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Sensitivity of Colding tool life equation on the dimensions of experimental dataset

In this work, 22 sets of cutting data and tool life for longitudinal turning of steel are analyzed using the Colding equation. When modeling tool life with a limited number of tool performance data points, the model error may be low for these points. Evaluating the model for test points not used when computing the model coefficients may give larger errors for these points. This work proves that the Colding model also provides sufficient precision when modelling data points not being used to create the model, and is therefore a well-functioning instrument for tool life modelling. The results also prove that for the selected data, the precision of the model can be greatly improved when the dimension of the data set is increased from 5 to 10 data points. Above 13 data points the precision improvements are negligible.

Keywords: *machining, tool life, turning, the Colding equation.*

INTRODUCTION

It is well known that a prediction of tool life is of great importance in modern industrial production involving machining operations. The time it takes for one tool to be considered worn out and the number of parts it can produce in combination with the tool change time for one or more tools engaged in the process plays a big role in the production costs. The life of the tool is governed by the combination of machinability of a workpiece material, tool properties, and the applied cutting data: cutting speed, v_c, feed, f, and depth of cut, a_p. A tool life normally decreases with an increase of cutting data, and the goal for any production is to find optimum cutting data either in regard to the minimum cost or in regard to highest production efficiency.

As an aid in finding optimal cutting data, many tool manufactures offer catalogue data or software applications to match the right tool to a specific workpiece material and operation in combination with cutting data suggestions.

These recommendations are made through accumulation of the data on the tool performance for different combinations of the tool and work materials while applying different cutting data, and accounting for the tool geometry and chip cross-section window of operation. This is a costly and time consuming process and little is known of the amount of data needed to allow for high quality cutting data recommendations.

The pioneering work of tool life modeling was made by Taylor [1]. A tool life equation is based on two constants and calculates the tool life for a chosen cutting speed or vice versa. Applying this equation the optimal economic life of a tool can be determined. The Taylor equation has proven to work very well in a limited range of cutting data, as shown in [2], because it does not include the data on chip cross-sectional parameters like feed, depth of cut, nose radius, etc. When creating cutting data recommendations for a larger range of cutting data normally used in machining practice, the chip thickness parameters need to be taken into consideration, and therefore there is a need for more complex tool life models.

The Colding equation, introduced by Bertil Colding [3, 4] and further developed by Lindström [5], has proven to adequately perform in predicting tool life in cases of such extended cutting data range, as previously shown, among others, by the authors of [6, 7]. The Colding model, as well as the Taylor model, is based on a curve fitting algorithm operating with five separate constants and has no direct link between the physical mechanisms of a tool wear in the cutting process and the chosen constants. To create a Colding model for one specific combination of a tool and workpiece material, a minimum of five tests is needed to be performed. Once the model and constants are established, the model error can then be calculated for these five or more data points.

OBJECTIVE AND PROBLEM DESCRIPTION

The Colding model represents a function in three dimensional spaces of tool life, cutting speed, and chip thickness. The function is established on a limited set of tool performance points from a selected range and interpolates tool life behaviour within this range. The question of accuracy of interpolation remains open. The aim of this work is to investigate the number of tool performance tests needed to create a Colding model that will model the cutting data and tool life with an acceptably small model error for a wide range of cutting data and tool life. To limit the cost of testing and minimize the needs for updating cutting data it is of great importance that the correct amount of data is collected from the start. With a limited number of tests there may be a risk of creating a tool life model that provides poor quality cutting data recommendations as a result of the interpolation or even frequently used extrapolation. The acquisition of the tool performance information leads to such expenses as workpiece material, tools, and operator time, and this pushes the tool manufacturers to limit the number of the tests performed. In this work, a large amount of cutting data and tool life obtained in machining tests has been used to create a Colding model. The model stability, its sensitivity and statistical variations are evaluated and presented by excluding selected data from the overall dataset.

BACKGROUND

The Colding equation with five constants published by B. Colding in 1981 [4] is, as the pioneering work by Taylor, essentially based on empirical curve fitting made between a tool life and cutting data:

$$v_c = e^{\left[K - \frac{(\ln h_e - H)^2}{4M} - N0 - L \ln h_e \ln T\right]}. \tag{1}$$

The equations can be regarded as an extension of the Taylor equation which can be clearly observed in studies of Lindström's reformulation of the Colding equation [5].

