3, ..., X_n$ and λ are equal to zero, i.e.,

$$\partial Z / \partial X_i = a_i - \lambda P_{X_i} = 0, \tag{9}$$

$$\partial Z / \partial \lambda = C^o - \sum_{i=1}^{n} P_{x_i} X_i = 0, \tag{10}$$

where, $i = 1,2,...,n$.

From (9)

$$P_{X_i} = \frac{a_i}{\lambda}. \quad (11)$$

From (10)

$$C^o = \sum_{i=1}^{n} P_{X_i} X_i. \quad (12)$$

Then, replace P_{X_j} of (12) by (11) to get:

$$C^o = P_{X_i} X_i + \sum_{i=1}^{n-1} \frac{a_j}{\lambda} X_j, \quad (13)$$

where $i \neq j$.

From (11)

$$\frac{1}{\lambda} = \frac{P_{X_i}}{a_i}. \quad (14)$$

Then, replace $\frac{1}{\lambda}$ of (13) by (14) to get:

$$X_i = \frac{C^o}{P_{X_i}} - \sum_{j=1}^{n-1} \frac{a_j}{a_i} X_j. \quad (15)$$

The next step is to test if this model statistically fits in the actual data, upon the mathematical model shown in the equation (15).

RESULTS

For the statistical test, one more variable, the total cost, C^o, was calculated upon the equation (1), in addition to the variables shown in Table 1 and 2. Then, in order to get the coefficients of the production function, shown in the equation (6), the equation (15) was made up with combinations of the input materials. In Table 3, various combinations of the variables for input materials are shown. Then, the statistical test was made with the data. Also in Table 3, the value of R^2 is shown on each combination of the input materials, which indicates how each model fits in the data.

As the result, the model of the production with lime and bentonite shows the best values of R^2. As shown in the model № 17 of Table 4, R^2 of the model for the equation (15) with the quantity of lime as the dependent variable is 0,8238, and R^2 of the model with the quantity of bentonite as the dependent variable is 0,7874, both of which satisfactory show the statistical fitting of the data on the mathematical model. More details of the statistical check of the model № 17 of Table 4 is shown in Table 5.

