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EVALUATION OF STRESS FIELD RECONSTRUCTION ERRORS NEAR THE CRACK TIP OF BODY UNDER PLANE STRAIN CONDITIONS

The purpose of this research has been to analyze the influence of the errors contained in the measured stress field components on the accuracy of Williams' series parameters. The relationships between relative errors of Williams' series parameters and input noise characteristics have been obtained. It has been shown that the constant shift in the input noise has much more profound impact on the accuracy of the first-order and higher order terms determination than the random noise variation.

Keywords: *fracture mechanics; Williams' coefficients accuracy; simulation; mixed-mode.*

Метою цього дослідження був аналіз впливу похибок, що містяться у виміряних компонентах поля напружень на точність визначення параметрів рядів Вільямса. Отримано залежності між відносними похибками параметрів рядів Вільямса і характеристиками вхідних шумів. Було показано, що постійне зміщення у вхідному шумі погіршує точність визначення перших членів і членів вищих порядків, значно більше, ніж випадковий шум.

Ключові слова: *механіка руйнування, похибки коефіцієнтів Вільямса, мішане навантаження.*

Introduction. Using higher order terms of the Williams' series [16] to characterize stress field around crack tip enables better understanding of fracture processes [5, 6, 15]. As it has been shown in the previous papers [2, 3, 8, 18], higher order coefficients can be used to characterize some essential features of the fracture process. On the other hand, neglecting the contribution of higher order terms deteriorates the accuracy of determination of the first-order terms, which defines stress intensity factors (SIFs) [11, 19]. Higher order terms in the Williams' series explain important features of stress field around crack tip. Hence, availability of data about determination accuracy of the corresponding coefficients is a key factor for their use in non–destructive testing technologies (NDT).

The reference literature [4, 10, 12–14] contains some research results of the input stress field noises influence on determination of the Williams' coefficients for loading modes I and II separately, which explained some essential features of their noise sensitivity. However, the real constructions are quite rarely subjected to pure I or II modes of loading. Actually, in practice there exists a mixed mode of loading, in particular mode I in combination with mode II or mode III. Some other researchers showed that the accuracy of SIFs determination essentially decreases when the other mode of loading additionally appears [7]. That is why, evaluation of noise sensitivity tendencies of the Williams' series parameters for the mixed mode of loading is important from a practical standpoint.

In general, any input data of the measured stress field used for derivation of the Williams' series parameters, should be considered as "noisy data" that contains some portion of errors. Any real measurement system has some inaccuracy, which can be presented as a composition of some random (pure noise) and constant (zero–level shift) components.

General investigation of noise sensitivity of the Williams' series parameters would be too cumbersome because of the presence of many independent coefficients and different determination techniques. We have selected for the purpose of the research only one practical case considered in [1], which is quite representative for the mixed mode (I+II) of loading, since the intensities of stress fields due to mode I and mode II are comparable. The Williams' series coefficients will be obtained from the full-field measurement data by means of least square fitting procedure, as it is widely used in NDT technique. This paper is a part of the project that deals with the development of a measuring part of the NDT system that is based on the use of the Williams' series parameters for the analysis of initiation and development of cracks in metal structures.

Brief explanation of simulation methodology. Evaluation of noise sensitivity of the Williams' series coefficients has been performed in the following way. First, pure stress field components σ_{xx}, σ_{yy}, σ_{xy} have been generated using the Williams' series formulae [14], truncated by N terms on the basis of the known coefficients values.

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix} = \sum_{n=1}^{N}\frac{n}{2}A_{I_n}\,r^{(n-2)/2}\begin{Bmatrix}\left\{2+(-1)^n+\frac{n}{2}\right\}\cos\left(\frac{n}{2}-1\right)\theta-\left(\frac{n}{2}-1\right)\cos\left(\frac{n}{2}-3\right)\theta\\ \left\{2-(-1)^n-\frac{n}{2}\right\}\cos\left(\frac{n}{2}-1\right)\theta+\left(\frac{n}{2}-1\right)\cos\left(\frac{n}{2}-3\right)\theta\\ -\left\{(-1)^n+\frac{n}{2}\right\}\sin\left(\frac{n}{2}-1\right)\theta+\left(\frac{n}{2}-1\right)\sin\left(\frac{n}{2}-3\right)\theta\end{Bmatrix}-$$