The Colding equation is based on five constants K, H, M, $N0$, and L where cutting speed v_c is a function of the tool life, T, and equivalent chip thickness, h_e. Equivalent chip thicknesses, h_e, as defined by R. Woxén [8], is a function of feed, f, depth of cut, a_p, major cutting angle, κ, and the nose radius of the tool, r_m:

$$h_e = \frac{a_p f}{\frac{a_p - r(1-\cos\kappa)}{\sin\kappa} + \kappa r_m + \frac{f}{2}}. \quad (2)$$

To create a set of Colding constants, the tool performance needs to be evaluated in at least five cutting data points. By varying cutting speed, v_c, feed, f, and depth of cut, a_p a window of cutting data can be created and tool life accordingly modelled. Extrapolation of the modelling results outside the cutting data test window is algorithmically incorrect, but is frequently practiced in the industry and therefore, extra care needs to be taken. A wear criterion, such as flank wear VB_{max} = 0.3 mm or maximum crater wear KT_{max} = 0.5 mm, is selected. The model does not take into account how this wear is developed, it only states the total engagement time before a specific wear criterion is met for the selected cutting data. It is possible to combine the Archad wear model [9] with the Colding model to allow for different wear criterion, as suggested by Ståhl et al. [10], although this is not discussed further in this work. Figure 1 shows how the Colding equation connects cutting data with the tool life.

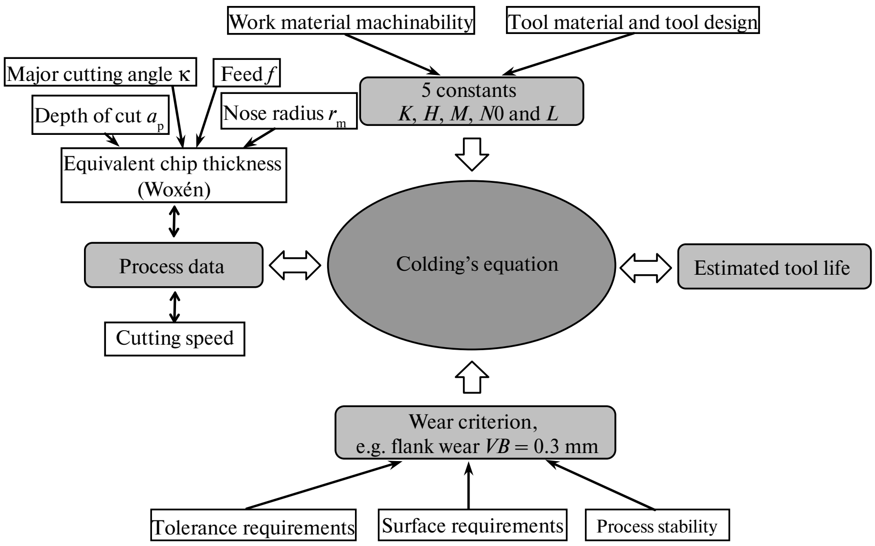

Fig. 1. The schematic of the Colding model design and its connection to cutting data, tool life and wear criterion.

B. Colding [11] and B. Hallert [12] used an ASEA automatic computer (Mod. FACIT EDB) to identify the number of tool life measurements needed to create a reliable tool life model. Eight different polynomial tool life models were tested with a range of two to nine model-constants. The Colding model with five constants (Eq. (1)) was yet not developed when this work was performed. It was concluded that for the polynomial relationship with 9 constants:

$$k + ax + bx^2 + cy + dy^2 + ez^2 - z + fxy + gyz + hxz = 0, \quad (3)$$

where $x = \ln h_e$, $y = \ln v_c$, and $z = \ln T$.

The number of tests should be at least about 25.

The ratio between the largest and smallest equivalent chip thickness should be about 10.

The test should be run using at least three equivalent chip thickness data while varying the cutting speed for the full cutting range where the tool life is linear for a specific equivalent chip thickness in the log T–log v_c plane.

