ISSN 2786-7102 (Online), ISSN 2786-7633 (Print)
https://doi.org/10.15407/srenergy2023.01.025

UDC 621.311.661.51

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GENERATION UNIT COMMITMENT MIXED INTEGER LINEAR MODEL FOR SIMULTANEOUS HEAT AND ELECTRIC DAILY LOAD COVERING

Abstract. *The unit commitment problem nowadays is widely used in the electric power sector. The problem was first time formulated in the 1940-s and still developing both methodologically and by including an additional number of technologies each of which has a different unique mathematical treatment corresponding to the specific technology's behavior. The common characteristic of the problem such as that is dedicated to the electricity production sector, hence the mathematical formulation is following pure electricity sector transformation but during the last years the Power-to-X technologies are implemented and their further development is expected in the future. This requires the advancement or at least modification of the problem formulation to meet possible exchange and usage between different types of energy within one integrated power system. The goal of the article is to further development of the existing versions of the unit commitment problem, which are dedicated to the operation of the generation in the power system by implementing additional equations allowing contemplation of the heat energy-producing technologies which are dedicated to cover a heat-energy load of the district heating systems. This should allow for conducting comprehensive studies of the simultaneous operation of electric- and heat-generating technologies to meet the energy demand of local energy systems, which is important for designing distributed generation mix, for example at a municipal level. The proposed mixed integer linear generation unit commitment model for simultaneous heat and electric daily load covering is described in the article. The proposed model in addition to the pure electric power balance also meets heat load using only-heat technologies (fuel boilers), combined heat and power units, and also industrial-scale electric boilers - which are converting electricity to heat energy.*

Keywords: mixed integer linear model, unit commitment problem, integrated power system, electric boilers, power-to-X technologies, conventional electricity generating technologies.

1. Introduction

The unit commitment problem nowadays is widely used in the electric power sector [1, 4, 5]. The problem was first time formulated in the 1940-s but the first algorithm developed to solve the unit commitment problem was proposed in 1959 [1]. The first mixed integer linear problem (MILP) regarding the unit commitment was formulated in 1962 [1, 3] after Bellman’s fundamental work on dynamic operation research was published [2]. The mathematical formulation is still developing both methodologically and by including an additional number of technologies each of which has a different unique mathematical treatment corresponding to the specific technology's behavior. The common characteristic of the problem such as that is dedicated to the electricity production sector, hence the mathematical formulation is following pure electricity sector transformation but during the last years the Power-to-X technologies are implemented and their further development is expected in the future. This requires the advancement or at least modification of the problem formulation to meet possible exchange and usage between different types of energy within one integrated power system. In general, the objective function of the unit commitment problem is formulated as follows [1]:

Total production costs = (Fuel cost + Start-up cost + Shut-down cost+ Maintenance cost) —> min. (1)

This formulation was published in 2018, but in 2022 the general formulation became more comprehensive [5]:

Min (Production cost + Start-up cost + Shut-down cost + Emission cost + Maintenance cost). (2)

Equation (2) includes “Production cost” instead of “Fuel cost” in (1), and this is more correct as during the operation of the power unit not only fuel but also other resources are consumed, e.g., pure water, chemicals etc. The “Emission cost” in (2) became more important in current times as a number of governments are adopting the decision to reach climate neutrality and fast penetration of renewable technologies, e.g., wind, solar photovoltaic, and biomass competing with conventional power units. But this parameter could be included in the constraints section of the unit commitment problem formulation as rather the limits for emissions are applied to the electric power generation, nor minimization of payment for unrestricted emissions amount.

The unit commitment problem “behavior” is mainly described in the constraints section. The typical constraints are [5, 6, 7]: Demand constraint – exact equilibrium between production (supply) and consumption for each period; Power generation limits – each unit should not exceed nominal installed capacity and should not be lower than technical generating limit; Power output ramping limits – period-to-period increase/decrease of power output should be within the technical possibility of the unit; Operation/idle time constraints – describes allowed operational modes behavior of each unit; Emission constraints – imposed to not exceed allowed emission volumes, e.g., from coal-fired units; Fuel constraints – limits available fuel on stock for each or for the group of fuel burning units for each time step. The list of the constraints and their formulation could vary depending on the goal of the study/calculations.

