UDC 621.391:519.72 https://doi.org/10.15407/vidbir2019.47.026

SYSTEMATIC ERROR OF LSM-ESTIMATION OF COVARIANCE COMPONENTS OF BIPERIODICALLY CORRELATED RANDOM SIGNALS

I. M. Javorskyj [1, 2], O. Y. Dzeryn [1], R. M. Yuzefovych [1, 3]

[1] **Karpenko Physico-Mechanical Institute of the NAS of Ukraine, Lviv;**

[2] **Telecommunication Institute of University of Technology and Life Science, Bydgoszcz, Poland;**

[3] **Lviv Polytechnic National University**

E-mail: roman.yuzefovych@gmail.com

Systematic error of the LSM-estimation for the covariance components of periodically correlated random signals (BPCRS) is analysed. It is show that this error is caused by the preliminary mean function estimation only. The formulae of the dependence of their error on the covariance component is obtained and asymptotic unbiasedness of estimation is proved. The partial case of BPCRS is considered and comparison with component estimators is given.

Keywords: *biperiodically correlated random signal, least squares method, covariance component LSM-estimation, systematic error.*

СИСТЕМАТИЧНА ПОХИБКА МНК-ОЦІНЮВАННЯ КОРЕЛЯЦІЙНИХ КОМПОНЕНТІВ БІПЕРІОДИЧНО КОРЕЛЬОВАНИХ ВИПАДКОВИХ СИГНАЛІВ

І. М. Яворський [1, 2], О. Ю. Дзерин [1], Р. М. Юзефович [1, 3]

[1] **Фізико-механічний інститут ім. Г. В. Карпенка НАН України, Львів;**

[2] **Інститут телекомунікації Технологічно-природничого університету, Бидгощ, Польща;**

[3] **Національний університет “Львівська політехніка”**

Поява дефектів у елементах обертових вузлів механічних систем призводить до стохастичної модуляції гармонічних складових вібраційних сигналів, властивості якої можна описати моделями у вигляді періодично та біперіодично корельованих випадкових процесів (БПКВП). Імовірнісні характеристики другого порядку цих класів нестаціонарних випадкових процесів є досить чутливими до зміни параметрів дефектів, тому використання діагностичних ознак, побудованих на їх основі, дає можливість виявляти дефекти на ранніх стадіях їх розвитку. Тому важливо забезпечити статистичну точність оцінювання імовірнісних характеристик, яка б відповідала вибраним рівням виявлення дефектів. Це означає, що потрібно сформувати належні вимоги до тих параметрів обробки, які визначають точність оцінювання, насамперед, до довжини реалізації і кроку дискретизації. Наведено аналітичні співвідношення, які визначають похибки оцінювання залежно від довжин реалізації, кроку дискретизації та параметрів сигналів БПКВП. Проаналізовано систематичну похибку МНК-оцінювання кореляційних компонентів БПКВП. Показано, що ця похибка зумовлена тільки попереднім оцінюванням математичного сподівання. Отримані формули її залежності від кореляційних компонентів сигналу і показана їх асимптотична незміщеність. На основі отриманих формул обчислено значення зміщень МНК-оцінок кореляційних компонент залежно від довжини реалізації та параметрів, які описують кореляційну структуру БПКВП, їх порівняно зі зміщенням за компонентного оцінювання.

Ключові слова: *біперіодично корельовані випадкові процеси, метод найменших квадратів (МНК), МНК-оцінки кореляційних компонентів, систематична похибка.*

The appearance of the faults in rotating elements of mechanical units induces the stochastic modulation of the harmonic components of vibration signals. The properties of this modulation can be described by the models in the form of periodically and biperiodically correlated random processes [1–3]. The probabilistic characteristics of the second order of these classes of nonstationary random processes are largely sensitive to the fault parameters, therefore the use of the diagnostic indicators formed on their basis allows us to detect the faults at the early initiation stage [1–4]. Proceeding from these facts we must provide such a statistical accuracy of probabilistic characteristics estimation which will necessarily match the chosen levels of the faults detection. It means that the proper requirements to the parameters of processing which determine the estimation error, first of all the length of realization and sampling interval should be formed. The choice of these values can be processed on the basis of the relations, that determine the estimation errors, the realization length, sampling interval and parameters of BPCRS.