It was also noted that the cost of conducting this testing would be significant and that a wear model with fewer constants is needed in order to limit the number and costs of testing.

EXPERIMENTAL SETUP

In this work, a total of 22 tool performance data points were evaluated when machining C45 E (SS 1672) in longitudinal turning according to ISO 3685:1993 using industry standard coated cemented carbide inserts. The Colding constants were calculated using the least squares method through the built-in feature Solver in the MS Excel® software with curve fitting and the minimization of deviation concerning the obtained measurement points for five or more tool performance data points. Also, the Matlab environment was used for calculations for which 1000 combinations of tool performance data points were randomly selected and Colding constants calculated using the least squares method through a built-in software feature based on an algorithm for data fitting developed by Levenberg-Marquardt [13, 14]. The full data set used to evaluate the Colding model is presented in Table 1.

Table 1. Measured tool performance data points when machining C45 E with cemented carbide inserts used to evaluate the Colding model

Test No.	Depth of cut, mm	Feed, mm/rev	Cutting speed, m/min	Chip thickness, mm	Tool life, min
1	3.5	0.50	260	0.416	7.65
2	3.5	0.50	245	0.416	9.51
3	3.5	0.50	230	0.416	13.17
4	3.5	0.50	215	0.416	17.55
5	3.5	0.50	200	0.416	20.34
6	3.5	0.50	185	0.416	30.24
7	3.5	0.50	170	0.416	33.85
8	3.5	0.50	150	0.416	71.03
9	2.0	0.35	355	0.266	10.05
10	2.0	0.15	490	0.119	12.24
11	2.0	0.25	410	0.194	14.34
12	1.5	0.20	455	0.146	14.17
13	3.0	0.20	430	0.169	18.70
14	2.0	0.25	420	0.194	9.06
15	2.0	0.35	365	0.266	7.00
16	1.5	0.30	405	0.214	11.20
17	2.5	0.40	330	0.317	4.64
18	2.0	0.25	420	0.194	9.66
19	2.0	0.35	365	0.266	10.65
20	1.5	0.30	405	0.214	13.45
21	2.5	0.35	330	0.279	13.29
22	2.5	0.40	330	0.317	10.74

The equivalent chip thicknesses (Eq. 2) in Table 1 range from 0.119 mm to 0.416 mm giving a ratio of approximately 3.5 between the smallest and the largest equivalent chip thickness. The cutting speed ranges from 150 m/min to 490 m/min, Fig. 2. All tests were performed with the major cutting angle $\kappa = 95°$ and nose radius $r_m = 0.8$ mm with no coolant applied.

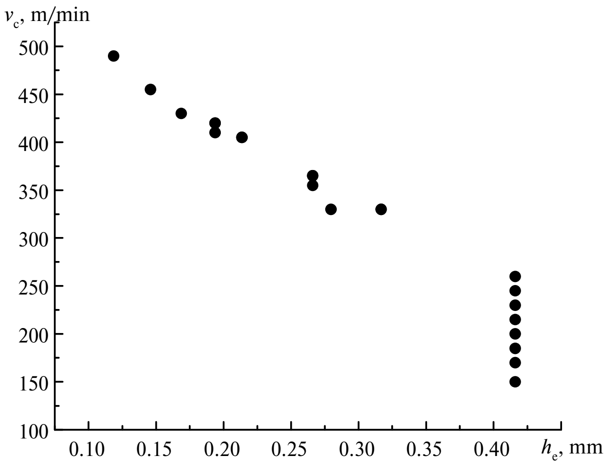

Fig. 2. The cutting data points plotted in the v_c–h_e plane.

The created models based on different measured tool performance data points were then evaluated based on the mean error ε_{err} in % between experimentally attained $v_{c,\,exp}$ and modeled cutting speed $v_{c,\,mod}$ for each model:

$$\varepsilon_{err} = \frac{100}{n} \sum_{j=1}^{j=n} \left| \frac{v_{c_i\,\exp_j} - v_{c_i\,\mathrm{mod}_j}}{v_{c_i\,\exp_j}} \right| . \quad (4)$$

All models were created with the same set of starting values, as shown in Table 2.