$$-\sum_{n=1}^{N}\frac{n}{2}A_{II_n}\,r^{(n-2)/2}\begin{Bmatrix}\left\{2-(-1)^n+\frac{n}{2}\right\}\sin\left(\frac{n}{2}-1\right)\theta-\left(\frac{n}{2}-1\right)sin\left(\frac{n}{2}-3\right)\theta\\ \left\{2+(-1)^n-\frac{n}{2}\right\}\sin\left(\frac{n}{2}-1\right)\theta+\left(\frac{n}{2}-1\right)\sin\left(\frac{n}{2}-3\right)\theta\\ -\left\{(-1)^n-\frac{n}{2}\right\}\cos\left(\frac{n}{2}-1\right)\theta-\left(\frac{n}{2}-1\right)\cos\left(\frac{n}{2}-3\right)\theta\end{Bmatrix},$$

where $A_{\mathrm{I}n}$, $A_{\mathrm{II}n}$ – Williams coefficients for I and II loading modes; r, θ – polar coordinates parameters centered at the crack tip.

Coefficients value applied have been obtained by Berto and Lazzarin [1] for thin welded lap joint. Tables 1 and 2 demonstrate the Williams' coefficients (for each mode separately) for the considered sample and the applied loading.

Table 1. AIn Williams' series coefficients used for stress field simulation according to [1]

$A_{\mathrm{I}1}$, MPa·mm$^{1/2}$	$A_{\mathrm{I}2}$, MPa	$A_{\mathrm{I}3}$, MPa·mm$^{-1/2}$	$A_{\mathrm{I}4}$, MPa·mm^{-1}
–1.73	4.87	–4.02	3.50

Table 2. AIIn Williams' series coefficients used for stress field simulation according to [1]

$A_{\mathrm{II}1}$, MPa·mm$^{1/2}$	$A_{\mathrm{II}2}$, MPa	$A_{\mathrm{II}3}$, MPa·mm$^{-1/2}$	$A_{\mathrm{II}4}$, MPa·mm^{-1}
–4.10	0	–1.00	1.02

In this work, the stress field components have been generated in the nodes of a square grid with the size of 100×100 regularly spaced points with the coordinates ranging from X = 0 mm, Y = 0 mm, overlapping the 2×2 mm area in a real scale. Straight 1 mm long crack (crack tip coordinates X_0 = 1 mm, Y_0 = 1 mm) was in parallel to X axis. The points within a square of 0,2 size of the crack length (10×10 grid steps centered at the crack tip) and the points on the crack face have been discarded from the data (Fig. 1).

This has been made since in real non–destructive measurements, data from that area do not satisfy the conditions of elastic and proportional deformations of the material and thus should be removed [5]. In such a way 9835 points of stress field components data for simulation of non-destructive testing data have been selected. It is known [9] that about 10000 points of measurement data is enough to receive reliable values for the first 10 Williams' coefficients.

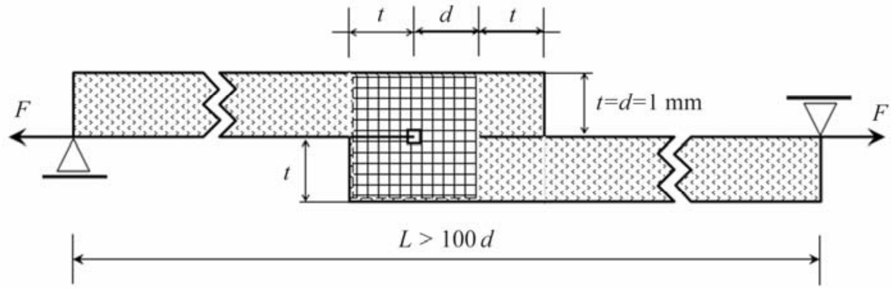

Fig. 1. Specimen geometry and loading scheme according to [1].

Procedure for derivation of the Williams' series coefficients needs the solution of the non-linear equations system (1). This solution has been obtained by using the least square fitting method as it was made in [19]. Stop-rule for software iterative procedure consisted in achieving a deviation of a crack tip position less than 5% of the grid step size during ten successive cycles of calculations (less than 1 micron in real coordinates). Iterative procedure reliability was tested by means of deriving the Williams' coefficients and crack tip position from pure stress field components with starting parameters values, shifted by 5% from their exact values. This test results have shown satisfactory reliability of the applied software. Convergent solution has been obtained less than in 20 iterations and the Williams' series parameters were derived with the errors below 10…5%.

