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PROPERTIES OF DISCRETE-TIME CONDITIONAL LINEAR CYCLOSTATIONARY RANDOM PROCESSES IN THE PROBLEMS OF ENERGY INFORMATICS

Abstract. *Modern challenges in the energy industry require comprehensive research in the field of energy informatics, which combines computer science, control systems, and energy management systems within a single methodology. An important area of energy informatics is the study of problems of systems and processes modeling in energy, including energy loads and consumption. Linear and conditional linear random processes (CLRP) are mathematical models of signals represented as the sum of a large number of random impulses occurring at random times. The energy consumption, vibration signals of energy objects, etc. can be modeled using this approach. A variant of the CLRP model with discrete time, taking into account the cyclic properties of energy consumption, has been investigated in the paper. The goal is to justify the conditions for the discrete-time CLRP to be a periodically correlated random process, as well as a cyclostationary process. It has been shown that the corresponding conditions depend on the periodicity of the probability distributions of the kernel and the generating white noise of the CLRP representation. To achieve the goal, the properties of mathematical expectation and covariance function of CLRP, as well as the method of characteristic functions, have been used. The paper proves that the discrete-time CLRP is a periodically correlated random sequence if the generating white noise has periodic mathematical expectation and variance, and the kernel is a periodically correlated random field. Based on the analysis of the multivariate characteristic function, it has been proven that the discrete-time CLRP is cyclostationary if the generating white noise is a cyclostationary process and the kernel is a cyclostationary random field. The properties of discrete-time conditional linear cyclostationary random processes are important for mathematical modeling, simulation, statistical analysis, and forecasting of energy consumption.*

Keywords: mathematical model, energy informatics, conditional linear random process, cyclostationary process, white noise, characteristic function.

1. Introduction

Problems and challenges in the modern energy industry are related to the volatility and partial controllability of renewable energy sources, the uncertainty of the energy consumers' behavior (which no longer necessarily corresponds to standard load profiles), decentralization, modification of the dynamic characteristics of sustainable energy systems, etc. [1]. There is a strong need for essential contributions from =++=the computer science community to overcome the above challenges. This can be done within the energy informatics framework, a highly interdisciplinary and dynamic field of research and development. Energy informatics combines computer science, control systems, and energy management systems in a single methodology [1–3]. Important directions of energy informatics are associated with the collection, analysis, deployment, and exploitation of energy status data, modeling, simulation, and prediction of the behavior of energy systems and processes [1], including energy loads and consumption mathematical modeling and computer simulation. Energy resources consumption (e.g., electric, gas, water consumption) analysis and simulation are also important tools for the problems of energy consumption behavior-based user segmentation, electricity consumption pattern (load profiles) analysis, energy consumption forecasting,

development of information measurement and information control systems in the electric power industry [4, 5].

The mathematical model, being the theoretical foundation of the structural, algorithmic, and technical implementation of the developed systems and technologies, should be adequate to the measuring process, represent the physical mechanism of its generation, and also be suitable for performing its theoretical analysis, solving problems of monitoring and diagnostics of energy facilities based on the results of the experiments. Linear and conditional linear random processes (CLRP) are the mathematical models satisfying the above demands, representing the investigated signals and processes as the sum of many random stochastically dependent impulses occurring at random Poisson times. The energy consumption, vibration signals of energy objects, etc. can be modeled using this approach.

The concept of a "conditionally linear random process" has been developed by Percy A. Pierre [6] in the context of his research on the problem of mathematical modeling of radar clutter. Continuous-time CLRP [6–8] is defined in the form of a stochastic integral of a random kernel driven by the process with independent increments. Different cases of the general approach have been investigated in [9, 10]. In the papers [7, 8] and others, the characteristic function method has been applied to continuous-time CLRP to study its probability distribution properties, including the conditions of its cyclostationarity (that is, the periodicity of finite-dimensional distribution functions, characteristic functions, or moment functions with respect to their time arguments), which is obviously important properties in the context of energy informatics because of cyclic nature of energy loads and consumption processes. This paper deals with the same properties but for the discrete-time variant of the model.