The Colding singularity has been discussed by the authors in previous publications [6, 7] and in this work there have been no limitations set to the Colding constants when modeling, allowing for the singularity to enter the h_e area for applicable cutting data.

Table 2. Starting values applied when modelling the Colding constants

Index	Value
K	6.0
H	–3.0
M	2.0
$N0$	0.3
L	–0.05

RESULTS AND DISCUSSION

The Colding model created with the curve fitting and no limitations to the Colding constants and the singularity for all 22 measured tool performance data points is presented in the v_c–h_e plane (Fig. 3) and in the T–v_c plane in Fig. 4. The rather high singularity can be noted in Fig. 3 at $h_e \approx 0.190$ mm.

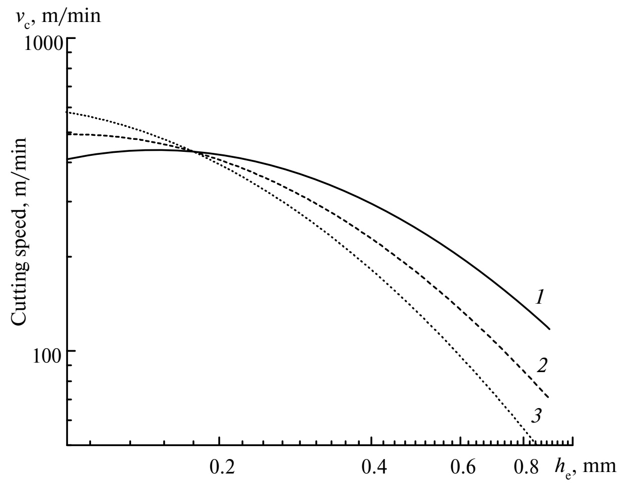

Fig. 3. The Colding model based on all 22 tool performance data points plotted in v_c–h_e plane with no limitations on the constants: tool life – 5 (*1*), 15 (*2*), 40 (*3*) min.

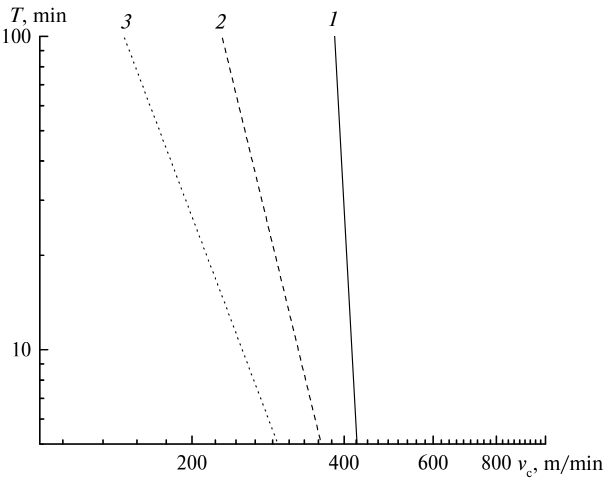

Fig. 4. The Colding model based on all 22 tool performance data points plotted in T–v_c plane with no limitations on the constants: chip thickness – 0.2 (*1*), 0.3 (*2*), 0.4 (*3*) mm.

Initially five measured tool performance data points, representing applicable cutting data, were selected according to a normal tool wear testing procedure. Additional tool performance data points were then selected and added to expand the cutting data window and verify data points within the cutting data window. The result is presented in Fig. 5, where all data are normalized to the results obtained with the largest dataset of all 22 tool performance data points, which is further on considered as best possible solution. It should be noted that this is a very practical approach and that the data in Fig. 5 are dependent on the order in which the chosen data points are added. The order of added data is presented in Table 3, where points 1, 6, 9, 11, and 22 make up the five initial tool performance data points used for creation of the initial Colding model. Thereafter one more point is added, in this case data point 2, and a new model is computed based on all previous data points (initial dataset) including the added point 2 and the error is then calculated.

Sequentially, all 22 tool performance data points are included in the model, following the order presented in Table 3. This test shows the importance of which tool performance data points are measured and included in the model. Each model is then tested on all data including tool performance data points not used to create the model. The mean error and the maximum error that can be found in the 22 tool performance data points are presented and normalized to the best possible model.