The LSM-estimator for the covariance components $\hat{B}_k(u)$ is characterized by the mean square $E\left[\hat{B}_k(u)-B_k(u)\right]^2$, which is determined by the sum of the variance of the estimator $D\left[\hat{B}_k(u)\right]=E\left[\hat{B}_k(u)-E\hat{B}_k(u)\right]$ and it is squared bias $\varepsilon\left[\hat{B}_k(u)\right]=E\hat{B}_k(u)-B_k(u)$

$$E\left[\hat{B}_k(u)-B_k(u)\right]^2=D\left[\hat{B}_k(u)\right]+\varepsilon^2\left[\hat{B}_k(u)\right].$$

The analysis of the variances of estimators $\hat{B}_k(u)$ was carried out in [5, 6]. This paper is dedicated to the analysis of the bias that is often called systematic error of estimation. The value $\sigma\left[\hat{B}_k(u)\right]=\sqrt{D\left[\hat{B}_k(u)\right]}$ is the root mean square deviation of estimator, and $\delta\left[\hat{B}_k(u)\right]=\sqrt{D\left[\hat{B}_k(u)\right]+\varepsilon^2\left[\hat{B}_k(u)\right]}$ is standard error.

Column matrix $\hat{\mathbf{B}}(u)=\left[\hat{B}_0(u),\hat{B}_1^c(u),...,\hat{B}_N^c(u),\hat{B}_1^s(u),...,\hat{B}_N^s(u)\right]^T$ with the elements that are LSM-estimators of covariance components, where N is the number of the highest covariance function, has the following form [5, 6]:

$$\hat{\mathbf{B}}(u)=\frac{\left[A_{rk}\right]^T}{|\mathbf{D}|}\tilde{\mathbf{B}}(u). \qquad (1)$$

Here $|\mathbf{D}|$ is a determinant of the matrix of the equation system. Solutions of system (1) are LSM-estimators of covariance components and A_{rk} are algebraic adjunctions of the matrix. The elements of the column matrix $\tilde{B}(u)$ are equal to

$$\tilde{B}_0(u)=\frac{1}{\theta}\int_0^\theta\zeta(t,u)\,dt,\quad \tilde{B}_l^c(u)=\frac{1}{\theta}\int_0^\theta\zeta(t,u)\cos\omega_l t\,dt,\quad \tilde{B}_l^s(u)=\frac{1}{\theta}\int_0^\theta\zeta(t,u)\sin\omega_l t\,dt,$$

where

$$\zeta(t,u)=\left[\xi(t)-\hat{m}(t)\right]\left[\xi(t+u)-\hat{m}(t+u)\right],$$

and $\hat{m}(t)$ is LSM-estimator of mean function of BPCRP [2].

The biases of LSM-estimators for the covariance components are represented by the formulae:

$$\varepsilon\left[\hat{B}_0(u)\right]=-\frac{1}{|\mathbf{D}|}\sum_{k=0}^{2N}\varepsilon\left[\tilde{B}_k(u)\right]A_{k+1,1}\,,\quad \varepsilon\left[\hat{B}_l^c(u)\right]=-\frac{1}{|\mathbf{D}|}\sum_{k=0}^{2N}\varepsilon\left[\tilde{B}_k(u)\right]A_{k+1,l+1}\,,$$

$$\varepsilon\left[\hat{B}_l^s(u)\right]=-\frac{1}{|\mathbf{D}|}\sum_{k=0}^{2N}\varepsilon\left[\tilde{B}_k(u)\right]A_{k+1,l+N+1}\,,\quad l=\overline{1,N}\,, \tag{2}$$

where

$$\varepsilon\left[\tilde{B}_0(u)\right]=\frac{1}{\theta}\int_0^{\theta}\varepsilon_{\zeta}(t,u)\,dt\,,\quad \varepsilon\left[\tilde{B}_k(u)\right]=\frac{1}{\theta}\int_0^{\theta}\varepsilon_{\zeta}(t,u)\cos\omega_k t\,dt\,,$$

$$\varepsilon\left[\tilde{B}_{k+l}(u)\right]=\frac{1}{\theta}\int_0^{\theta}\varepsilon_{\zeta}(t,u)\sin\omega_k t\,dt\,,$$

and

$$\varepsilon_{\zeta}(t,u)=E\left[\overset{\circ}{\hat{m}}(t)\overset{\circ}{\xi}(t+u)+\overset{\circ}{\hat{m}}(t+u)\overset{\circ}{\xi}(t)-\overset{\circ}{\hat{m}}(t)\overset{\circ}{\hat{m}}(t+u)\right]. \tag{3}$$