The discrete-time CLRP, represented as a stochastic sum with a stationary generative white noise has been introduced and analyzed in [6]. The general case of discrete-time CLRP has been defined in [11], and the properties of its mathematical expectation and covariance function have been analyzed. There are theoretical papers where the central limit problem in relation to randomly weighted sums of random variables [12], linear processes with random coefficients [13], etc. have been investigated. In the literature, however, there is no analysis of the properties of multidimensional probability distributions or moment functions for the general case of a discrete-time conditional linear random process, which can be used in the applied problems of mathematical modeling of cyclostationary (or periodically correlated) [14] signals and processes.

The concept of periodically correlated random sequences has been introduced in [15] and their spectral properties have been investigated. The statistical analysis methods of such processes have been considered in [14, 16–18]. The relationships between periodically correlated, cyclostationary, and linear random sequences (in the form of autoregressive moving average models with periodic coefficients) have been studied by many authors, including [14, 16, 19]. The corresponding applications for modeling the signals in energy and diagnostics of energy equipment have been represented in [14, 16, 18].

The main goal of the paper is to obtain the conditions for discrete-time CLRP to be periodically correlated using the properties of its moment functions, and to characterize the conditions for discrete-time CLRP to be cyclostationary using the characteristic function method.

2. Discrete-time conditional linear periodically correlated random processes

We start our analysis with the definition of continuous-time CLRP [7, 8]. A real-valued continuous-time conditional linear random process $\xi(\omega, t), \omega \in \Omega, t \in (-\infty, \infty)$ (where $\{\Omega, \mathsf{F}, \mathbf{P}\}$ is some probability space) is defined in the following form:

$$\xi(\omega, t) = \int_{-\infty}^{\infty} \varphi(\omega, \tau, t) d\eta(\omega, \tau), \tag{1}$$

where $\varphi(\omega, \tau, t), \tau, t \in (-\infty, \infty)$ is a real-valued *stochastic* kernel of CLRP; $\eta(\omega, \tau), \tau \in (-\infty, \infty)$ is a stochastically continuous Hilbert process with independent increments, satisfying the following conditions: $\mathbf{E}\eta(\omega, \tau) = a(\tau) < \infty$ and $\mathrm{Var}\big[\eta(\omega, \tau)\big] = b(\tau) < \infty$, $\forall \tau$; random functions $\varphi(\omega, \tau, t)$ and $\eta(\omega, \tau)$ are stochastically independent.

A real-valued discrete-time conditional linear random process $\xi_t(\omega)$, $t \in \mathbf{Z}$, $\omega \in \Omega$ is defined as a random sequence in the following form [11]:

$$\xi_t(\omega) = \sum_{\tau=-\infty}^{\infty} \varphi_{\tau,t}(\omega)\zeta_\tau(\omega),\tag{2}$$

where $\varphi_{\tau,t}(\omega)$, $\tau, t \in \mathbf{Z}$ is a real-valued random function (kernel), which can be considered a random matrix as well as a two-dimensional random field on $\mathbf{Z}^2$;

$\zeta_\tau(\omega)$, $\tau \in \mathbf{Z}$ is a sequence of infinitely divisible independent random variables (infinitely divisible discrete-time white noise);

random functions $\varphi_{\tau,t}(\omega)$ and $\zeta_\tau(\omega)$ are stochastically independent.

Let us denote the mathematical expectation and variance of the above white noise $\zeta_\tau(\omega)$ as follows:

$$\mathbf{E}\zeta_\tau(\omega) = a_\tau < \infty, \ \mathrm{Var}\left[\zeta_\tau(\omega)\right] = \sigma_\tau^2 < \infty, \ \forall \tau.$$

The sum (2) is assumed to be exist in the mean-square convergence sense [11]. The relationship between models (1) and (2) has been also analyzed in [11].