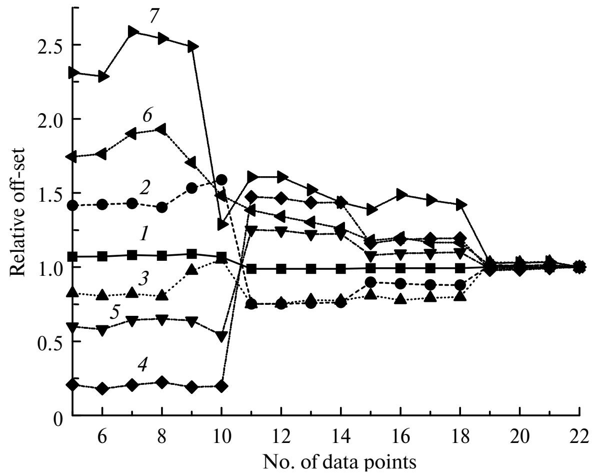

Fig. 5. The Colding constants and errors when sequentially increasing the number of measured tool performance data points included in the tool life model: K_{norm} (*1*), H_{norm} (*2*), M_{norm} (*3*), $N0_{\text{norm}}$ (*4*), L_{norm} (*5*), mean error (*6*), max error (*7*).

Table 4 presents the final set of the Colding constants when all data are included. It also presents the mean error and the maximum error found in the set of data when modeling. The model mean error when using 7 tool performance data points to create the Colding model was calculated to 4.0% showing the risk of not including enough data. The largest error found for an individual tool performance data point for this model was 18.2 %. The identical error, but for the model created with all 22 tool performance data points was 2.5 times smaller. The best possible model including all 22 tool performance data points has a mean error of 2.11 % and the maximum error found in the set of data points is 7.0 %.

When the number of tool performance data points is limited one could probably reduce the risk of creating a poor model by limiting the constants, controlling the singularity and in some cases extrapolate data to the left of the maximum point of the Colding curve, also known as the h-line.

In order to further evaluate the amount of data needed to create a well-functioning tool life model 1000 randomly created datasets with tool performance data points was subjected to Colding modelling. No limitations were set on the selection criteria thus covering both larger and smaller windows of cutting data and investigating the related accuracy for cases of interpolation and extrapolation. Tests were performed using 7, 9 and 13 tool performance data points to create 1000 unique data sets for each test. Figure 6 presents the variation of the K constant dependent on the selected data. The K constant was chosen because it gives the value of $v_c = e^K$ at the extreme point of the Colding plot corresponding to a tool life of 1 min. The corresponding mean error when testing each model on all 22 measured tool performance data points is presented in Fig. 7. The highest mean error for 7 tool performance data points found was 13.0 %, 10 data sets 10.5 % and 13 data sets 6.5 %.

Table 3. The order of added data points. The error presented is the mean average error when the model is tested on all 22 tool performance data points

Test No.	Equivalent chip thickness, mm	Cutting speed, m/min	Measured tool life, min	Error, %
1	0.416	260	7.65	
6	0.416	185	30.24	
9	0.266	355	10.05	
11	0.194	410	14.34	
22	0.317	330	10.74	3.68
2	0.416	245	9.51	3.72
7	0.416	170	33.85	4.01
16	0.214	405	11.20	4.07
12	0.146	455	14.17	3.60
17	0.317	330	4.64	3.13
21	0.279	330	13.29	2.92
3	0.416	230	13.17	2.83
8	0.416	150	71.03	2.75
18	0.194	420	9.66	2.66
13	0.169	430	18.70	2.49
20	0.214	405	13.45	2.53
4	0.416	215	17.55	2.46
5	0.416	200	20.34	2.46
10	0.119	490	12.24	2.13
14	0.194	420	9.06	2.13
15	0.266	365	7.00	2.14
19	0.266	365	10.65	2.11

Table 4. The Colding constants for all measured data and the mean error and maximum errors presented

Index	Value
K	6.136
H	–1.331
M	0.610
$N0$	0.499
L	–0.289
Mean error, %	2.11
Max error, %	7.02