The goal of the article is to further development of the existing versions of the unit commitment problem, which are dedicated to the operation of the generation in the power system by implementing additional equations allowing contemplation of the heat energy-producing technologies which are dedicated to cover a heat-energy load of the district heating systems. This should allow for conducting comprehensive studies of the simultaneous operation of electric- and heat-generating technologies to meet the energy demand of local energy systems, which is important for designing distributed generation mix, for example at a municipal level. The proposed mixed integer linear generation unit commitment model for simultaneous heat and electric daily load covering is described in the article. The proposed model in addition to the pure electric power balance also meets heat load using only-heat technologies (fuel boilers), combined heat and power units, and also industrial-scale electric boilers, which are converting electricity to heat energy.

The MILP unit commitment problem for the Integrated power system of Ukraine was formulated in several papers [8, 9] which are aimed at correctly describing specifics of power generation units used in Ukraine and their typical operation modes. These mathematical models are corresponding only to the electric power sector in the sense that only the electric power balance (equilibrium of production and consumption for each time step) is taken into account, but while covering the demand for electric power the combined heat and power (CHP) units simultaneously participate with all other exactly electric power units. To include CHP in formulation only volumes of electric energy generated by them are taken into account and the volume of heat power is neglected. As CHPs are operated according to the demand for heat consumption by households, tertiary, and industry sectors this approach is artificial and far from real life. Of course, the levels of electric power consumption also depend on the atmospheric temperature, but in Ukraine, exactly heat generation technologies are widely used to meet heat energy demand. Hence, to more correctly include CHPs in electric power balance also other heat energy production units, such as fuel boilers, should be considered, due to CHPs and fuel boilers together covering heat energy demand in reality. This requires taking into consideration within the mathematical model formulation not only the electric power balance but also the heat energy balance, considering the possibility of each technology participating in supplying an appropriate type of energy to meet the appropriate type of energy demand.