The mathematical expectation $\mathbf{E}\xi_t(\omega)$ and covariance function R_{t_1,t_2}, $t_1, t_2 \in \mathbf{Z}$ of a discrete-time conditional linear random process (2) is represented as:

$$\mathbf{E}\xi_t(\omega) = \sum_{\tau=-\infty}^{\infty} \phi_{\tau,t} a_\tau,\tag{3}$$

$$R_{t_1,t_2} = \sum_{\tau=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} R_{\tau,s;t_1,t_2}^\varphi a_\tau a_s + \sum_{\tau=-\infty}^{\infty} \mathbf{E}\left(\varphi_{\tau,t_1}(\omega)\varphi_{\tau,t_2}(\omega)\right)\sigma_\tau^2,\tag{4}$$

where $\phi_{\tau,t} = \mathbf{E}\varphi_{\tau,t}(\omega)$ is the mathematical expectation of the kernel of discrete-time CLRP;

$R_{\tau,s;t_1,t_2}^\varphi = \mathbf{E}\left(\overset{\circ}{\varphi}_{\tau,t_1}(\omega)\overset{\circ}{\varphi}_{s,t_2}(\omega)\right)$ is the covariance function of the kernel of a discrete-time conditional linear

random process ($\overset{\circ}{\varphi}_{\tau,t}(\omega) = \varphi_{\tau,t}(\omega) - \mathbf{E}\varphi_{\tau,t}(\omega)$ is the centered kernel).

Let there exist the least integer number (period) $T > 1$ such that white noise $\zeta_\tau(\omega)$ has periodic mathematical expectation and variance, that is,

$$\mathbf{E}\zeta_\tau(\omega) = a_\tau = a_{\tau+T} \text{ and } \mathrm{Var}\left[\zeta_\tau(\omega)\right] = \sigma_\tau^2 = \sigma_{\tau+T}^2,\tag{5}$$

and mathematical expectation and covariance function of the kernel has the following properties:

$$\phi_{\tau,t} = \phi_{\tau+T,t+T}, \quad R_{\tau,s;t_1,t_2}^\varphi = R_{\tau+T,s+T;t_1+T,t_2+T}^\varphi,\tag{6}$$

then discrete-time CLRP (2) is *periodically correlated* in a random sequence.

To prove this, we should analyze the properties of mathematical expectation and covariance function of discrete-time conditional linear random process under conditions (5) and (6). Thus, we have the following property of mathematical expectation:

$$\mathbf{E}\xi_t(\omega) = \sum_{\tau=-\infty}^{\infty} \phi_{\tau,t} a_\tau = \sum_{\tau=-\infty}^{\infty} \phi_{\tau+T,t+T} a_{\tau+T} = \sum_{s=-\infty}^{\infty} \phi_{s,t+T} a_s = \mathbf{E}\xi_{t+T}(\omega).$$

Taking into account $\mathbf{E}\left(\varphi_{\tau,t_1}(\omega)\varphi_{\tau,t_2}(\omega)\right) = R_{\tau,\tau;t_1,t_2}^\varphi + \phi_{\tau,t_1}\phi_{\tau,t_2}$, and denoting $\tau+T = \tau_1$, $s+T = s_1$, we obtain the following property of the covariance function:

$$R_{t_1,t_2} = \sum_{\tau=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} R_{\tau+T,s+T;t_1+T,t_2+T}^\varphi a_{\tau+T} a_{s+T} + \sum_{\tau=-\infty}^{\infty}\left(R_{\tau+T,\tau+T;t_1+T,t_2+T}^\varphi + \phi_{\tau+T,t_1+T}\phi_{\tau+T,t_2+T}\right)\sigma_{\tau+T}^2 =$$

$$= \sum_{\tau_1=-\infty}^{\infty}\sum_{s_1=-\infty}^{\infty} R_{\tau_1,s_1;t_1+T,t_2+T}^\varphi a_{\tau_1} a_{s_1} + \sum_{\tau_1=-\infty}^{\infty}\left(R_{\tau_1,\tau_1;t_1+T,t_2+T}^\varphi + \phi_{\tau_1,t_1+T}\phi_{\tau_1,t_2+T}\right)\sigma_{\tau_1}^2 = R_{t_1+T,t_2+T}.$$

The mathematical expectation and covariance function of discrete-time CLRP is periodical functions with respect to their time arguments. Thus, the investigated process is periodically correlated [14, 15] if conditions (5) and (6) hold.