Let us concretize formulae (2) for BPCRP

$$\xi(t)=\xi_c(t)\cos\lambda_{11}t+\xi_s(t)\sin\lambda_{11}t\,, \tag{4}$$

where $\xi_c(t)$ and $\xi_s(t)$ are jointly stationary connected random processes, and $\lambda_{11}=\frac{2\pi}{T_1}+\frac{2\pi}{T_2}$, T_1 and T_2 are the periods of nonstationarity.

Covariance function of process (4) is the following

$$b(t,u)=B_{00}(u)+B_{22}^c(u)\cos\lambda_{22}t+B_2^s(u)\sin\lambda_{22}t\,, \tag{5}$$

where

$$B_{00}(u)=\frac{1}{2}\left[R_c(u)+R_s(u)\right]\cos\lambda_{11}u+R_{cs}^-(u)\sin\lambda_{11}u\,,$$

$$B_{22}^c(u)=\frac{1}{2}\left[R_c(u)-R_s(u)\right]\cos\lambda_{11}u+R_{cs}^+(u)\sin\lambda_{11}u\,,$$

$$B_{22}^s(u)=R_{cs}^+(u)\cos\lambda_{11}u+\frac{1}{2}\left[R_s(u)-R_c(u)\right]\sin\lambda_{11}u\,,$$

and $R_c(u)=E\overset{\circ}{\xi}_c(t)\overset{\circ}{\xi}_c(t+u)$, $R_s(u)=E\overset{\circ}{\xi}_s(t)\overset{\circ}{\xi}_s(t+u)$, $\overset{\circ}{\xi}_c(t)=\xi_c(t)-m_c$, $\overset{\circ}{\xi}_s(t)=\xi_s(t)-m_s$, $R_{cs}^+(u)$ even and $R_{cs}^-(u)$ odd parts of cross-covariance function $R_{cs}(u)=E\overset{\circ}{\xi}_c(t)\overset{\circ}{\xi}_s(t+u)$.

The LSM-estimators of the covariance components are determined by the relations:

$$\hat{B}_{00}(u)=\frac{1}{|\mathbf{D}|}\sum_{l=1}^{3}A_{l1}\tilde{B}_l(u)\,,\quad \hat{B}_{22}^c(u)=\frac{1}{|\mathbf{D}|}\sum_{l=1}^{3}A_{l2}\tilde{B}_l(u)\,,\quad \hat{B}_{22}^s(u)=\frac{1}{|\mathbf{D}|}\sum_{l=1}^{3}A_{l3}\tilde{B}_l(u)\,.$$

The determinant $|\mathbf{D}|$ in this case is:

$$|\mathbf{D}| = \begin{vmatrix} 1 & c_{02} & a_{02} \\ c_{20} & c_{22} & a_{22} \\ a_{02} & a_{22} & s_{22} \end{vmatrix},$$

and its elements are equal to

$$c_{rk} = \frac{1}{\theta}\int_0^\theta \cos\lambda_{rr}t\cos\lambda_{kk}tdt\,,\quad s_{rk} = \frac{1}{\theta}\int_0^\theta \sin\lambda_{rr}t\sin\lambda_{kk}tdt\,,\quad a_{rk} = \frac{1}{\theta}\int_0^\theta \cos\lambda_{rr}t\sin\lambda_{kk}tdt.$$

The elements of matrix $\tilde{B}(u) = \left[\tilde{B}_1(u), \tilde{B}_2(u), \tilde{B}_3(u)\right]^T$ are equal to

$$\tilde{B}_0(u) = \frac{1}{\theta}\int_0^\theta \zeta(t,u)\,dt\,,\quad \tilde{B}_2(u) = \frac{1}{\theta}\int_0^\theta \zeta(t,u)\cos\lambda_{22}tdt\,,\quad \tilde{B}_3(u) = \frac{1}{\theta}\int_0^\theta \zeta(t,u)\sin\lambda_{22}tdt\,.$$