2. Mathematical model formulation

The MILP unit commitment problem proposed in this paper is formulated with the use of the below-listed sets, parameters, and variables. T — set of periods, as the one day is considered then set consists 24 elements (a day with 1 hour time resolution); K — set of all power units of all power stations including thermal power plants (TPP), nuclear power plants (NPP), combined heat and power stations (CHP), autoproducers (APr), hydro units of hydroelectric power stations (HPS) and units of hydro pumping storages stations (HPSS), and also wind (Wind) and photovoltaic (PV) power stations; K^{PS} — set of hydro pumped storages stations (namely – Kyiv, Dnister and Tashlyk HPSS), for each HPSS the set of hydro units is determined — U^{KPS}; K^{TPP} — set of thermal power plants, for each TPP the set of power units is determined — U^{TPP}; K^{CHP} — set of CHP plants, for each CHP the set of power units is determined — U^{CHP}; K^{R} — the set of renewable generators (Wind and PV); M^{HB} — the set of fuel burning boilers producing the heat energy (mostly natural gas burning boilers in Ukraine); M^{EHB} — the set of industry scale electric boilers producing the heat energy; P_t^W — the variable corresponding to the curtailed electric power production by Wind, constant MW during one-time step (MWh); P_t^{PV} — the variable corresponding to the curtailed electric power production by PV, constant MW during one-time step (MWh); $P_{kt}{}^{G}$ — electric power produced by technologies included in the set $K \setminus (K^{PS} \cup K^{TPP} \cup K^{R} \cup K^{CHP})$, constant MW during one-time step (MWh); $P_{kut}{}^{TPP}$ — the variable corresponding to the electric power produced by TPP's power unit, constant MW during one-time step (MWh); $P_{kut}{}^{CHP\_E}$ — the variable corresponding to the electric power produced by CHP's power unit, constant MW during one-time step (MWh); $P_{kut}{}^{CHP\_H}$ — heat energy produced by CHP's power unit, constant MW during one-time step (MWh); $P_{mt}{}^{HB\_H}$ — the variable corresponding to the heat energy produced by fuel fired boiler, constant MW during one-time step (MWh); $P_{mt}{}^{EHB\_H\_P}$ — the variable corresponding to the heat energy produced by electric boiler, constant MW during one-time step (MWh); $y_{kut}{}^{G}$ — binary variable, equal to 1 (one), if hydro unit is operating in generation mode, otherwise is 0 (zero); $P_{kut}{}^{G}$ — the variable corresponding to the electric power produced by HPSS in generation mode, constant MW during one-time step (MWh); $y_{kut}{}^{P}$ — binary variable, equal to 1 (one), if hydro unit is operating in pumping mode, otherwise is 0 (zero); $P_{kut}{}^{P}$ — electric power consumed by hydro unit of HPSS in pumping mode, constant MW during one-time step (MWh); z_t^I — the variable corresponding to imported electric power, constant MW during one-time step (MWh); z_t^E — the variable corresponding to exported electric power, constant MW during one-time step (MWh); c^{W} — parameter corresponding to the price of electricity produced by wind farms, USD/MWh; c^{PV} — parameter corresponding to the price of electricity produced by solar photovoltaic plants, USD/MWh; $c_{kt}{}^{G}$ — parameter corresponding to the price of electricity produced by technologies from set $K \setminus (K^{PS} \cup K^{TPP} \cup K^{R} \cup K^{CHP})$, USD/MWh; $c_{kut}{}^{TPP}$ — parameter corresponding to the price of electricity produced by each type of thermal power plant units, USD/MWh; $c_{kut}{}^{CHP\_E}$ — parameter corresponding to the price of electric energy produced by combined heat and power plant units, USD/MWh; $c_{kut}{}^{PSG}$ — parameter corresponding to the price of electric energy produced by hydro pumping storages station units, USD/MWh; $c_{kut}{}^{CHP\_H}$ — parameter corresponding to the price of heat energy produced by combined heat and power plant units, USD/MWh; $c_{mt}{}^{HB\_H}$ — parameter corresponding to the price of the heat energy produced by fuel burning boilers, USD/MWh; $c_{mt}{}^{EHB\_H}$ — parameter corresponding to the price of the heat energy produced by electric boilers, USD/MWh.

The proposed objective function is minimizing the production cost of both electricity and heat energy:

$$
\begin{aligned}
&\sum_{t=1}^{T}[P_t^W c^W + P_t^{PV} c^{PV} + \sum_{k=1}^{K\setminus(K^{PS}\cup K^{TPP}\cup K^{R}\cup K^{CHP})} P_{kt}^{G} c_{kt}^{G} + \sum_{k=1}^{K^{TPP}} \sum_{u=1}^{U^{TPP}} P_{kut}^{TPP} c_{kut}^{TPP} + \sum_{k=1}^{K^{CHP}} \sum_{u=1}^{U^{CHP}} P_{kut}^{CHP\_E} c_{kut}^{CHP\_E}]+ \\
&+\sum_{t=1}^{T}[\sum_{k=1}^{K^{PS}} \sum_{u=1}^{U^{KPS}} y_{kut}^{G} P_{kut}^{G} c_{kut}^{PSG} + z_t^I c_t^I - z_t^E c_t^E]+ \qquad (3) \\
&+\sum_{t=1}^{T}[\sum_{k=1}^{K^{CHP}} \sum_{u=1}^{U^{CHP}} P_{kut}^{CHP\_H} c_{kut}^{CHP\_H} + \sum_{m=1}^{M^{HB}} P_{mt}^{HB\_H} c_{mt}^{HB\_H} + \sum_{m=1}^{M^{EHB}} P_{mt}^{EHB\_H\_P} c_{mt}^{EHB\_H}] \to \min.
\end{aligned}
$$

There are two equilibrium equations (constraints) are needed to provide the electric and heat energy production-consumption balance for each time step. The first equation is for electric energy (4), and the second one is for heat energy (5).