Normalization of the input noise for simulation of measurements has been performed in the following way. The ranges of stress values for each generated component have been analyzed, and the maximum range value (64.07 MPa for σ_{xx}) has been selected for determination of the normalization coefficient. It is known that for reliable measurements the range of the measured value should overlap 90% of the system measuring range. 5% of the measuring system range should be left on both - the lower and the upper limits of the measured value to guarantee reliability of measurements. In such circumstances, normalization coefficient for standard deviation for input stress noises was set equal to one per cent $\sigma_{1\%}$ = 64.07/90 = 0.7119 MPa from the measurement system range. Noises for the stress field components have been generated as normally distributed independent random sets of 9835 double precision numbers with standard deviations equal to 1; 2; 3; 4; 5% of the system range, with zero average and 1; 2; 3; 4; 5% of the measurement system range with +1% average value of the shift separately for each stress field component. The 1% value of the average shift has been selected in view of the fact that elastic properties of the majority of the used structural materials are guaranteed with close level of precision. In such a way we have received 10 sets of field noises for input stress field components.

Then, noise fields have been added to "pure" stress field components and "noisy" stress field components have been obtained. These field components have been used for simultaneous derivation of the Williams' coefficients and the crack tip position in one procedure. In such a way we have received 10 sets of the Williams' series coefficients and crack tip positions for each specified level of noise in the input stress field components. After this, we have reconstructed stress field components based on the Williams' series formulae (1) using the values of coefficients and crack tip position coordinates, obtained from the "noisy" stress field components. Comparison of the stress field components, reconstructed with the Williams' series based on "noisy" parameters with the original stress field components is important for the following reasons:

• to make sure that the stress field components, generated by the Williams' series based on the "noisy" parameters, restore pure stress field components accurately enough.

• to assess the optimal level of input stress measurements accuracy;

• to assess the optimal number of the Williams' series terms;

• to assess the optimal number of the stress measurement points used for derivation of the Williams' series parameters.

Deviations of the Williams' series parameters and the reconstructed stress field components have been analyzed to show their noise sensitivity.

Numerical results and their interpretation. Deviations of the crack tip coordinates determined as the Williams' series parameters did not exceed 0.8% (8 micron in real scale) for all the used data sets. Such a low sensitivity to input noise allows using crack tip coordinates derived as the Williams' series parameters as an effective indicator for monitoring of crack growth.

Deviations of the Williams' series coefficients, derived from artificially "noisy" stress field components are presented in Figs. 2–5.

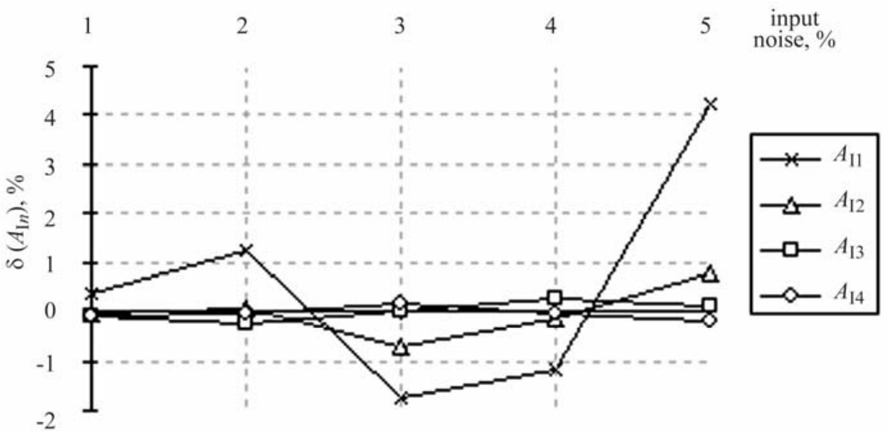

Fig. 2. Relations between deviations of the Williams' coefficients related to loading mode I and the noise level in the input stress field components for pure random noises.