The LSM-estimator of mean function of the random process (4) can be rewritten as:

$$\hat{m}(t) = \sum_{r=-1,1} m_r e^{i\lambda_{rr}t}\,,$$

where

$$m_1 = \frac{1}{2|\mathbf{M}|}\left[\tilde{m}_c M_{11} + \tilde{m}_s M_{21} - i\left(\tilde{m}_c M_{12} + \tilde{m}_s M_{22}\right)\right],\quad m_{-1} = \overline{m_1}\,,$$

herewith the determinant $|\mathbf{M}|$:

$$|\mathbf{M}| = \begin{vmatrix} c_{11} & a_{11} \\ a_{11} & s_{11} \end{vmatrix}.$$

The values M_{rk} are the algebraic adjunctions of $|\mathbf{M}|$, and values $\tilde{m}_c$ and $\tilde{m}_s$ are equal to

$$\tilde{m}_c = \frac{1}{\theta}\int_0^\theta \xi(t)\cos\lambda_{11}tdt\,,\quad \tilde{m}_s = \frac{1}{\theta}\int_0^\theta \xi(t)\sin\lambda_{11}tdt\,.$$

Let us consider the functions

$$\begin{Bmatrix} h_1^c(t,\theta) \\ h_1^s(t,\theta) \end{Bmatrix} = \frac{1}{\theta}\int_0^\theta b(t,s-t)\begin{Bmatrix} \cos\lambda_{11}s \\ \sin\lambda_{11}s \end{Bmatrix} ds\,,$$

$$\begin{Bmatrix} h_2^c(t,\theta) \\ h_2^s(t,\theta) \end{Bmatrix} = \frac{1}{\theta}\int_0^\theta b(t+u,s-t-u)\begin{Bmatrix} \cos\lambda_{11}s \\ \sin\lambda_{11}s \end{Bmatrix} ds\,,$$

$$\begin{Bmatrix} I_c(\theta) \\ I_s(\theta) \end{Bmatrix} = \frac{1}{\theta^2}\int_0^\theta\int_0^\theta b(s_1,s_2-s_1)\begin{Bmatrix} \cos\lambda_{11}s_1\cos\lambda_{11}s_2 \\ \sin\lambda_{11}s_1\sin\lambda_{11}s_2 \end{Bmatrix} ds_1 ds_2\,,$$

$$I_{cs}(\theta) = \frac{1}{\theta^2}\int_0^\theta\int_0^\theta b(s_1,s_2-s_1)\cos\lambda_{11}s_1\sin\lambda_{11}s_2 ds_1 ds_2\,,$$

$$H_1(\theta) = \frac{1}{2|\mathbf{M}|}\left[I_c(\theta)\left(M_{11}^2 - M_{12}^2\right) + I_s(\theta)\left(M_{21}^2 - M_{22}^2\right) + 2I_{cs}(\theta)\left(M_{11}M_{21} - M_{12}M_{22}\right)\right],$$

$$H_2(\theta)=\frac{1}{|\mathbf{M}|}\left[M_{11}M_{12}I_c(\theta)+M_{21}M_{22}I_s(\theta)+I_{cs}(\theta)(M_{11}M_{22}+M_{12}M_{21})\right],$$

$$H_3(\theta)=\frac{1}{2|\mathbf{M}|}\left[I_c(\theta)\left(M_{11}^2+M_{12}^2\right)+I_s(\theta)\left(M_{22}^2+M_{21}^2\right)+2I_{cs}(\theta)(M_{11}M_{21}+M_{22}M_{12})\right].$$

In this case for the value (3) we have:

$$\varepsilon_\zeta(t,u)=\frac{1}{|\mathbf{M}|}\Big[h_1^c(t,\theta)\left[M_{11}\cos\lambda_{11}(t+u)+M_{12}\sin\lambda_{11}(t+u)\right]+h_1^s(t,\theta)\times$$

$$\times\left[M_{21}\cos\lambda_{11}(t+u)+M_{22}\sin\lambda_{11}(t+u)\right]+h_2^c(t,\theta)\left[M_{11}\cos\lambda_{11}t+M_{12}\sin\lambda_{11}t\right]+$$

$$+h_2^s(t,\theta)(M_{21}\cos\lambda_{11}t+M_{22}\sin\lambda_{11}t)-H_1(\theta)\cos\lambda_{11}(2t+u)-$$

$$-H_2(\theta)\sin\lambda_{11}(2t+u)-H_3(\theta)\cos\lambda_{22}u\Big].$$

Let us introduce the notations:

$$\hat{B}_1(u)=\hat{B}_{00}(u),\quad \hat{B}_2(u)=\hat{B}_{22}^c(u),\quad \hat{B}_3(u)=\hat{B}_{22}^s(u). \tag{6}$$