$$P_t^W + P_t^{PV} + \sum_{k=1}^{K\backslash(K^{PS}\cup K^{TPP}\cup K^R\cup K^{CHP\_E})} P_{kt}^G + \sum_{k=1}^{K^{TPP}} \sum_{u=1}^{U^{TPP}} P_{kut}^{TPP} + \sum_{k=1}^{K^{CHP}} \sum_{u=1}^{U^{CHP}} P_{kut}^{CHP\_E} +$$
$$+\sum_{k=1}^{K^{PS}} \sum_{u=1}^{U^{KPS}} \left(y_{kut}^G P_{kut}^G - y_{kut}^P P_{kut}^P \right) - \sum_{m=1}^{M^{EHB}} P_{mt}^{EHB\_E\_C} + z_t^I - z_t^E = D_t^E ;\forall t \in T; \tag{4}$$

$$\sum_{k=1}^{K^{CHP}} \sum_{u=1}^{U^{CHP}} P_{kut}^{CHP\_H} + \sum_{m=1}^{M^{HB}} P_{mt}^{HB} + \sum_{m=1}^{M^{EHB}} P_{mt}^{EHB\_H\_P} = D_t^H ;\forall t \in T, \tag{5}$$

where D_t^E — the parameter corresponding to the electric energy demand in the energy system, constant MW during one-time step (MWh); D_t^H — the parameter corresponding to the heat energy demand in the district heating, constant MW during one-time step (MWh); $P_{mt}^{EHB\_E\_C}$ — the variable corresponding to the electric energy consumed by the electric boiler from the grid for the heating the carrier (water), constant MW during one-time step (MWh).

The constraints regarding the operation of various electricity-generating technologies for the electric load covering are described in detail in [8, 9], hence below is a mathematical formulation (behavior description) of the operation of the heat-generating technologies. The mathematical models dedicated to the optimization of the operation of heat-generating technologies themselves are described in many papers, e.g. detailed description is represented in [10] published as a result of the European Union’s Horizon 2020 research and innovation programme. But the common practice is the use of separate mathematical models and appropriate tools for the district heating and electric grids, which does not exactly correspond to the real situation "on the ground". Actually, such separate development of district heating and electric grids often results in inefficient operation of CHP as an electricity producer first-of-all in summer-time. And instead, the usage of electricity for heating is often limited by the local electric grids, which are not corresponding to maintaining the appropriate load for the transformation of electricity to heat. And that is why the simultaneous modeling of the unit commitment for electric and heating load covering could be a promising technique first of all for the local district heating and electric grid planning, but also could be used for the whole power system state level as some generators' installed capacity is much bigger as required at the local level.

The rapid implementation of solar photovoltaic generation at the local level is an important driver for the use of electric boilers as the usual situation of the excess of electricity produced by solar technologies exactly during the day-time. At the level of the energy system, the common practice is to temporarily curtail the excess of solar generation, but another way is to use it for the heating or at least the hot water supply. Of course, the usage of this excess electricity for heating should be in equilibrium with the heat energy demand curve, which is required to be stored for some time in the heat storage, and as shown in the [11, 12] the actual district heating grid storage capacity is enough both in winter-time and in summer to accumulate and store about 3 GW of heat energy for several hours, that is the realistic base for implementation of electric boilers into district heating grids.

The constraints that describe the use of electric boilers consist of two types of equations: the first one sets the limits of power that could be consumed (electric power) or produced (heat energy) during appropriate periods of a day (6); the second one requires fulfillment of one-day energy balance the sense of which is all electricity that we consume for heating purposes should be outputted to the district heat for storing and then wholly consumed by users for heating purposes (7).

$$P_{mt}^{EHB\_E\_C} \le P_m^{MAX\_C} ;\forall m \in M ;\forall t \in \left[t_{start}^{night} ... t_{end}^{night} ; t_{start}^{day} ... t_{end}^{day} \right] \subset T;$$

$$P_{mt}^{EHB\_E\_C} \ge P_m^{MIN\_C}; \forall m \in M; \forall t \in \left[t_{start}^{night} ... t_{end}^{night}; t_{start}^{day} ... t_{end}^{day} \right] \subset T; \tag{6}$$

$$P_{mt}^{EHB\_H\_P} \le P_m^{MAX\_P}; \forall m \in M; \forall t \in \left[1 ... t_{start}^{night} - 1; t_{end}^{night} + 1 ... t_{start}^{day} - 1; t_{end}^{day} + 1 ... t^{LAST} \right] \subset T;$$