The evaluation of the number of tool performance data points needed to create accurate Colding constants was further investigated by creating 1000 random combinations of data sets with number of tool performance data points included from 5 to 17. The errors for these models are presented in Fig. 8. Line *1* represents the fraction, given in %, of the number of models for which the randomly selected dataset generates a model error of 5.11%, i.e. additional 3 % to the best possible 2.11 % created on all 22 tool performance data points (see Table 3). It can be noted that the accuracy increases drastically from 5 to 9 tool performance data points included and for a set of 11 tool performance data points only 5 % of the Colding models will have a model error of 5.11 % or larger. Line *2* represents the averaged maximum error found for all 1000 combinations and the line *3* represents the average model error for all 1000 combinations. All errors presented are errors when testing the models on all 22 available tool performance data points, also those excluded when creating each model.

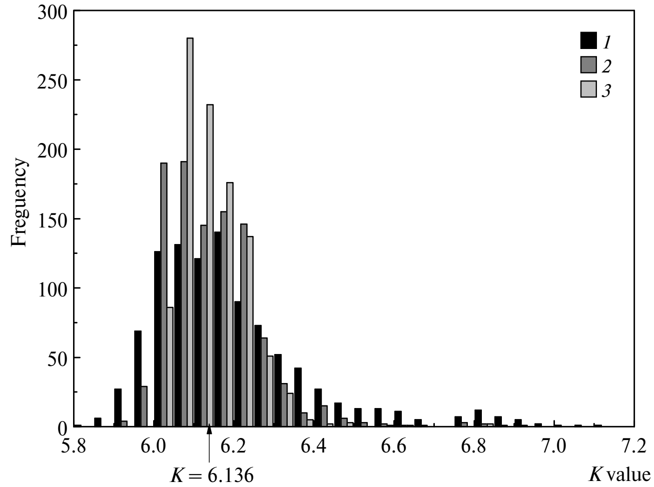

Fig. 6. A histogram plot of the K constant for 1000 combinations of data sets randomly selected using 7 (*1*), 10 (*2*), and 13 (*3*) tool performance data points in the tool life model.

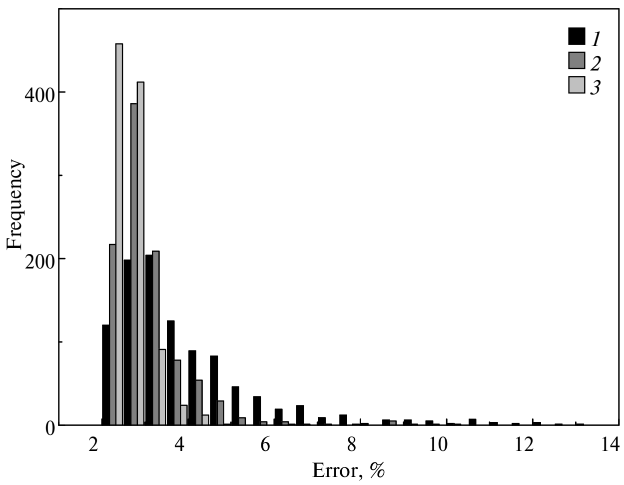

Fig. 7. A histogram plot of the mean error for 1000 combinations of data sets randomly selected using 7 (*1*), 10 (*2*), and 13 (*3*) tool performance data points.

Figure 9 plots the absolute largest error found in one single tool performance data point among the given 1000 combinations of data sets. This plot illustrates that more than 1000 combinations are needed as the error is not strictly decreasing for an increase of tool performance data points. The total number of combinations when operating with 5 data points out of 22 is 26334 and 705432 when operating with 11 out of 22.

