

CT IMAGE DENOISING BASED ON LOCALLY ADAPTIVE THRESHOLDING

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Abstract. The noise in reconstructed X-ray Computed Tomography (CT) slices is complex, non-stationary and indefinitely distributed. Subsequent image processing is needed in order to achieve a good-quality medical diagnosis. It requires a sufficiently great ratio between the detailed contrasts and the noise component amplitude. This paper presents an adaptive method for noise reduction in CT images, based on the local statistical evaluation of the noise component in the domain of Repagular Wavelet Transformation (RWT). Considering the spatial dependence of the noise strength, the threshold constant for processing the high frequency coefficients in the proposed shrinkage method is a function of the local standard deviation of the noise for each pixel of the image. Experimental studies have been conducted using different images in order to evaluate the effectiveness of the proposed algorithm.

Keywords: repagular wavelet transform, statistical noise reduction, X-ray computed tomography.

INTRODUCTION

Because of its high productivity in obtaining images having very good spatial resolution, X-ray CT plays a central role in clinical diagnostics. The improvement of this resolution is directly related to the radiation dose. However, its increase poses a certain risk to the patient's health. On the other hand, lowering this dose can significantly reduce the diagnostic value of the tomographic image obtained. The protocol optimization for the corresponding clinical study is based on the balance between preserving the fine details of the image, the noise level and the radiation dose for the patient. Therefore, the development of algorithms for noise reduction in CT images obtained at a reduced radiation dose, while maintaining the detailed information, is a topical task. The statistical fluctuations of the X-ray quanta reaching the corresponding detector are a source of quantum noise in the projection data. This noise, combined with the noise generated by the electronics of workstations, is propagated by means of the reconstruction algorithms in the CT slices. The methods for noise reduction in CT images obtained at a low radiation dose are mainly divided into three groups [1]: methods in the projection data domain; methods in the image domain; image and iterative-reconstructive methods.

The algorithms developed in the Radon space most often use iterative, optimization and filtering methods before the application of the Filtered Backprojec-

tion. The low values of the signal-to-noise ratio in the sonogram lead to the loss of structural information.

The methods for reducing the noise in the reconstructed CT slices can be divided into two subgroups: spatial-domain denoising techniques and transform-domain denoising techniques. The linear filters used in the spatial domain are not very effective for the preservation of the small details in the image, because its diagnostic value is reduced. Significantly better results are obtained with methods based on anisotropic diffusion. Its iterative character allows for improved computational efficiency, but speckled or mosaic effects appear in the denoised CT images [6–7]. Noise reduction algorithms in the transformation domain are based on different multiscale transformations, with the possible use of noise assessment [8–11]. CT image is decomposed in the low and high frequency sub-bands of the scale space. Usually, the high-frequency wavelet coefficients are subjected to threshold processing because the noise components are located mainly in these subbands. The denoised image is obtained by means of inverse multiscale transformation.

The third group of CT noise reduction methods refers to iterative reconstruction using iterative and optimization algorithms that are implemented within the two spaces for obtaining CT images. The advantage is the direct use of the statistics of the noise component in the projection data during the implementation of the algorithm, but the problem is the increased computational value [12–14]. With the improvement of workstations, the iterative reconstruction is a preferred alternative to the Filtered Backprojection. This helps to solve the problem of reduced radiation dose while preserving the diagnostic properties of the image [15–16].

The goal of this paper is to present a locally adaptive, shrinkage algorithm for noise reduction in CT images, based on the individual reconstruction of a pair of intersecting subsets of projection data and threshold processing in the RWT field. The rest of the report is organized as follows: section 2 provides a brief description of the motivation for the proposed method. The multiscale RWT used in this paper is presented in Section 3, and the following section provides a detailed description of the implementation of the announced noise reduction algorithm in CT images. Section 5 presents the outcome of the experimental studies carried out, as well as some comparative assessments. The last section summarizes the results obtained and the conclusions drawn.