$$P_{mt}^{EHB\_H\_P} \ge P_m^{MIN\_P}; \forall m \in M; \forall t \in \left[1 ... t_{start}^{night} - 1; t_{end}^{night} + 1 ... t_{start}^{day} - 1; t_{end}^{day} + 1 ... t^{LAST} \right] \subset T, \tag{7}$$

where $P_m^{MAX\_C}$ — the parameter corresponding to the maximum electric power limit that could be consumed by an electric boiler for heating purposes from the electric grid, MW; $P_m^{MIN\_C}$ — the parameter corresponding to the minimum electric power limit that should be consumed by an electric boiler for heating purposes from the electric grid (usually equal to 0 (zero), MW; $P_m^{MAX\_P}$ — the parameter corresponding to the maximum heat energy that could be consumed by a user from the heating district grid this heat was earlier produced by the electric boiler, and hence stored in the district heating grid, MW; $P_m^{MIN\_P}$ — the parameter corresponding to the minimum heat energy that could be consumed by a user from the heating district grid (usually equal to 0 (zero) this heat was earlier produced by the electric boiler, and hence stored in the district heating grid, MW; t_{start}^{night} — the time-step during a night-time (index of set T) is a parameter corresponding to the allowed starting time of electric boiler heating the heat carrier consuming the electricity from the grid; t_{end}^{night} — the time-step during a night-time (index of set T) is a parameter corresponding to the time when the electric boiler should stop consuming the electricity from the grid and hence stop heating the heat carrier; t_{start}^{day} — the time-step during a day-time (index of set T) is a parameter corresponding to the allowed starting time of electric boiler heating the heat carrier consuming the electricity from the grid; t_{end}^{day} — the time-step during a day-time (index of set T) is a parameter corresponding to the time when the electric boiler should stop consuming the electricity from the grid and hence stop heating the heat carrier; t^{LAST} — the last time-step during a day-time (maximum index of set T).

The breakdown of the set T into two periods is explained by the usual modes of the power system – during night-time, the power system has an excess of electric power, and during this period the additional consumption of electric power is a factor that allows increasing electric grid stability; during the day-time especially under large penetration of renewables which are producing peaking power (usually solar generation) also there is an excess of electric power which should be consumed or curtailed. The consumption of electric energy between these periods is forbidden for electric boilers, for the reason of ensuring electric grid stability.

The last constraint requires the exact equilibrium between the energy volume that was consumed by electric boilers for heating the heat carrier during the whole day and that one consumed by the users from the district heating grid where it is stored (8).

$$\sum_{t=1}^{T} \sum_{m=1}^{M^{EHB}} P_{mt}^{EHB\_E\_C} = \sum_{t=1}^{T} \sum_{m=1}^{M^{EHB}} P_{mt}^{EHB\_H\_P}. \tag{8}$$

According to (8) an exact equilibrium between the consumption of energy (i.e. electricity) and the energy production (i.e. heat energy) is required, which assumes the absence of heat losses during its storage, but if the heat energy stores for a long time the losses will be not equal to a zero, especially in winter. This would be corrected by applying appropriate parameter corresponding to the specific heat losses according to the air temperature at the right part of the constraint (8), but according to the equations (6, 7) it is assumed that all stored heat energy is consumed over the next several hours, and in that case, this coefficient is close to 1(one) and has been not presented in (8).

3. Results of calculation and discussion

The calculations were made for three scenarios. In the first scenario, the Hydro Pumped Storages (HPS) are used to cover the electricity load, and electric boilers (ElB) are not used to cover the heat load –

ElBNo&HPSYes Scenario. In the second scenario, the HPS are not used to cover the electricity load, and electric boilers are used to cover the heat load, and hence consume electricity from the electric grid – ElBYes&HPSNo Scenario. In the third scenario, both HPS and electric boilers are used – ElBYes&HPSYes scenario. To demonstrate the action of the proposed model the actual data regarding electric load covering for the 14 of July 2018 was used. The heat load has been constructed according to the typical heat load for the summer season based on [10]. The input data regarding hour-to-hour electricity and heat energy consumption, and generation profiles of NPP, autoproducers, wind, and solar generation are presented in Tab. 1.