3. Discrete-time conditional linear cyclostationary random processes

Let $\mathsf{F}_\varphi \subset \mathsf{F}$ be σ-subalgebra, generated by a random matrix $\varphi_{\tau,t}(\omega)$, satisfying with probability 1 the following conditions (which are important for the convergence of (2) by distribution):

$$\sum_{\tau=-\infty}^{\infty}\left|\varphi_{\tau,t}(\omega)a_\tau\right| < \infty \ \text{ and } \ \sum_{\tau=-\infty}^{\infty}\left|\varphi_{\tau,t}(\omega)\right|^2\sigma_\tau^2 < \infty, \ \forall t.$$

The m-dimensional characteristic function of discrete-time CLRP (2) is represented as:

$$f_\xi(u_1,u_2,...,u_m;t_1,t_2,...,t_m) = \mathbf{E}\left(\exp\left[i\sum_{k=1}^{m}u_k\xi_{t_k}(\omega)\right]\right) = \mathbf{E}f_\xi^{\mathsf{F}_\varphi}(\omega,u_1,u_2,...,u_m;t_1,t_2,...,t_m),$$

where $f_\xi^{\mathsf{F}_\varphi}(\omega,u_1,u_2,...,u_m;t_1,t_2,...,t_m) = \mathbf{E}\left(\exp\left[i\sum_{k=1}^{m}u_k\xi_{t_k}(\omega)\right]\Big|\mathsf{F}_\varphi\right)$ is conditional with respect to σ-subalgebra F_φ characteristic function of discrete-time CLRP (2).

Taking into account that elements $\varphi_{\tau,t}(\omega)\zeta_\tau(\omega)$ in the sum (2) are conditionally F_φ-independent infinitely divisible random variables (with respect to σ-subalgebra F_φ), the random function $f_\xi^{\mathsf{F}_\varphi}(\omega,u_1,u_2,...,u_m;t_1,t_2,...,t_m)$ is represented as:

$$
\begin{aligned}
f_\xi^{\mathsf{F}_\varphi}&(\omega,u_1,u_2,...,u_m;t_1,t_2,...,t_m) = \\
&= \exp\Bigg[i\sum_{k=1}^{m}u_k\sum_{\tau=-\infty}^{\infty}\varphi_{\tau,t_k}(\omega)a_\tau + \\
&+ \sum_{\tau=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{m}u_k\varphi_{\tau,t_k}(\omega)} - 1 - ix\sum_{k=1}^{m}u_k\varphi_{\tau,t_k}(\omega)\right)\frac{d_xK(x;\tau)}{x^2}\Bigg],
\end{aligned}
\tag{7}
$$

$$t_k \in \mathbf{Z}, u_k \in \mathbf{R}, k = \overline{1,m},$$

where $K(x;\tau), x \in (-\infty,\infty), \tau \in \mathbf{Z}$ is a Poisson jump spectrum in Kolmogorov's form of the infinitely divisible white noise $\zeta_\tau(\omega)$.

Probability distribution of discrete-time conditional linear random process belongs to the class of mixtures of infinitely divisible distributions.