MOTIVATION AND JUSTIFICATION

Generally, a certain statistical model of the distribution of the noise component in the image is used in post-processing methods for noise reduction in CT slices and it may lead to inaccurate reflection of the actual situation. The approximate description of the physical process of obtaining the projection data can be achieved with Poisson statistics. However, the analytical description of the noise in CT images is complicated by the fact that reconstruction methods generate a correlation in the noisy data [17]. Therefore, [18,19] propose noise reduction by means of a wavelet threshold method based on the correlation between two CT images that are equivalent in terms of the information they contain.

The method this paper proposes is motivated by the announced in [19] Adaptive Wavelet Shrinkage (AWShrink) algorithm of noise reduction by means of several CT reconstructions. The authors use the fact that the noise in two CT

images, equivalent in terms of information content, is uncorrelated, unlike their structural information. The threshold processing in the method is based on the local evaluation of the noise in the two images, obtained through their high frequency wavelet coefficients. The noise-estimated image is obtained from the corresponding averaged low-frequency coefficients, as well from the modified high-frequency coefficients by means of the inverse wavelet transformation.

REPAGULAR WAVELETTRANSFORM

The wavelet transformation used in this paper is the RWT, introduced by Poliakova and Krylov [20] in order to increase the accuracy when separating image contours. A family of functions that are localized at one point is used as a basic wavelet. Its scale change occurs not by a dilation operator, as in the usual wavelet transformation, but by using functions of different regularity.

If $s(t)$ is a signal with limited energy and $\psi(t, a)$ is the base wavelet (with a regularity parameter a , its continuous RWT can be set with the convolution $W_s^r(a, b) = s(t)\overline{\psi_a(b)}$, where $\overline{\psi_a(t)} = \psi(-t, a)$. In the applications of the RWT for basic wavelet, the function $\tilde{\psi}(t, a) = 2^{-ta}$ is often used and the corresponding convolution is realized using the filter

$$\{G_a^\Psi(n)\}_{n=0}^{2(N_a-1)} = \{-2^{-(N_a-1)a}, \dots, -2^{-2a}, -2^{-a}, -1, 1, 2^{-a}, 2^{-2a}, \dots, 2^{-(N_a-1)a}\}$$

where $2N_a$ is the number of filter coefficients. The values of the parameter a of the RWT can be set by the formula $a = 2^{-j}\alpha_0$, where α_0 is any fixed number from the interval (0,1), and $j = 0, 1, 2, \dots$. The change of j results in modifying the regularity parameter, i.e. the transition from one scale level to another in RWT is done by changing the parameter a .

THE PROPOSED METHOD

A Brief Overview of the Method

The overview of the proposed method can be seen in the block diagram in Fig. 1.

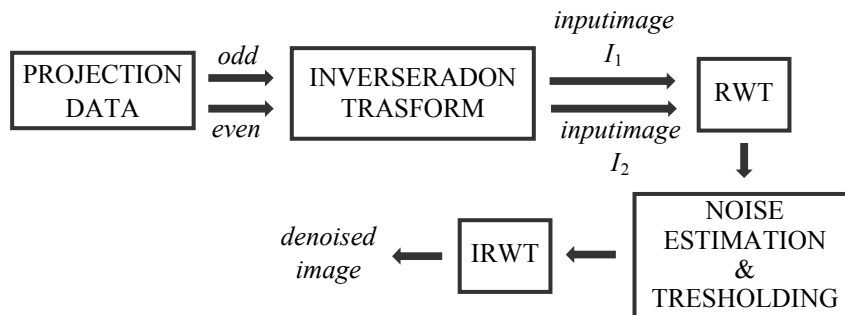


Fig. 1. A block diagram of the proposed method for reducing noise in CT images

The first step consists in obtaining the couple of input images that differ only in terms of their noise components. Their generation is realized by separate reconstructions of two projection sets, equivalent in terms of information. One of the possibilities of obtaining these projections is the double scan of the object

under identical conditions and using half of the total radiation dose. Another option is the use of two non-intersecting subsets of projection data, whose overlapping represents the entire set of projections. An example of such subsets are respectively the sets of odd and even projections obtained at a single scan. In this case, it is assumed that the noise between the adjacent parallel projections is uncorrelated, since the cross disturbances of the detector are negligible. The averaged image obtained from the pair of reconstructed images corresponds to the reconstruction from the full set of projections. These methods are CT variants with a single source and it is necessary to take into account certain factors, such as the reconstruction method, the sufficient number of projections, etc. [21].