Table 1. The input data for the calculations

Time step (hour)	Heat energy consumption, MWh	Electric energy consumption, GWh	NPP generation, GWh	Autoproducers generation, GWh	Wind generation, GWh	Solar PV generation, GWh
1	1363.2	9.27	3.92	0.06	0.19	0
2	1168.5	8.93	3.92	0.06	0.13	0
3	1168.5	8.72	3.92	0.06	0.10	0
4	1168.5	8.65	3.92	0.06	0.08	0
5	1168.5	8.52	3.92	0.06	0.08	0.03
6	779.0	8.74	3.92	0.06	0.07	0.10
7	779.0	9.04	3.92	0.06	0.07	0.35
8	779.0	9.21	3.92	0.06	0.03	0.87
9	1947.4	9.92	3.92	0.06	0.01	1.48
10	1947.4	10.36	3.92	0.06	0.04	2.21
11	2336.9	10.53	3.92	0.06	0.05	2.50
12	2336.9	10.73	3.92	0.06	0.12	2.63
13	1947.4	10.68	3.92	0.06	0.16	2.58
14	1947.4	10.70	3.92	0.06	0.21	2.16
15	1752.7	10.50	3.92	0.06	0.20	1.99
16	1752.7	10.48	3.92	0.06	0.15	1.78
17	1168.5	10.55	3.92	0.06	0.17	1.46
18	1168.5	10.58	3.92	0.06	0.19	1.03
19	2531.7	10.44	3.92	0.06	0.17	0.32
20	2531.7	10.21	3.92	0.06	0.08	0.09
21	2726.4	10.32	3.92	0.06	0.05	0
22	2726.4	10.65	3.92	0.06	0.09	0
23	2921.2	10.07	3.92	0.06	0.06	0
24	1947.4	9.49	3.92	0.06	0.08	0
Sum	**42064.8**	**237.3**	**94.1**	**1.5**	**2.6**	**21.6**

The MILP mathematical model is written in MathProg [13] language, a freeware version of AMPL. As a solver, the GNU GLPK [14] program is used. The overall time for one calculation with standard table PC is about 5 seconds. The result of calculations according to each scenario is the schedule of power output for each technology used to cover electricity and heat load during the day. The Tab. 2 and 3 are representing the results of calculations for the third scenario when electric boilers with a 500 MW total capacity are implemented and simultaneously all existing in Ukraine Hydro Pumping Storage Stations are used. Similar calculations were made for the first and second scenarios and the comparison of all scenarios is represented in Tab. 4.

Table 2. The results of calculations for covering electric load according to the ElBYes&HPSYes scenario

Time step (hour)	Combined heat and power, GWh	Thermal Power Plant, GWh	Hydro generation, GWh	Hydro Pumping Stations (pumping), GWh	Hydro Pumping Stations (generation), GWh	Electric Boilers (consumption), GWh
1	1.4	4.0	0.2	-	-	0.5
2	1.3	3.8	0.2	-	-	0.5
3	1.3	3.6	0.2	0.4	-	0.2
4	1.4	3.4	0.7	0.4	-	0.5

Continuation of Table 2.

Time step (hour)	Combined heat and power, GWh	Thermal Power Plant, GWh	Hydro generation, GWh	Hydro Pumping Stations (pumping), GWh	Hydro Pumping Stations (generation), GWh	Electric Boilers (consumption), GWh
5	1.3	3.3	0.8	0.4	-	0.5
6	1.2	3.1	0.3	-	0.3	0.3
7	1.1	2.9	0.3	-	0.3	-
8	1.2	2.9	0.4	0.4	0.3	-
9	1.3	3.1	0.7	0.6	-	-
10	1.2	3.2	0.3	0.6	-	-
11	1.4	3.4	0.2	0.4	-	0.5
12	1.4	3.6	0.2	0.6	-	0.5
13	1.4	3.7	0.5	1.1	-	0.5
14	1.4	3.5	0.9	0.9	-	0.5
15	1.3	3.3	0.4	0.5	0.3	0.5
16	1.2	3.1	0.2	0.1	0.3	-
17	1.1	3.1	0.2	0.1	0.7	-
18	1.2	3.1	0.4	-	0.8	-
19	1.3	3.3	0.4	-	1.0	-
20	1.4	3.5	0.5	-	0.7	-
21	1.4	3.7	1.0	-	0.2	-
22	1.4	3.7	1.4	-	-	-
23	1.4	3.5	1.1	-	-	-
24	1.3	3.3	0.8	-	-	-
Sum	**31.1**	**81.0**	**12.1**	**6.7**	**5.1**	**4.9**