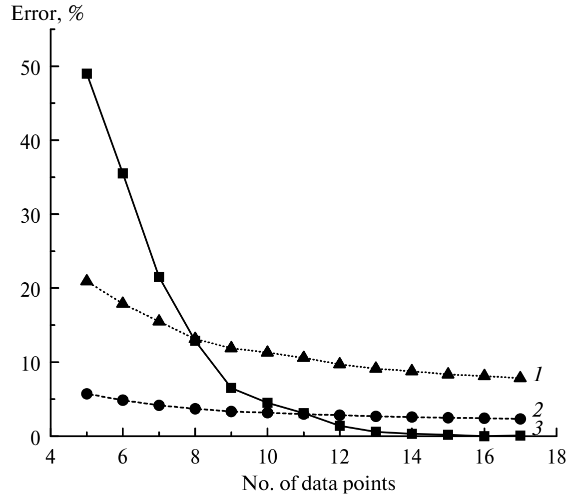

Fig. 8. Model errors for data set dimensions of 5 to 17 points; line *1* represents the ratio of models with an error over 5.11 %, line *2* represents the averaged max error found in 1000 combinations and the line *3* represents the mean model error.

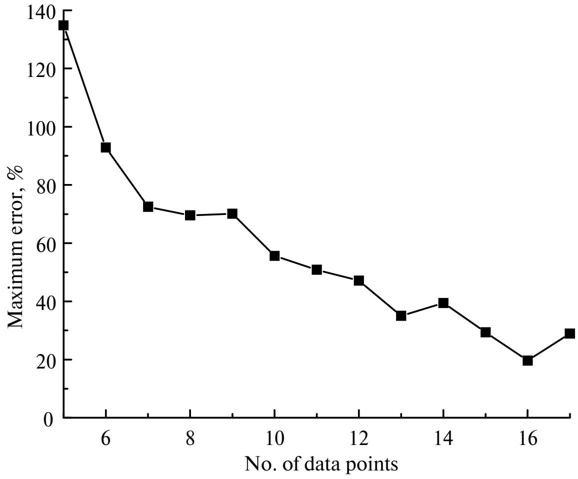

Fig. 9. A largest error found in an individual data point within 1000 combinations of datasets and Colding constants.

The cutting speed for a machining operation can be evaluated by selecting, Fig. 1, targeted tool life and chip thickness ($T = 15$ min and $h_e = 0.25$ mm for the example given below) and respective calculation via Colding equation (Eq. 1). A histogram of the cutting speed for 1000 randomly selected combinations of cutting data points creating the Colding constants when using 7, 10 and 13 tool performance points in the tool life model is presented in Fig. 10. Table 5 presents the mean value of the suggested cutting speed as well as the standard deviation. When operating with 7 tool performance data points, 95 % of the models will estimate the cutting speed within 362±27.9 m/min and when operating with 13 tool performance data points, the model provides the cutting speed of 358±14.3 m/min for 95 % of the models. The variation can be recalculated into relative possible error given in percent. For 7, 10, and 13 tool performance data points used in the model, the relative variation will be ±7.7, ±5.7 and ±4.0 % respectively. This can alternatively be equated to the case where 13 randomly selected tool performance data points will provide accuracy of no less than 4 % with the probability of 95 %.

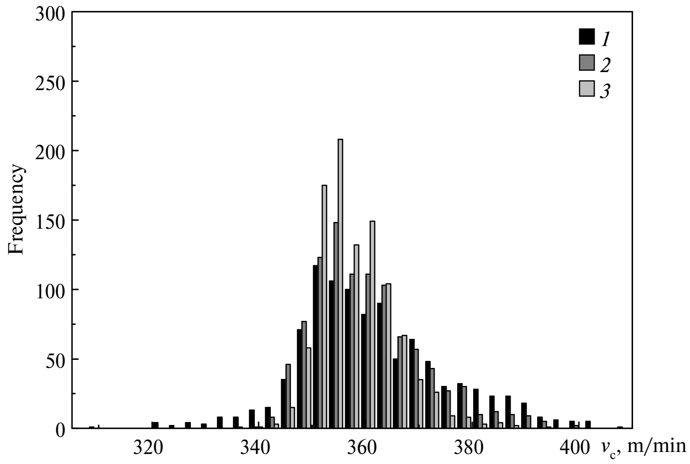

Fig. 10. The distribution of the modelled cutting speed for a turning operation (h_e = 0.25 mm and T = 15 min) for 1000 sets of Colding constants when using 7 (*1*), 10 (*2*), and 13 (*3*) tool performance data points.