In case there is a dual-source CT scanner, obtaining the input images is not a problem, if the pair of tubular detector systems uses the same protocol for scanning and reconstruction [22].

The statistical evaluation of the noise in the reconstituted pair of images, and hence in their averaged image, is obtained by means of their RWT coefficients. The denoised image is obtained through the inverse RWT applied to the low-frequency and the corrected high-frequency coefficients of the averaged image, respectively. The resulting image corresponds to the reconstruction of the complete set of projections but with an improved signal-to-noise ratio.

Noise Assessment

Let us denote with I_1 and I_2 the pair of images obtained by separate reconstruction of the even and odd numbered projections of a given scan, and the total number of the received projections is an even number. The noise in the pair of projection sets is uncorrelated and, through the reconstruction methods, it appears as such in the CT slices. In addition, the noise components of the images are assumed to have zero averages and approximately equal standard deviations (at a given position of the pixel), as the number of the quantas that produces them is approximately equal. Let us note that the noise level in each of the images increases by a factor $\sqrt{2}$ compared to the noise level in the reconstructed image corresponding to the full set of projections [23].

Because the structural information is common to both images, their difference $D = I_1 - I_2$ can be used to estimate the noise in each of them, and therefore in their averaged image $I = 0,5(I_1 + I_2)$. Taking into account the zero covariance between the pair of random variables, then $\sigma(I_1) = \sigma(I_2) = 2^{-0,5} \sigma(D)$ and $\sigma(I) = 2^{-0,5} \sigma(I_1) = 2^{-1} \sigma(D)$.

Let $w_{j,m}(\cdot)$ be the coefficient of the m -th pixel, at a scale of j ($j > 0, m \in \mathbb{R}$), obtained through RWT. Since the transformation is linear, the equality $w_{j,m}(D) = w_{j,m}(I_1) - w_{j,m}(I_2)$ is valid. As it has been noted above, the noise in CT images is non-stationary, i.e. its power is spatially dependent, which implies a local evaluation of its standard deviation. Therefore, for each pixel m , a neighbourhood Ω_m is selected that contains n pixels. Since the noise has a zero average, then $\sigma_{j,m}(D) = \sqrt{n^{-1} \sum_{n \in \Omega_m} w_{j,m}(D)^2}$ and, finally, for the standard deviation

of the noise in the averaged image I , we obtain: $\sigma_{j,m}(D) = \sqrt{n^{-1} \sum_{n \in \Omega_m} w_{j,m}(D)^2}$.

Determining the Threshold Mask

There are different methodologies for selecting threshold values in shrinkage methods. The nonlinear functions of the “hard” and “soft” thresholds proposed in [24] are most often used. The noise-estimation algorithm includes: wavelet decomposition of the signal; estimation of the coefficients from the high frequency bands, by means of a certain threshold value; obtaining the estimated signal through the inverse transform. A basic idea in noise reduction shrinkage methods is the use of the rarity of the multiscale transformations. The carriers of the structural information of the image are a small number of coefficients, and noise reduction is based on the shrinkage of the rest to zero.

A problem for hard threshold processing are the small changes in the image due to the discontinuity of the function used. The proposed method uses a continuous function of the soft threshold, and the estimated coefficients of the averaged image are $w_{j,m}^0(I) = \text{sgn}(w_{j,m}(I))\max(|w_{j,m}(I)| - \eta_{j,m}(r), 0)$, where $\eta_{j,m}(r) = 2^{-1}r\sigma_{j,m}(r)$, and $r > 0$ is a threshold parameter that regulates the amount of the suppressed noise.

RESULTS AND COMPARATIVE ANALYSIS

Two types of measures are used to evaluate the quality of the denoised CT images. They are based on:

- a) Pixel Difference Measurement — Mean Absolute Error (MAE) and Peak Signal-to-Noise Ratio (PSNR);
- b) Human Visual Measurement — Structural Similarity Index (SSIM) and Universal Image Quality Index (UIQI).

The algorithms for calculating the listed measures are implemented in Matlab.