Table 3. The results of calculations for covering heat load according to the ElBYes&HPSYes scenario

Time step (hour)	Combined heat and power, MWh	Natural Gas fired boilers, MWh	Other boilers incl. biomass, MWh	Electric Boilers (supply already earlier generated and stored heat energy), MWh
1	500	276	400	-
2	500	186	315	-
3	500	96	400	-
4	500	81	400	-
5	500	102	400	-
6	460	45	127	-
7	420	45	80	88
8	458	45	80	50
9	498	383	400	500
10	460	800	80	441
11	500	1250	400	-
12	500	1250	400	-
13	500	860	400	-
14	500	860	400	-
15	500	686	400	-
16	462	244	400	500
17	422	225	80	295
18	447	445	80	50
19	487	978	400	500
20	500	945	400	500
21	500	1139	400	500
22	500	1139	400	500
23	500	1335	400	500
24	500	445	336	500
Sum	**11.6**	**13.9**	**7.6**	**4.9**

According to the results (table 4) the most feasible third scenario (ElBYes&HPSYes scenario) as it requires minimum usage of gas-fired boilers to cover heat energy load, hence minimum natural gas consumption. At the same time, according to this scenario, the about minimum operation of gas-fired CHP is needed causing additional minimization of natural gas consumption. The interesting result is that with the absence of Hydro Pumping Station operations (ElBYes&HPSNo scenario) also possible to guarantee the coverage of electric and heat energy load, but in that case, the usage of coal-fired Thermal Power Plants is maximum causing additional emissions of greenhouse gases and hazardous air pollution. The last result is showing that the implementation of electric boilers into the electric grid serves as a source of additional power grid flexibility, causing its safer operation.

Table 4. The comparison of results of calculations according to the scenarios

Technology	**Scenario**		
	ElBNo & HPSYes	**ElBYes & HPSNo**	**ElBYes & HPSYes**
	Electricity load covering, GWh per day		
Combined heat and power	32.1	27.1	31.1
Thermal power plants	75.0	82.6	81.0
Hydro	12.1	13.0	12.1
Electric Boilers (consumption)	0	5.3	4.9
	Heat energy load covering, GWh per day		
Combined heat and power	12.0	11.6	11.6
Natural Gas fired boilers	17.9	16.6	13.9
Other boilers incl. biomass	8.0	6.4	7.6
Electric Boilers (supply already earlier generated and stored heat energy)	0	5.3	4.9

4. Conclusions

The unit commitment problem nowadays is widely used in the electric power sector, and is still developing both methodologically and by including an additional number of technologies each of which has a different unique mathematical treatment corresponding to the specific technology's behavior. The penetration of the Power-to-X technologies during last years requires the advancement or at least modification of the problem formulation to meet possible exchange and usage between different types of energy within one integrated power system. The proposed mixed integer linear generation unit commitment model is a further development of the existing versions of the unit commitment problem, which are dedicated to the operation of the generation in the power system by implementing additional equations allowing contemplation of the heat energy-producing technologies which are dedicated to cover a heat-energy load of the district heating systems. This allows for conducting comprehensive studies of the simultaneous operation of electric- and heat-generating technologies to meet the energy demand of local energy systems, which is important for designing distributed generation mix, for example at a municipal level. The proposed model in addition to the pure electric power balance also meets heat load using only-heat technologies (fuel boilers), combined heat and power units, and also industrial-scale electric boilers – which are converting electricity to heat energy.