Then the biases of the estimator of the covariance components are represented by the general formula:

$$\varepsilon\left[\hat{B}_k(u)\right]=-\frac{1}{|\mathbf{D}|}\Bigg[A_{1k}\left[\frac{1}{\theta}\int_0^\theta\varepsilon_\zeta(t,u)\,dt\right]+A_{2k}\left[\frac{1}{\theta}\int_0^\theta\varepsilon_\zeta(t,\mathrm{u})\cos\lambda_{22}t\,dt\right]+$$

$$+A_{3k}\left[\frac{1}{\theta}\int_0^\theta\varepsilon_\zeta(t,u)\sin\lambda_{22}t\,dt\right]\Bigg],\quad k=1,2,3. \tag{7}$$

The analysis of biases is cumbersome, that is why we'll consider the case when $u=0$ only.

$$\varepsilon_\zeta(t,0)=\frac{1}{|\mathbf{M}|}\Big[2h_c(t,\theta)(M_{11}\cos\lambda_{11}t+M_{12}\sin\lambda_{11}t)+2h_s(t,\theta)(M_{21}\times$$

$$\times\cos\lambda_{11}t+M_{22}\sin\lambda_{11}t)-H_1(\theta)\cos\lambda_{22}t-H_2(\theta)\sin\lambda_{22}t-H_3(\theta)\Big]. \tag{8}$$

Hence

$$\varepsilon(\theta)=\frac{1}{\theta}\int_0^\theta\varepsilon_\zeta(t,0)\,dt=\frac{1}{|M|}\Big[2\left[M_{11}I_c(\theta)+M_{12}I_{sc}(\theta)\right]+2\left[M_{21}I_{cs}(\theta)+M_{22}I_s(\theta)\right]-$$

$$-H_1(\theta)c_{20}-H_2(\theta)a_{02}-H_3(\theta)\Big]. \tag{9}$$

Let us simplify the expressions $I_c(\theta)$, $I_s(\theta)$ and $I_{cs}(\theta)$, which are included in this relation, setting the variable $u=s-t$ and changing the order of integration we find:

$$I_c(\theta)=\frac{1}{\theta^2}\int_0^\theta\int_{-t}^{\theta-t}b(t,u)\cos\lambda_{11}t\cos\lambda_{11}(t+u)\,du\,dt=$$

$$=\frac{1}{\theta^2}\int_0^\theta\int_0^{\theta-u}b(t,u)\left[\cos\lambda_{11}u+\cos\lambda_{11}(2t+u)\right]dt\,du.$$

After substituting this expression into formulae (5) and integrating over t in the first approximation we have:

$$I_c(\theta)=\frac{1}{2\theta}\int_0^{\theta}\left(1-\frac{u}{\theta}\right)\left[2B_{00}(u)\cos\lambda_{11}u+B_{22}^{c}(u)\cos\lambda_{11}u-B_{22}^{s}(u)\sin\lambda_{11}u\right]du\ .$$

Analogically for $I_s(\theta)$:

$$I_s(\theta)=\frac{1}{\theta^2}\int_0^{\theta}\int_0^{\theta-u}b(t,u)\left[\cos\lambda_{11}u-\cos\lambda_{11}(2t+u)\right]du\ .$$

In the first approximation:

$$I_s(\theta)=\frac{1}{2\theta}\int_0^{\theta}\left(1-\frac{u}{\theta}\right)\left[2B_{00}(u)\cos\lambda_{11}u-B_{22}^{c}(u)\cos\lambda_{11}u+B_{22}^{s}(u)\sin\lambda_{11}u\right]du\ .$$