T a b l e 4 . R^2 of the linear functions

№	Model of equation (6)	Model of equation (15)	R^2
1	$Q = a_1 * X_{\text{lime}} + a_2 * X_{\text{bentonite}} + a_3 * X_{\text{electricity}} + a_4 * X_{\text{ore}} + a_5 * X_{\text{gas}}$	$X_{\text{lime}}=\alpha_1+\alpha_2*C^o/P_{\text{lime}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{electricity}}+\alpha_5*X_{\text{ore}}+\alpha_6*X_{\text{gas}}$	0.3645
		$X_{\text{bentonite}}=\alpha_1+\alpha_2*C^o/P_{\text{bentonite}}+\alpha_3*X_{\text{lime}}+\alpha_4*X_{\text{electricity}}+\alpha_5*X_{\text{ore}}+\alpha_6*X_{\text{gas}}$	0.2611
		$X_{\text{electricity}}=\alpha_1+\alpha_2* C^o/P_{\text{electricity}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{lime}}+\alpha_5*X_{\text{ore}}+\alpha_6*X_{\text{gas}}$	0.1801
		$X_{\text{ore}}=\alpha_1+\alpha_2* C^o/P_{\text{ore}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{electricity}}+\alpha_5*X_{\text{lime}}+\alpha_6*X_{\text{gas}}$	0.1015
		$X_{\text{gas}}=\alpha_1+\alpha_2* C^o/P_{\text{gas}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{electricity}}+\alpha_5*X_{\text{lime}}+\alpha_6*X_{\text{ore}}$	0.1364
2	$Q = a_1 * X_{\text{lime}} + a_2 * X_{\text{bentonite}} + a_3 * X_{\text{electricity}} + a_4 * X_{\text{ore}}$	$X_{\text{lime}}=\alpha_1+\alpha_2* C^o/P_{\text{lime}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{electricity}} +\alpha_5*X_{\text{ore}}$	0.3559
		$X_{\text{bentonite}}=\alpha_1+\alpha_2* C^o/P_{\text{bentonite}}+\alpha_3*X_{\text{lime}}+\alpha_4*X_{\text{electricity}} +\alpha_5*X_{\text{ore}}$	0.2582
		$X_{\text{electricity}}=\alpha_1+\alpha_2* C^o/P_{\text{electricity}}+\alpha_3*X_{\text{lime}}+\alpha_4*X_{\text{bentonite}} +\alpha_5*X_{\text{ore}}$	0.1150
		$X_{\text{ore}}=\alpha_1+\alpha_2* C^o/P_{\text{ore}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{electricity}} +\alpha_5*X_{\text{lime}}$	0.0880
3	$Q = a_1 * X_{\text{lime}} + a_2 * X_{\text{bentonite}} + a_3 * X_{\text{gas}} + a_4 * X_{\text{ore}}$	$X_{\text{lime}}=\alpha_1+\alpha_2* C^o/P_{\text{lime}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{gas}} +\alpha_5*X_{\text{ore}}$	0.2525
		$X_{\text{bentonite}}=\alpha_1+\alpha_2* C^o/P_{\text{bentonite}}+\alpha_3*X_{\text{lime}}+\alpha_4*X_{\text{gas}} +\alpha_5*X_{\text{ore}}$	0.1638
		$X_{\text{gas}}=\alpha_1+\alpha_2* C^o/P_{\text{gas}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{lime}} +\alpha_5*X_{\text{ore}}$	0.0691
		$X_{\text{ore}}=\alpha_1+\alpha_2* C^o/P_{\text{ore}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{gas}} +\alpha_5*X_{\text{lime}}$	0.1034
4	$Q = a_1 * X_{\text{lime}} + a_2 * X_{\text{bentonite}} + a_3 * X_{\text{electricity}} + a_4 * X_{\text{gas}}$	$X_{\text{lime}}=\alpha_1+\alpha_2* C^o/P_{\text{lime}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{electricity}} +\alpha_5*X_{\text{gas}}$	0.6109
		$X_{\text{bentonite}}=\alpha_1+\alpha_2* C^o/P_{\text{bentonite}}+\alpha_3*X_{\text{lime}}+\alpha_4*X_{\text{electricity}} +\alpha_5*X_{\text{gas}}$	0.8124
		$X_{\text{electricity}}=\alpha_1+\alpha_2* C^o/P_{\text{electricity}}+\alpha_3*X_{\text{lime}}+\alpha_4*X_{\text{bentonite}} +\alpha_5*X_{\text{gas}}$	0.1753
		$X_{\text{gas}}=\alpha_1+\alpha_2*C^o/P_{\text{gas}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{electricity}} +\alpha_5*X_{\text{lime}}$	0.1413
5	$Q = a_1 * X_{\text{electeicity}} + a_2 * X_{\text{bentonite}} + a_3 * X_{\text{gas}} + a_4 * X_{\text{ore}}$	$X_{\text{electricity}}=\alpha_1+\alpha_2*C^o/P_{\text{electricity}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{gas}} +\alpha_5*X_{\text{ore}}$	0.1343