Let there exist the least integer number (period) $T > 1$ such that the following holds:
- random matrices $\varphi_{\tau,t}(\omega)$, $\tau,t \in \mathbf{Z}$ and $\varphi_{\tau+T,t+T}(\omega)$ are stochastically equivalent in the wide sense, that is, their finite-dimensional cumulative distribution functions are equal:

$$\mathbf{P}\left(\bigcap_{i=1}^{n}\bigcap_{j=1}^{m}\left\{\omega:\varphi_{\tau_j,t_j}(\omega) < x_{ij}\right\}\right) = \mathbf{P}\left(\bigcap_{i=1}^{n}\bigcap_{j=1}^{m}\left\{\omega:\varphi_{\tau_j+T,t_j+T}(\omega) < x_{ij}\right\}\right), x_{ij} \in \mathbf{R},\tag{8}$$

- infinitely divisible white noise $\zeta_\tau(\omega), \tau \in \mathbf{Z}$ is stochastically periodic in the sense of

$$a_\tau = a_{\tau+T}, \quad d_xK(x;\tau) = d_xK(x;\tau+T).\tag{9}$$

Then the characteristic function $f_\xi(u_1,u_2,...,u_m;t_1,t_2,...,t_m)$ is periodic by its time arguments, that is,

$$f_\xi(u_1,u_2,...,u_m;t_1,t_2,...,t_m) = f_\xi(u_1,u_2,...,u_m;t_1+T,t_2+T,...,t_m+T).\tag{10}$$

The random process satisfying the property (10) is called cyclostationary of the order m [14]. Thus, under the conditions (8) and (9) the sequence (2) is discrete-time conditional linear cyclostationary random processes. Obviously, the white noise $\zeta_\tau(\omega)$, satisfying (9) is also cyclostationary, that is, its one-dimensional characteristic function is T-periodic by time argument. It should be also noted that if $T = 1$ then (2) is strict-sense stationary conditional linear random process.

The above statement can be proven analyzing the properties of conditional characteristic function (7) of discrete-time conditional linear random process under the conditions (8) and (9).

First of all, it can be noted that

$$\text{Law}\left(\sum_{k=1}^{m}u_k\sum_{\tau=-\infty}^{\infty}\varphi_{\tau,t_k}(\omega)a_\tau\right)=\text{Law}\left(\sum_{k=1}^{m}u_k\sum_{\tau=-\infty}^{\infty}\varphi_{\tau+T,t_k+T}(\omega)a_{\tau+T}\right)=$$

$$=\text{Law}\left(\sum_{k=1}^{m}u_k\sum_{\tau=-\infty}^{\infty}\varphi_{s,t_k+T}(\omega)a_s\right),$$

where $s=\tau+T$, and also we use the notation $\text{Law}(\xi(\omega))=\text{Law}(\eta(\omega))$ if given random variables $\xi(\omega)$ and $\eta(\omega)$ have the same probability distributions.

Further, the second part of the expression (7) has the same property, that is,

$$\text{Law}\left(\sum_{\tau=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{m}u_k\varphi_{\tau,t_k}(\omega)}-1-ix\sum_{k=1}^{m}u_k\varphi_{\tau,t_k}(\omega)\right)\frac{d_xK(x;\tau)}{x^2}\right)=$$

$$=\text{Law}\left(\sum_{\tau=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{m}u_k\varphi_{\tau+T,t_k+T}(\omega)}-1-ix\sum_{k=1}^{m}u_k\varphi_{\tau+T,t_k+T}(\omega)\right)\frac{d_xK(x;\tau+T)}{x^2}\right)=$$

$$=\text{Law}\left(\sum_{\tau=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{m}u_k\varphi_{s,t_k+T}(\omega)}-1-ix\sum_{k=1}^{m}u_k\varphi_{s,t_k+T}(\omega)\right)\frac{d_xK(x;s)}{x^2}\right).$$

From the above analysis it follows that probability distribution of the random F_φ-conditional characteristic function $f_\xi^{\mathsf{F}_\varphi}(\omega,u_1,u_2,...,u_m;t_1,t_2,...,t_m)$ is periodic by its time arguments, that is,

$$\text{Law}(f_\xi^{\mathsf{F}_\varphi}(\omega,u_1,u_2,...,u_m;t_1,t_2,...,t_m))=\text{Law}(f_\xi^{\mathsf{F}_\varphi}(\omega,u_1,u_2,...,u_m;t_1+T,t_2+T,...,t_m+T))$$

Taking into account that unconditional characteristic function of discrete-time CLRP is equal to $f_\xi(u_1,u_2,...,u_m;t_1,t_2,...,t_m)=\mathbf{E}f_\xi^{\mathsf{F}_\varphi}(\omega,u_1,u_2,...,u_m;t_1,t_2,...,t_m)$, we can state that (10) holds, and investigated process is cyclostationary of the order m.