Two experiments are conducted in order to evaluate the effectiveness of the proposed method for reducing noise in CT images.

Test CT Image

A medical Matlab 16-bit monochrome 3-D CT image (ankle) with 512×512 resolution is used as a test image. Wavelet filtering of image CT-MONO2-16-ankle.dcm is performed, to which Gaussian noise has been added. Each of the images obtained is subjected to the Radon transform in order to obtain and separate the projection data, numbered by odd and even numbers respectively. At the next stage, the corresponding reconstructed CT slices are decomposed by RWT, at a pre-selected level of decomposition.

MAE, PSNR and SIMM for the CT-MONO2-16-ankle.dcm image, at the respective noise levels

$\sigma, \%$	AWShrink			Proposed		
	MAE	PSNR, dB	SIMM	MAE	PSNR, dB	SIMM
$\sigma=10$	1,574	30,43	0,7	0,778	39,56	0,83
$\sigma=15$	1,585	28,82	0,59	0,966	36,39	0,72
$\sigma=20$	1,696	27,86	0,48	1,33	33,49	0,66
$\sigma=25$	1,732	27,43	0,43	1,594	30,77	0,59
$\sigma=30$	1,781	26,44	0,32	1,67	28,89	0,54

AWShrink is used for the comparative analysis of the proposed method. Table provides the corresponding values for three of the four of those quality measures obtained when using the two methods.

Figures 2 and 3 show the results obtained when using the wavelet noise reduction methods, whose standard deviation is $\sigma = 10$ and $\sigma = 20$ respectively.

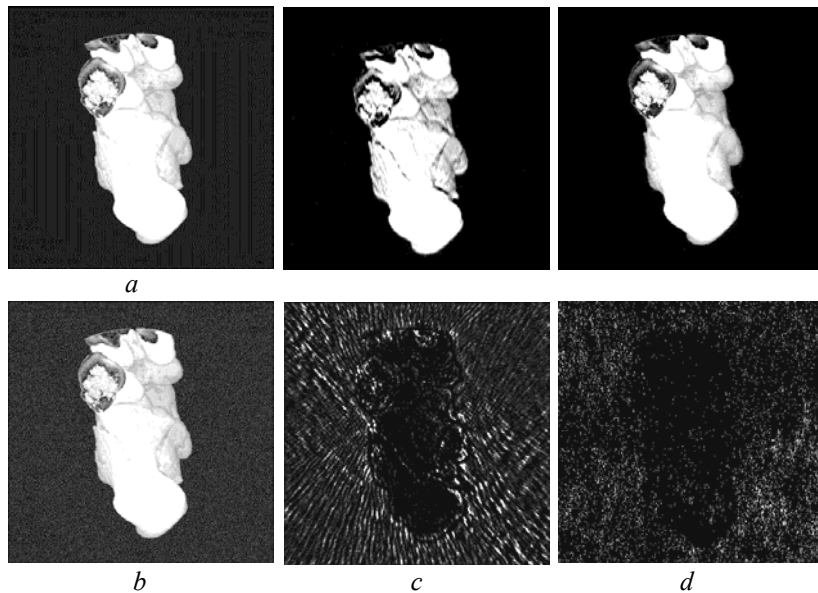


Fig. 2. Denoised results of CT-MONO2-16-ankle.dcm. (noise level $\sigma = 10$) obtained by the two wavelet-shrinkage methods: *a* — Original image; *b* — Noisy image, $\sigma = 10$; *c* — Denoised image and the corresponding residual information, obtained through A-Shrink; *d* — Denoised image and the corresponding residual information, obtained through the proposed method