		$X_{\text{bentonite}}=\alpha_1+\alpha_2*C^o/P_{\text{bentonite}}+\alpha_3*X_{\text{electricity}}+\alpha_4*X_{\text{gas}} +\alpha_5*X_{\text{ore}}$	0.2153
		$X_{\text{gas}}=\alpha_1+\alpha_2*C^o/P_{\text{gas}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{electricity}} +\alpha_5*X_{\text{ore}}$	0.1211
		$X_{\text{ore}}=\alpha_1+\alpha_2*C^o/P_{\text{ore}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{gas}} +\alpha_5*X_{\text{electricity}}$	0.0855
6	$Q = a_1 * X_{\text{electeicity}} + a_2 * X_{\text{lime}} + a_3 * X_{\text{gas}} + a_4 * X_{\text{ore}}$	$X_{\text{electricity}}=\alpha_1+\alpha_2*C^o/P_{\text{electricity}}+\alpha_3*X_{\text{lime}}+\alpha_4*X_{\text{gas}} +\alpha_5*X_{\text{ore}}$	0.1116
		$X_{\text{lime}}=\alpha_1+\alpha_2*C^o/P_{\text{lime}}+\alpha_3*X_{\text{electricity}}+\alpha_4*X_{\text{gas}} +\alpha_5*X_{\text{ore}}$	0.2986
		$X_{\text{gas}}=\alpha_1+\alpha_2*C^o/P_{\text{gas}}+\alpha_3*X_{\text{lime}}+\alpha_4*X_{\text{electricity}} +\alpha_5*X_{\text{ore}}$	0.1323
		$X_{\text{ore}}=\alpha_1+\alpha_2*C^o/P_{\text{ore}}+\alpha_3*X_{\text{lime}}+\alpha_4*X_{\text{gas}} +\alpha_5*X_{\text{electricity}}$	0.1302
7	$Q = a_1 * X_{\text{lime}} + a_2 * X_{\text{bentonite}} + a_3 * X_{\text{electricity}}$	$X_{\text{lime}}=\alpha_1+\alpha_2*C^o/P_{\text{lime}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{electricity}}$	0.8239
		$X_{\text{bentonite}}=\alpha_1+\alpha_2*C^o/P_{\text{bentonite}}+\alpha_3*X_{\text{lime}}+\alpha_4*X_{\text{electricity}}$	0.6287
		$X_{\text{electricity}}=\alpha_1+\alpha_2*C^o/P_{\text{electricity}}+\alpha_3*X_{\text{lime}}+\alpha_4*X_{\text{bentonite}}$	0.0965
8	$Q = a_1 * X_{\text{lime}} + a_2 * X_{\text{bentonite}} + a_3 * X_{\text{ore}}$	$X_{\text{lime}}=\alpha_1+\alpha_2*C^o/P_{\text{lime}}+\alpha_3*X_{\text{bentonite}}+\alpha_4*X_{\text{ore}}$	0.2488
		$X_{\text{bentonite}}=\alpha_1+\alpha_2*C^o/P_{\text{bentonite}}+\alpha_3*X_{\text{lime}}+\alpha_4*X_{\text{ore}}$	0.1561
		$X_{\text{ore}}=\alpha_1+\alpha_2*C^o/P_{\text{ore}}+\alpha_3*X_{\text{lime}}+\alpha_4*X_{\text{bentonite}}$	0.0826
9	$Q = a_1 * X_{\text{bentonite}} + a_2 * X_{\text{electricity}} + a_3 * X_{\text{gas}}$	$X_{\text{bentonite}} =\alpha_1+\alpha_2*C^o/P_{\text{bentonite}}+\alpha_3*X_{\text{gas}}+\alpha_4*X_{\text{electricity}}$	0.8159
		$X_{\text{electricity}}=\alpha_1+\alpha_2*C^o/P_{\text{electricity}}+\alpha_3*X_{\text{bentonite}}+\alpha_4* X_{\text{gas}}$	0.1285
		$X_{\text{gas}}=\alpha_1+\alpha_2*C^o/P_{\text{gas}}+\alpha_3*X_{\text{electricity}}+\alpha_4*X_{\text{bentonite}}$	0.1050
10	$Q = a_1 * X_{\text{ore}} + a_2 * X_{\text{electricity}} + a_3 * X_{\text{gas}}$	$X_{\text{ore}} =\alpha_1+\alpha_2*C^o/P_{\text{ore}}+\alpha_3*X_{\text{gas}}+\alpha_4*X_{\text{electricity}}$	0.4047
		$X_{\text{electricity}}=\alpha_1+\alpha_2*C^o/P_{\text{electricity}}+\alpha_3*X_{\text{ore}}+\alpha_4* X_{\text{gas}}$	0.0990
		$X_{\text{gas}}=\alpha_1+\alpha_2*C^o/P_{\text{gas}}+\alpha_3*X_{\text{electricity}}+\alpha_4*X_{\text{ore}}$	0.1183
11	$Q = a_1 * X_{\text{lime}} + a_2 * X_{\text{electricity}} + a_3 * X_{\text{gas}}$	$X_{\text{lime}} =\alpha_1+\alpha_2*C^o/P_{\text{lime}}+\alpha_3*X_{\text{gas}}+\alpha_4*X_{\text{electricity}}$	0.8002
		$X_{\text{electricity}}=\alpha_1+\alpha_2*C^o/P_{\text{electricity}}+\alpha_3*X_{\text{lime}}+\alpha_4* X_{\text{gas}}$	0.1010
		$X_{\text{gas}}=\alpha_1+\alpha_2*C^o/P_{\text{gas}}+\alpha_3*X_{\text{electricity}}+\alpha_4*X_{\text{lime}}$	0.1418