4. Discussion

As it was already mentioned above the key element of the methodology of application of continuous-time linear and conditional linear random processes for the mathematical modelling of electricity loads is a representation of investigated process as a sum of a large amount of random impulses (loads related to individual consumers) occurring at random Poisson time moments. In context of linear random processes, the impulses (with random duration and amplitude) are independent. The authors of the paper [20] investigated electricity consumption data at the level of individual households, measured by modern information systems based on smart meters. According to the results of experiments it has been shown [20] that the processes of electricity consumption of individual households are stochastically dependent. That is why, we state that mathematical modelling of electricity loads process $\xi(\omega,t)$ in the form of conditional linear random process (considering the stochastic dependency between the individual impulses-loads) is more adequate to investigated object.

The resulting representation follows from (1) (where process with independent increments is nonhomogeneous Poisson counting process) and has the following form:

$$\xi(\omega,t)=\sum_{k=-\infty}^{\infty}\varphi(\omega,\tau_k(\omega),t),$$

where $...<\tau_{k-1}(\omega)<\tau_k(\omega)<\tau_{k+1}(\omega)<...$ are the times of a Poisson process, which are equal to the times of random impulses $\varphi(\omega,\tau_k(\omega),t)$ occurrence ($\varphi(\omega,\tau_k(\omega),t)=0$ if $t<\tau_k(\omega)$), and random functions $...,\varphi(\omega,\tau_{k-1},t)$, $\varphi(\omega,\tau_k,t)$, $\varphi(\omega,\tau_{k+1},t),...$ are stochastically dependent (at fixed nonrandom times $...<\tau_{k-1}<\tau_k<\tau_{k+1}<...$).

Analyzing the cyclic behaviour of energy consumers of residential areas or enterprises, it can be shown that the investigated electricity loads process $\xi(\omega, t)$ will be cyclostationary [7, 8, 14] with a period of $T_0 = 24$ hours, that is, its m-dimensional ($m \geq 1$) characteristic function is periodic by its time arguments.

Now, let us consider the discrete-time random process

$$\xi_t(\omega) = \int_{(t-1)h}^{th} \xi(\omega, s)ds, t \in \mathbf{Z}, h = \frac{T_0}{T}, T \in \mathbf{N},$$

which for each $t \in \mathbf{Z}$ equals to *electricity consumption* during time interval $\big[(t-1)h, th\big]$. If $h = 1$ hour, then $\xi_t(\omega), t \in \mathbf{Z}$ is *hourly electricity consumption*, which can be modelled as discrete-time conditional linear cyclostationary random processes (or discrete-time conditional linear periodically correlated random processes when we need to consider only its moment functions up to second order) with the period T.

The characteristics of representation (2), that is probability properties of the kernel and generating white noise, as well as moment (3), (4) and characteristic (7) functions can be used in the different areas in energy informatics, such as electricity consumption monitoring and forecasting, computer simulation, identification of the electricity consumption profile features etc.