The proposed MILP generation unit commitment mathematical model is written in MathProg language, a freeware version of AMPL. As a solver, the GNU GLPK program is used. The overall time for one calculation with a standard table PC is about 5 seconds. The result of calculations according to each scenario is the schedule of power output for each technology used to cover electricity and heat load during the day.

According to the results of calculations the most feasible scenario is corresponding the simultaneous use Electric Boilers and Hydro Pumping Storage Stations, and this scenario requires minimum usage of gas-fired boilers to cover heat energy load, hence minimum natural gas consumption. At the same time, according to this scenario, the about minimum operation of gas-fired CHP is needed causing additional minimization of natural gas consumption. The interesting result is that with the absence of Hydro Pumping

Station operations (the use of Electric Boilers, but not use Hydro Pumping Storages scenario) also possible to guarantee the coverage of electric and heat energy load, but in that case, the usage of coal-fired Thermal Power Plants is maximum causing additional emissions of greenhouse gases and hazardous air pollution. This result is showing that the implementation of electric boilers into the electric grid serves as a source of additional electric grid flexibility, causing its safer operation.

Acknowledgments

I highly appreciate input from research fellows of General Energy Institute NAS of Ukraine, namely Ph.D. Tetiana Nechaeva, Ph.D. Volodymyr Derii, Ph.D. Oleksandr Teslenko for consultancies regarding heat load profile construction, estimation of the adequacy of the heat energy producing technologies modeling, and for discussion of the results obtained.

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МОДЕЛЬ ЦІЛОЧИСЛОВОГО МАТЕМАТИЧНОГО ПРОГРАМУВАННЯ ЗАВАНТАЖЕННЯ ГЕНЕРУВАЛЬНИХ ПОТУЖНОСТЕЙ ДЛЯ ОДНОЧАСНОГО ПОКРИТТЯ ГРАФІКІВ ЕЛЕКТРИЧНОГО ТА ТЕПЛОВОГО НАВАНТАЖЕННЯ

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Анотація. *Моделі оптимального завантаження генерувальних потужностей для покриття графіків навантаження наразі широко використовуються для досліджень електроенергетичних систем. Математичне формулювання моделі вперше було здійснено у 1940-х роках, але і зараз ці математичні моделі розвиваються як методологічно, так і за рахунок включення до їх складу додаткових (нових) технологій, кожна з яких представлена специфічними (унікальними) рівняннями, що відображають особливості функціонування технологій. Загальною характеристикою моделі є те, що вона присвячена лише сектору електроенергетики, тобто розвиток моделі відбувається по мірі розвитку саме технологій електроенергетики, але протягом останніх років до складу електроенергетичних систем впроваджуються технології Power-to-X, і очікується, що цей процес буде продовжений. Це вимагає вдосконалення або принаймні модифікації математичного формулювання оптимізаційної задачі для врахування можливого обміну та використання між різними видами енергії в одній інтегрованій енергосистемі. Метою статті є подальший розвиток існуючих версій моделі оптимального завантаження генерувальних потужностей для покриття графіків навантаження електроенергетичної системи, за рахунок використання додаткових рівнянь, які дозволяють розглядати технології виробництва теплової енергії, призначені для покриття графіків теплового навантаження систем централізованого теплопостачання. Це повинно дозволити проводити комплексні дослідження одночасної роботи технологій виробництва електро- та теплової енергії для покриття попиту місцевих енергетичних систем, що є актуальним для формування структури розподіленої генерації, наприклад, на рівні окремого населеного пункту або громади. У статті запропонована модель цілочислового математичного програмування завантаження генерувальних потужностей для одночасного покриття графіків електричного та теплового навантаження. Запропонована модель крім суто електричного балансу забезпечує покриття теплового навантаження із використанням технологій генерування теплової енергії (паливні котли), теплових електроцентралей, а також промислових електричних котлів, які перетворюють електроенергію в тепло.*

Ключові слова: модель цілочислового математичного програмування, задача оптимального завантаження генеруючих потужностей, об’єднана енергосистема, електрокотли, технології power-to-X, традиційні технології генерації електроенергії.

Надійшла до редколегії: 27.01.2023