Continue of table 4

12	$Q = a_1 * X_{\text{ore}} + a_2 * X_{\text{bentonite}} + a_3 * X_{\text{gas}}$	$X_{\text{ore}} = \alpha_1 + \alpha_2 * C^o / P_{\text{ore}} + \alpha_3 * X_{\text{gas}} + \alpha_4 * X_{\text{bentonite}}$	0.1309
		$X_{\text{bentonite}} = \alpha_1 + \alpha_2 * C^o / P_{\text{bentonite}} + \alpha_3 * X_{\text{ore}} + \alpha_4 * X_{\text{gas}}$	0.1160
		$X_{\text{gas}} = \alpha_1 + \alpha_2 * C^o / P_{\text{gas}} + \alpha_3 * X_{\text{bentonite}} + \alpha_4 * X_{\text{ore}}$	0.0599
13	$Q = a_1 * X_{\text{ore}} + a_2 * X_{\text{bentonite}} + a_3 * X_{\text{electricity}}$	$X_{\text{ore}} = \alpha_1 + \alpha_2 * C^o / P_{\text{ore}} + \alpha_3 * X_{\text{electricity}} + \alpha_4 * X_{\text{bentonite}}$	0.0637
		$X_{\text{bentonite}} = \alpha_1 + \alpha_2 * C^o / P_{\text{bentonite}} + \alpha_3 * X_{\text{ore}} + \alpha_4 * X_{\text{electricity}}$	0.2088
		$X_{\text{electricity}} = \alpha_1 + \alpha_2 * C^o / P_{\text{electricity}} + \alpha_3 * X_{\text{bentonite}} + \alpha_4 * X_{\text{ore}}$	0.0727
14	$Q = a_1 * X_{\text{ore}} + a_2 * X_{\text{lime}} + a_3 * X_{\text{gas}}$	$X_{\text{ore}} = \alpha_1 + \alpha_2 * C^o / P_{\text{ore}} + \alpha_3 * X_{\text{gas}} + \alpha_4 * X_{\text{lime}}$	0.1665
		$X_{\text{lime}} = \alpha_1 + \alpha_2 * C^o / P_{\text{lime}} + \alpha_3 * X_{\text{ore}} + \alpha_4 * X_{\text{gas}}$	0.1939
		$X_{\text{gas}} = \alpha_1 + \alpha_2 * C^o / P_{\text{gas}} + \alpha_3 * X_{\text{lime}} + \alpha_4 * X_{\text{ore}}$	0.0680
15	$Q = a_1 * X_{\text{ore}} + a_2 * X_{\text{electricity}} + a_3 * X_{\text{lime}}$	$X_{\text{ore}} = \alpha_1 + \alpha_2 * C^o / P_{\text{ore}} + \alpha_3 * X_{\text{electricity}} + \alpha_4 * X_{\text{lime}}$	0.1192
		$X_{\text{lime}} = \alpha_1 + \alpha_2 * C^o / P_{\text{lime}} + \alpha_3 * X_{\text{ore}} + \alpha_4 * X_{\text{electricity}}$	0.2903
		$X_{\text{electricity}} = \alpha_1 + \alpha_2 * C^o / P_{\text{electricity}} + \alpha_3 * X_{\text{lime}} + \alpha_4 * X_{\text{ore}}$	0.0443
16	$Q = a_1 * X_{\text{ore}} + a_2 * X_{\text{bentonite}} + a_3 * X_{\text{lime}}$	$X_{\text{ore}} = \alpha_1 + \alpha_2 * C^o / P_{\text{ore}} + \alpha_3 * X_{\text{bentonite}} + \alpha_4 * X_{\text{lime}}$	0.0826
		$X_{\text{lime}} = \alpha_1 + \alpha_2 * C^o / P_{\text{lime}} + \alpha_3 * X_{\text{ore}} + \alpha_4 * X_{\text{bentonite}}$	0.2488
		$X_{\text{bentonite}} = \alpha_1 + \alpha_2 * C^o / P_{\text{bentonite}} + \alpha_3 * X_{\text{lime}} + \alpha_4 * X_{\text{ore}}$	0.1561
17	$Q = a_1 * X_{\text{lime}} + a_2 * X_{\text{bentonite}}$	$X_{\text{lime}} = \alpha_1 + \alpha_2 * C^o / P_{\text{lime}} + \alpha_3 * X_{\text{bentonite}}$	0.8238