Table 5. Statistical analysis of calculated cutting speed v_c for a turning operation (h_e = 0.25 and T = 15 min) for 7, 10 and 13 tool performance data points

No. of data points	7	10	13
Mean value, m/min	362	360	358
Standard deviation, m/min	13.9	10.3	7.2
95 % of the models, m/min	±27.9	±20.5	±14.3
95 % of the models, %	±7.7	±5.7	±4.0

Table 6 presents the ratio of models in % that have a mean model error larger than 4 % and alternatively larger than 10 % when tested on the 22 measured data points.

CONCLUSIONS

For this extended data set of experimentally measured tool performance in longitudinal turning, modeled with the Colding tool life equation, a number of conclusions can be made:

– for a randomly selected 1000 combinations of model constants the computed model error does not exceed 10 % if 10 tool performance data points or more are employed;

– when selecting 13 tool performance data points, only 2.3 % out of 1000 randomized models have an error exceeding 4 %. The largest error for an individual tool performance data point error is however approx. 35 % with the mean max error below 10 %.

The model is improving dramatically when enlarging the dimension of the dataset from 5 to 10 experimental tool performance data points. Above 13 data points the model improvement is only marginal.

Table 6. The fraction of models resulting in error exceeding 4 % and alternatively 10 %

No. of data points	The fraction of models, %, for the error, %	
	> 4	> 10
5	72.9	8.4
6	59.3	3.4
7	42.1	2.1
8	30.9	0.8
9	19.0	0.4
10	15.5	0
11	8.8	0
12	5.4	0
13	2.3	0
14	1.4	0
15	0.6	0
16	0.1	0
17	0.1	0

When using 13 randomly picked tool performance data points we will be 95 % sure to not add a model error of more than 4 % as a result of poor selection of modeled tool performance data points.

It should be noted that all 1000 data sets in each test have been randomly selected. With a more careful selection of tool performance data points, as suggested by Colding and Hägglund [11, 6], the authors of this work believe that the result can be greatly improved. Figure 5 shows how a real selection of data points could be made and one can note that already when selecting 10 data point the mean error and max error is decreased significantly.

This work has proven that the Colding equation is a well-functioning tool life model also when tested on data not being used to create the model.

This work is solely based on analyzing one set of data with 22 measured cutting data points and tool lives. Further statistical analysis is needed with a more general perspective to create a greater understanding of the Colding tool life model and its use and limitations.

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Проаналізовано 22 набору режимів різання і стійкість інструменту при поздовжньому точінні стали при застосуванні моделі Колдінга. При моделюванні стійкості інструменту при обмеженій кількості даних про робочі характеристики помилка моделі може бути незначною в заданих точках. Оцінка моделі для тестових точок, які не використовуються при обчисленні коефіцієнтів моделі, може показати більші помилки в цих точках. Доведено, що модель Колдінга забезпечує достатню точність при моделюванні даних, що не використовуються для створення моделі, і тому може бути застосо-

вана для моделювання періоду стійкості інструменту. Результати також доводять, що для даних, що використовуються, точність моделі може бути значно поліпшена при збільшенні набору точок з 5 до 10, а при збільшенні понад 13 точок поліпшення точності моделювання незначні.

Ключові слова: *обробка, стійкість інструменту, точіння, рівняння Колдінга.*

Проанализированы 22 набора режимов резания и стойкость инструмента при продольном точении стали с применением модели Колдинга. При моделировании стойкости инструмента при ограниченном количестве данных о рабочих характеристиках ошибка модели может быть незначительной в заданных точках. Оценка модели для тестовых точек, не используемых при вычислении коэффициентов модели, может показать бóльшие ошибки в этих точках. Доказано, что модель Колдинга обеспечивает достаточную точность при моделировании данных, не используемых для создания модели, и поэтому может быть применена для моделирования периода стойкости инструмента. Результаты также доказывают, что для используемых данных точность модели может быть значительно улучшена при увеличении набора точек с 5 до 10, а при увеличении более 13 точек улучшения точности моделирования незначительны.

Ключевые слова: *обработка, стойкость инструмента, точение, уравнение Колдинга.*

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