The perspective theoretical research should be related to the analysis of random matrix $\varphi_{\tau,t}(\omega)$ from the point of view of periodically correlated or cyclostationary random field on $\mathbf{Z}^2$ [14]. In this context, the theoretical results of the current paper can be extended analyzing the different combinations of components $\varphi_{\tau,t}(\omega)$ and $\zeta_\tau(\omega)$ of discrete-time conditional linear cyclostationary random process. That is, the following cases can be considered:

- $\varphi_{\tau,t}(\omega)$ is cyclostationary random field (in general, with different periods by τ and t) and $\zeta_\tau(\omega)$ is cyclostationary white noise;
- $\varphi_{\tau,t}(\omega)$ is cyclostationary random field and $\zeta_\tau(\omega)$ is stationary white noise;
- $\varphi_{\tau,t}(\omega)$ is cyclostationary random field with the same period $T = 1$ by both time arguments and $\zeta_\tau(\omega)$ is cyclostationary white noise.

The perspective applied research is based on using the random coefficient periodic autoregressive model (as a particular case of discrete-time conditional linear cyclostationary random process) for the problems of estimation, identification, computer simulation, and forecasting in energy.

5. Conclusions

The properties of discrete-time conditional linear random processes have been analyzed in the context of applications for mathematical modelling of electricity consumption and other important problems of energy informatics.

Using the analysis of the expressions of mathematical expectation and covariance function of discrete-time CLRP it has been shown that process is periodically correlated if the kernel of its representation is periodically correlated random field and generating white noise is periodically correlated random process.

Using the method of characteristic functions, it has been proven that discrete-time CLRP is cyclostationary if the kernel of its representation is cyclostationary random field and generating white noise is cyclostationary random process.

The results can be used for the mathematical modelling, theoretical analysis of probability characteristics of electricity, gas, water, and other energy resources consumptions, vibration signals of energy facilities, etc.

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ВЛАСТИВОСТІ УМОВНИХ ЛІНІЙНИХ ЦИКЛОСТАЦІОНАРНИХ ВИПАДКОВИХ ПРОЦЕСІВ З ДИСКРЕТНИМ ЧАСОМ У ЗАДАЧАХ ЕНЕРГЕТИЧНОЇ ІНФОРМАТИКИ

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Анотація. *Сучасні проблеми та виклики в енергетиці вимагають проведення комплексних досліджень у галузі енергетичної інформатики, що об'єднує в рамках єдиної методології комп'ютерні науки, системи керування та системи енергоменеджменту. Важливим напрямом енергетичної інформатики є вивчення проблем моделювання систем та процесів в енергетиці, включно з процесами енергонавантаження та енергоспоживання. Лінійні та умовні лінійні випадкові процеси (УЛВП) є математичними моделями сигналів, представлених у вигляді суми великого числа випадкових імпульсів, що виникають у випадкові моменти часу. Саме таким чином можна моделювати процеси споживання енергоресурсів (електро-, газо-, водоспоживання), вібраційні сигнали енергооб'єктів та ін. У роботі досліджена модель УЛВП з дискретним часом, що дозволяє враховувати циклічні властивості енергоспоживання. Метою роботи є обґрунтування умов, за яких УЛВП з дискретним часом буде періодично корельованим процесом, а також циклостаціонарним випадковим процесом. Показано, що відповідні умови залежать від властивостей періодичності ймовірнісних розподілів ядра та породжуючого білого шуму в зображенні УЛВП. Для досягнення мети використано властивості математичного сподівання та кореляційної функції УЛВП, а також метод характеристичних функцій. У роботі доведено, що УЛВП з дискретним часом є періодично корельованою випадковою послідовністю, якщо породжуючий білий шум має періодичні математичні сподівання та дисперсію, а ядро є періодично корельованим випадковим полем. На основі аналізу багатовимірної характеристичної функції доведено, що УЛВП з дискретним часом є циклостаціонарним, якщо породжуючий білий шум є циклостаціонарним процесом, а ядро є циклостаціонарним випадковим полем. Охарактеризовані властивості умовних лінійних циклостаціонарних випадкових процесів з дискретним часом є важливими для вирішення задач математичного, комп'ютерного моделювання, статистичного аналізу та прогнозування споживання енергоресурсів.*
Ключові слова: математична модель, енергетична інформатика, умовний лінійний випадковий процес, циклостаціонарний процес, білий шум, характеристична функція.