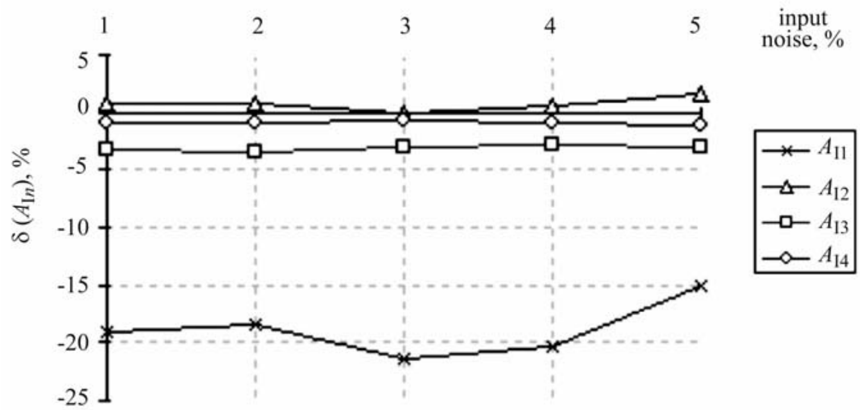

Fig. 3. Relations between deviations of the Williams' coefficient, related to loading mode I and the noise level in the input stress field components with noises, shifted by 1%.

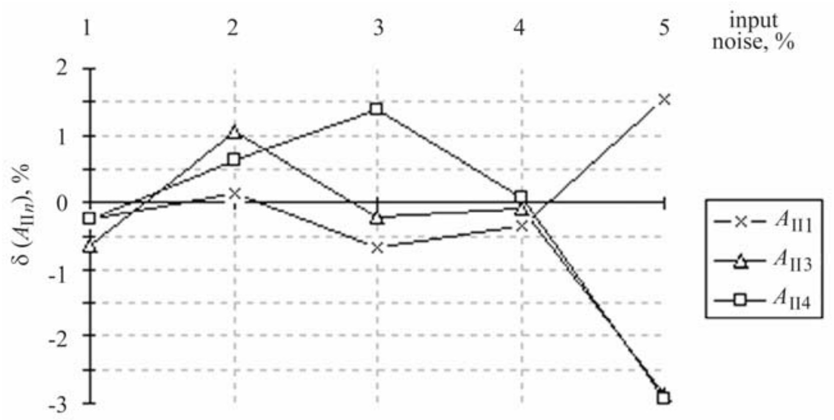

Fig. 4. Relations between deviations of the Williams' coefficients related to loading mode II and the noise level in the input stress field components for pure random noises.

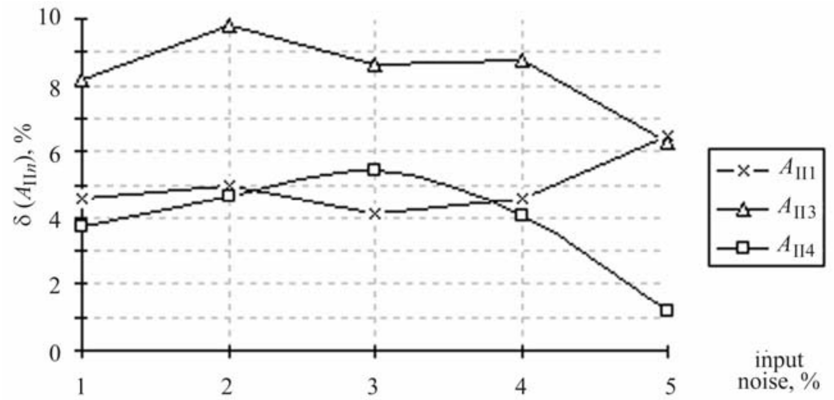

Fig. 5. Relations between deviations of the Williams' coefficients related to loading mode II and the noise level in the input stress field components with noises, shifted by 1%.

As shown in Figs. 2–5, the existence of noises with shifted mean value in the input data essentially deteriorates the accuracy of the Williams' coefficients determination. Obtained error values of all the coefficients for shifted input noises demonstrate dominant influence of this shift on the error value. The error values of the Williams' coefficients have not revealed strong dependence from the standard deviation of random noise in input stress field components. This can be explained by the indirect noises averaging as a result of application of the least square fitting procedure of about 10 000 data points. It should be also pointed out that the use of the least square fitting method for solving the nonlinear equation system can result in constant shifts in output data values [17]. Such influence can explain irregularities in the obtained results, especially when the preset standard deviations of the input noises were strong. We have not noticed any essential difference between the dependencies for noise sensitivity coefficients related to I and II modes of loading.