		$X_{\text{bentonite}} = \alpha_1 + \alpha_2 * C^o / P_{\text{bentonite}} + \alpha_3 * X_{\text{lime}}$	0.7874
18	$Q = a_1 * X_{\text{ore}} + a_2 * X_{\text{bentonite}}$	$X_{\text{ore}} = \alpha_1 + \alpha_2 * C^o / P_{\text{ore}} + \alpha_3 * X_{\text{bentonite}}$	0.1071
		$X_{\text{betonie}} = \alpha_1 + \alpha_2 * C^o / P_{\text{bentonite}} + \alpha_3 * X_{\text{ore}}$	0.1045
19	$Q = a_1 * X_{\text{ore}} + a_2 * X_{\text{lime}}$	$X_{\text{ore}} = \alpha_1 + \alpha_2 * C^o / P_{\text{ore}} + \alpha_3 * X_{\text{lime}}$	0.1517
		$X_{\text{lime}} = \alpha_1 + \alpha_2 * C^o / P_{\text{lime}} + \alpha_3 * X_{\text{ore}}$	0.1889
20	$Q = a_1 * X_{\text{ore}} + a_2 * X_{\text{bentonite}}$	$X_{\text{ore}} = \alpha_1 + \alpha_2 * C^o / P_{\text{ore}} + \alpha_3 * X_{\text{electricity}}$	0.4564
		$X_{\text{electricity}} = \alpha_1 + \alpha_2 * C^o / P_{\text{electricity}} + \alpha_3 * X_{\text{ore}}$	0.0240
21	$Q = a_1 * X_{\text{ore}} + a_2 * X_{\text{gas}}$	$X_{\text{ore}} = \alpha_1 + \alpha_2 * C^o / P_{\text{ore}} + \alpha_3 * X_{\text{gas}}$	0.9760
		$X_{\text{gas}} = \alpha_1 + \alpha_2 * C^o / P_{\text{gas}} + \alpha_3 * X_{\text{ore}}$	0.0564
22	$Q = a_1 * X_{\text{lime}} + a_2 * X_{\text{electricity}}$	$X_{\text{lime}} = \alpha_1 + \alpha_2 * C^o / P_{\text{lime}} + \alpha_3 * X_{\text{electricity}}$	0.8213
		$X_{\text{electricity}} = \alpha_1 + \alpha_2 * C^o / P_{\text{electricity}} + \alpha_3 * X_{\text{lime}}$	0.0239
23	$Q = a_1 * X_{\text{lime}} + a_2 * X_{\text{gas}}$	$X_{\text{lime}} = \alpha_1 + \alpha_2 * C^o / P_{\text{lime}} + \alpha_3 * X_{\text{gas}}$	0.9973
		$X_{\text{gas}} = \alpha_1 + \alpha_2 * C^o / P_{\text{gas}} + \alpha_3 * X_{\text{lime}}$	0.0671
24	$Q = a_1 * X_{\text{bentonite}} + a_2 * X_{\text{electricity}}$	$X_{\text{bentonite}} = \alpha_1 + \alpha_2 * C^o / P_{\text{bentonite}} + \alpha_3 * X_{\text{electricity}}$	0.8347
		$X_{\text{electricity}} = \alpha_1 + \alpha_2 * C^o / P_{\text{electricity}} + \alpha_3 * X_{\text{bentonite}}$	0.0597
25	$Q = a_1 * X_{\text{bentonite}} + a_2 * X_{\text{gas}}$	$X_{\text{bentonite}} = \alpha_1 + \alpha_2 * C^o / P_{\text{bentonite}} + \alpha_3 * X_{\text{gas}}$	0.9985
		$X_{\text{gas}} = \alpha_1 + \alpha_2 * C^o / P_{\text{gas}} + \alpha_3 * X_{\text{bentonite}}$	0.0340
26	$Q = a_1 * X_{\text{electricity}} + a_2 * X_{\text{gas}}$	$X_{\text{electricity}} = \alpha_1 + \alpha_2 * C^o / P_{\text{electricity}} + \alpha_3 * X_{\text{gas}}$	0.8974
		$X_{\text{gas}} = \alpha_1 + \alpha_2 * C^o / P_{\text{gas}} + \alpha_3 * X_{\text{electricity}}$	0.1321