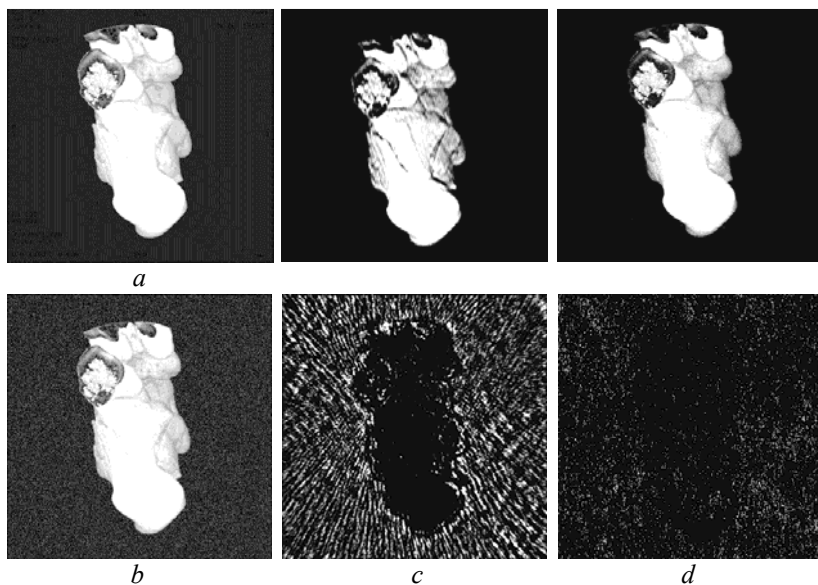


Fig. 3. Denoised results of CT-MONO2-16-ankle.dcm. (noise level $\sigma = 20$) obtained by the two wavelet-shrinkage methods: *a* — Original image; *b* — Noisy image, $\sigma = 20$; *c* — Denoised image and the corresponding residual information, obtained through AW-Shrink; *d* — Denoised image and the corresponding residual information, obtained through the proposed method

Real CT Data

The purpose of the next experiment is to evaluate visually the effectiveness of the proposed method, as well as to confirm it by means of the quantitative measure UIQI. The comparative analysis is again conducted by means of the AWSHrink, using real CT images of the three groups of studied organs: pancreas, colon and thorax. These images are obtained from publicly available medical databases (<https://www.cancerimagingarchive.net/>) and are in DICOM format.

Figures 4–6 present the respective base CT slices and their denoised images, accompanied by the values obtained for UIQI. In addition, the corresponding images containing the residual information, which has been removed from the image when applying these two algorithms, are presented for each pair.

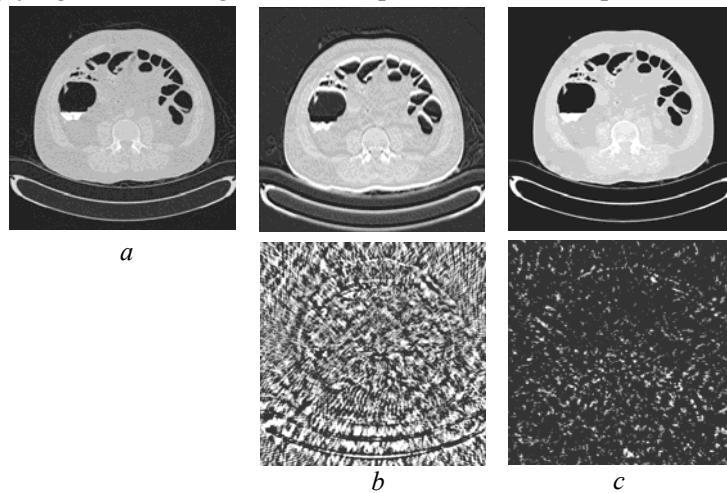


Fig. 4. Results from the reduction of noise in the “Pancreas” image, obtained by the two wavelet-shrinkage methods: *a* — Input test CT image; *b* — Denoised image ($UIQI = 0,632$) and the corresponding residual information, obtained through AWSHrink; *c* — Denoised image ($UIQI = 0,918$) and the corresponding residual information, obtained through the proposed method