Higher error values for A_{I1} coefficient (Figs. 2, 3) are explained by its lower (if compared with the other coefficients of type I) absolute value. A_{I2} (T-stress related) coefficient, which defines a constant value existing in σ_{xx} stress field component, does not compensate a constant shift in the input noise. Its low error values can be explained by the fact that it contributes only to σ_{xx} component, so it is not affected by the noises added to the other stress field components.

Deviations of the stress field components, reconstructed from the "noisy" Williams' series parameters from the original stress field components are presented in Figs. 6 and 7.

As shown in Fig. 6, the existence of pure random noises in the input data generates some deviations in stress field components reconstructed by the Williams' series. Such deviations are much lower than the set input noise levels and start increasing when the input noise level exceeds 3%. As shown in Fig. 7, the existence of noises with constant 1% shift in the input data generates deviations of about 5…6% in the stress field components, reconstructed by the Williams' series. Moreover, a constant component in the input data has a dominating influence on the accuracy of reconstruction of the stress field components by the Williams' series similarly as in the case of the series coefficients errors.

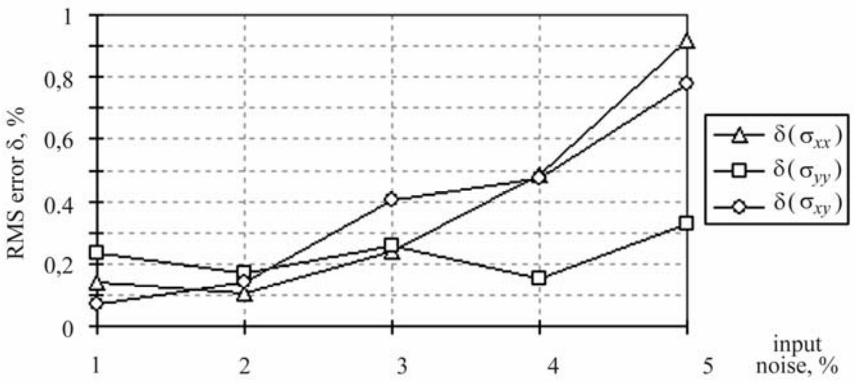

Fig. 6. Relations between deviations of the stress field components reconstructed using the "noisy" Williams' series parameters and the noise level in the input stress field components for pure random noises.

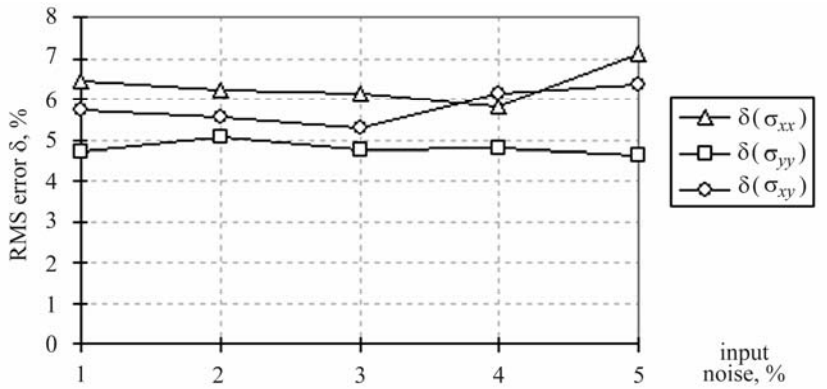

Fig. 7. Relations between deviations of the stress field components reconstructed using the "noisy" Williams' series parameters and the noise level in the input stress field components with noises, shifted by 1%.

CONCLUSION

Examination of the input noise impact in stress field components on the accuracy of the Williams' series parameters derivation has shown that the presence of the constant shift in the input noises can strongly affect the accuracy of the obtained parameters. Introducing a constant shift equal to 1% from measurement system dynamic range into the input noise results in about ten times increase of coefficients determination errors and relative deviations of the reconstructed field components. The crack tip coordinates, obtained as the Williams' series parameters, have the lowest sensitivity to noises.

Input stress field data used for derivation of the Williams' series parameters should be provided with the minimal possible level of the constant error component.

Alternatively, this component should be sufficiently compensated by some additional preprocessing techniques.

The use of the least square method for derivation of the Williams' series parameters reduces the influence of the random input noises in the stress field data by indirect averaging and generates some constant shift in the output data.

The crack tip coordinates obtained as the Williams' series parameters can be used as reliable indicator for monitoring the crack growth in non–destructive testing techniques.

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