In Table 5, the T-statistics of each independent variable, the Akaike Information Criterion (AIC) and Shwartz Criterion don't show sufficient statistical fitting. According to the mathematical model of the equation (16), the coefficient, $C^o \big/ P_{X_i}$, should be 1.0, but in Table 5, the coefficients of $C^o \big/ P_{\lim e_i}$ and $C^o \big/ P_{\text{bentonit}}$ are 0,8368 and 0,7794. In this analysis, approximation is taken for the further steps of the analysis, and they are both assumed to be 1.0.

Table 5. Statistical test on the linear model of production function with lime and bentonite

Model	Dependent Variable	Independent Variable	Coefficient $\alpha_1, \alpha_2, \ldots$	T-Statistics	R^2	AIC	Schwartz
$X_{lime}=\alpha_1+\alpha_2 * C^o / P_{lime}+\alpha_3 * X_{bentonite}$	Quantity of Lime (X_{lime})	Interception	10600	2.0064	0.8238	17.730	17.861
		Total cost (C^0)/Lime price (P_{lime})	0.8368	11.581			
		Quantity of Bentonite ($X_{bentonite}$)	−0.7455	−9.8316			
$X_{bentonite}= =\alpha_{1+}+\alpha_2 * C^o / P_{bentonite}+\alpha_3 * X_{lime}$	Quantity of Bentonite ($X_{bentonite}$)	Interception	17038	2.7269	0.7874	18.153	18.285
		Total cost (C^0)/Bentonite price ($P_{bentonite}$)	0.7794	10.271			
		Quantity of Lime (X_{lime})	−1.100869	−9.573509			

The next step is to estimate the weighting factors, which are indicated as the coefficients a_i, where $i=1,2,\ldots,n$ of the equation (6).

When

$$\frac{a_j}{a_i}=\alpha_{ij}, \tag{16}$$

where, α_{ij} is the observed value of the coefficient that is obtained by the linear regression analysis, as shown in Table 5.

From (15) and (16)

$$X_i=\frac{C^o}{P_{X_i}}-\sum_{j=1}^{n-1}\alpha_{ij}X_j, \tag{17}$$

where

$$\frac{\sum_{j=1}^{n-1}a_j}{a_i}=\sum_{j=1}^{n}\alpha_{ij}. \tag{18}$$

From (7)

$$\sum_{i=1}^{n}a_i=a_i+\sum_{j=1}^{n-1}a_j=1. \tag{19}$$

Then, from (18) and (19)

$$\frac{1-a_i}{a_i}=\sum_{j=1}^{n-1}\alpha_{ij}, \tag{20}$$

$$1-a_i=a_i\sum_{j=1}^{n-1}\alpha_{ij}, \tag{21}$$

$$a_i\left(\sum_{j=1}^{n-1}\alpha_{ij}+1\right)=1. \tag{22}$$

Therefore

$$a_i=\frac{1}{1+\sum_{j=1}^{n-1}\alpha_{ij}}. \tag{23}$$

From the equation (17) and the values of the coefficients of lime and bentonite in Table 4, the following 2 equations are obtained:

$$X_{\text{lime}}=\frac{C^o}{P_{\text{lim}e}}-0,74546\times X_{\text{bentonit}}, \tag{24}$$

$$X_{\text{bentonit}}=\frac{C^o}{P_{\text{bentonit}}}-1,10087\times X_{\text{lime}}. \tag{25}$$