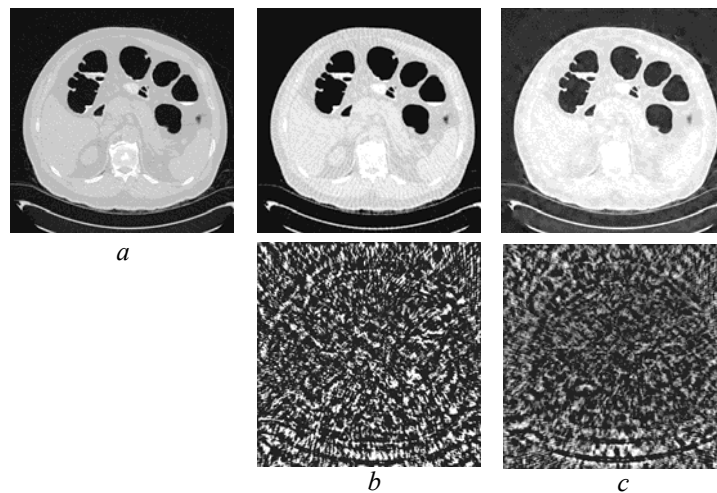


Fig. 5. Results from the reduction of noise in the “Colon” image, obtained by the two wavelet-shrinkage methods: *a* — Input test CT image; *b* — Denoised image ($UIQI = 0,742$) and the corresponding residual information, obtained through AWSHrink; *c* — Denoised image ($UIQI = 0,862$) and the corresponding residual information, obtained through the proposed method

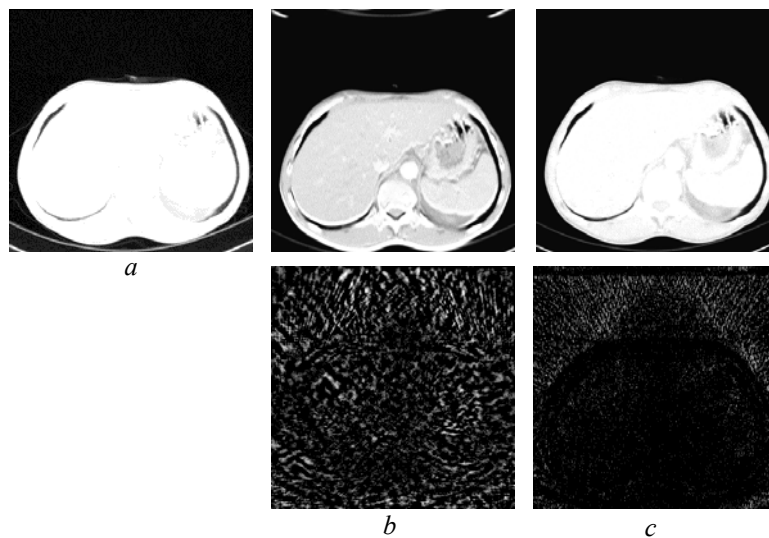


Fig. 6. Results from the reduction of noise in the “Chest” image, obtained by the two wavelet-shrinkage methods: *a* — Input test CT image; *b* — Denoised image ($UIQI = 0,646$) and the corresponding residual information, obtained through AWSHrink; *c* — Denoised image ($UIQI = 0,967$) and the corresponding residual information, obtained through the proposed method

Summarized Results

The results obtained from the two experiments, connected with the comparative analysis and shown in Table and Figures 1–6, lead to the following conclusions:

- For each of the test images obtained at different values of σ the proposed method achieves lower values for MAE as well as higher values for PSNR and SSIM.
- In percentage terms, this advantage over AWSHrink (in respect of the individual measures) is as follows: MAE (32%), PSNR (20%) and SSIM (33%).
- Compared to AWSHrink, the proposed method retains more textural details, not only in the images used in the two experiments, but also in other tested images.
- For each of the test CT images, the proposed method achieves better results for the quality measure UIQI (in percentage – 36%).

CONCLUSION

Based on indirect statistical estimation in the frequency domain, an adaptive threshold algorithm for noise reduction in CT images is proposed. The proposed method is implemented in the RWT domain and is based on the correlation between two CT images, which are equivalent in terms of information content. The continuous function of the soft threshold is used for threshold processing of the high frequency wavelet coefficients of the averaged image.

For the comparative analysis, two experiments are performed, in which the proposed method shows better results in terms of the measures evaluating the qualities of the noise-estimated image.

The assessment of the noise in the projection data and in the corresponding reconstructed slice, as well as its reduction, is among the important tasks in the area of diagnostic X-ray CT. Its solution can be included in the strategy of minimizing the radiation risk to the patient and improving the quality of the diagnostic image. The proposed adaptive wavelet shrinkage method is applicable when post-processing the reconstructed projection data.

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