With the equation (23) and the values of the coefficients in the equations (24) and (25), the following production function is obtained:

$$Q=0,5729\,X_{\text{lime}}+0,4760\,X_{\text{bentonit}}. \tag{26}$$

The correlation between the quantity of the final product and the calculated values upon the equation (26) is shown in Table 6. With data of 36 months from January 2008 to December 2010, the statistical values don't show any fitting of the calculated value in the actual data. However, with the data of 12 months from January to December 2008, the statistical indicators show the improvement. The actual value of the final product quantity is 26,88 times larger than the calculated value, but the behavior in time series over 12 months show proportional rise and fall of the product, and therefore it shows a predictability of the final product upon quantity of bentonite and lime, as shown in Fig. 6. In this period, the first 12 months, the most of the prices of the input materials are stable as shown in Fig. 2 and Fig. 3, and it shows that the stable prices improved the predictability by the obtained production function in the equation (26).

Table 6. Correlation between the final product quantity and the calculated value

№	Dependent Variable	Independent Variable	Coefficient	T-Statistics	R^2	AIC	Schwartz	Durbin-Watson
1	Final product quantity	Interception	1053544	3.8250	0.0005	25.500	25.588	2.0003
		Calculated Q	–0.7414	–0.1323				
2	Final product quantity	Interception	–272693.4	–0.4470	0.3153	24.989	25.070	1.4704
		Calculated Q	26.8764	2.1457				

In Table 5 data is from January 2008 to December 2010. In Table 6 data is from January 2008 to December 2008.

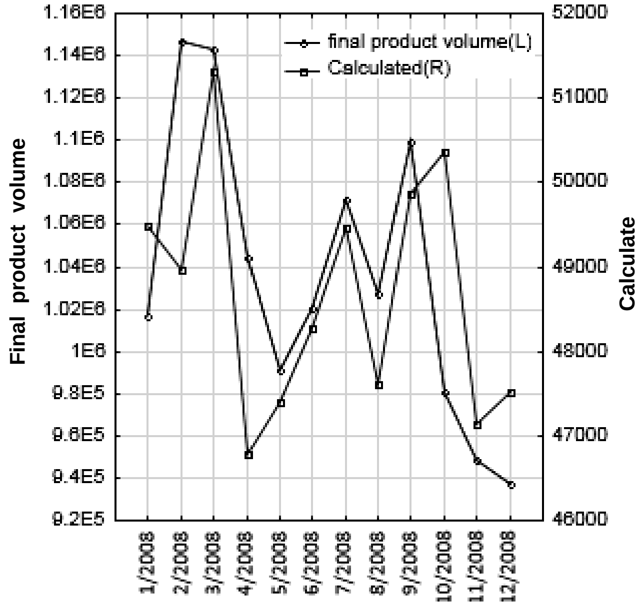

Рис. 6. Comparison of the quantity of final product and the calculated value in 2008

CONCLUSIONS AND RECOMMENDATIONS

Upon the analysis of the given data of the alloy production in Dnepropetrovsk, it is concluded that the productivity of the manufacturing process can be predicted by the linear form of the production function, as long as the prices of the input materials are stable.

Fewer numbers of input variables can predict the quantity of the final products. In this analysis, only the quantities of bentonite and lime are the input variables of the production function, given that the prices are stable; and, the other input materials and utilities, ore, electricity and gas were not used.

On this analysis, the obtained quantity of the final product by the obtained utility function needs to be multiplied by the factor of about 27, because of the fewer input variables included in the production function.

Further research and analysis are needed for different production systems and products, to compare the results with this analysis.

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