For the function $I_{cs}(\theta)$ we obtain in the first approximation:

$$I_{cs}(\theta)=\frac{1}{2\theta}\int_0^{\theta}\left(1-\frac{u}{\theta}\right)\left[B_{22}^{s}(u)\cos\lambda_{11}u+B_{2}^{c}(u)\sin\lambda_{11}u\right]du\ .$$

Taking into account relation (8) for the second and the third components of the value (7) we obtain:

$$\varepsilon_c(\theta)=\frac{1}{\theta}\int_0^{\theta}\varepsilon_{\zeta}(t,0)\cos\lambda_{22}t\,dt=\frac{1}{|M|}\Big[M_{11}\left[I_c(\theta)+I_c^{(2)}(\theta)\right]+M_{22}\left[I_s^{(1)}(\theta)-I_s(\theta)\right]+$$
$$+M_{12}\left[I_{sc}(\theta)+I_{sc}^{(1)}(\theta)\right]+M_{12}\left[I_{cs}^{(1)}(\theta)-I_{cs}(\theta)\right]-$$
$$-H_1(\theta)c_{22}-H_2(\theta)a_{22}-H_3(\theta)c_{02}\Big], \qquad (10)$$

$$\varepsilon_s(\theta)=\frac{1}{\theta}\int_0^{\theta}\varepsilon_{\zeta}(t,0)\sin\lambda_{22}t\,dt=\frac{1}{|M|}\Big[M_{11}\left[I_{cs}^{(1)}(\theta)+I_{cs}(\theta)\right]+M_{22}\left[I_{sc}(\theta)-I_{sc}^{(1)}(\theta)\right]+$$
$$+M_{12}\left[I_c(\theta)-I_c^{(1)}(\theta)\right]+M_{21}\left[I_{cs}^{(1)}(\theta)+I_s(\theta)\right]-$$
$$-H_1(\theta)a_{22}-H_2(\theta)s_{22}-H_3(\theta)a_{02}\Big], \qquad (11)$$

where

$$\begin{Bmatrix} I_c^{(1)}(\theta) \\ I_s^{(1)}(\theta) \end{Bmatrix}=\frac{1}{\theta^2}\int_0^{\theta}\int_0^{\theta}b(t,s-t)\begin{Bmatrix}\cos\lambda_{11}s\,\cos\lambda_{33}t \\ \sin\lambda_{11}s\,\sin\lambda_{33}t\end{Bmatrix}ds\,dt\ ,$$

$$\begin{Bmatrix} I_{cs}^{(1)}(\theta) \\ I_{sc}^{(1)}(\theta) \end{Bmatrix}=\frac{1}{\theta^2}\int_0^{\theta}\int_0^{\theta}b(t,s-t)\begin{Bmatrix}\cos\lambda_{11}s\,\sin\lambda_{33}t \\ \sin\lambda_{11}s\,\cos\lambda_{33}t\end{Bmatrix}ds\,dt\ .$$

It can be easity seen that in the first approximation $I_c^{(1)}(\theta)=I_s^{(1)}(\theta)$ and $I_{cs}^{(1)}(\theta)=-I_{sc}^{(1)}(\theta)$. Then the bias of the estimators of covariance components of random process in the first approximation is determined by the formula

$$\varepsilon\left[\hat{B}_k(\theta)\right]=-\frac{1}{|D|}\left[A_{1k}\varepsilon_0(\theta)+A_{2k}\varepsilon_c(\theta)+A_{3k}\varepsilon_s(\theta)\right],\quad k=1,2,3. \qquad (12)$$

This value is obtained only by the preliminary calculation of the BPCRP mean function. Dependence of these values on covariance function of the process is determined by formulae (9), (10) and (11), and dependence on realization length θ is represented by the determinants $|\mathbf{M}|$, $|\mathbf{D}|$ and theirs algebraic adjunctions. In the asymptotic $\theta\to\infty$ these determinants degenerate into diagonal:

$$|\mathbf{D}|\to\begin{vmatrix}1 & 0 & 0\\ 0 & \frac{1}{2} & 0\\ 0 & 0 & \frac{1}{2}\end{vmatrix}=\frac{1}{4},\quad |\mathbf{M}|\to\begin{vmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{vmatrix}=\frac{1}{4}.$$

Substituting the value of determinants and their adjunctions into expressions (9), (10) and (11), we obtain for the biases of estimators of covariance components:

$$\varepsilon\left[\hat{B}_{00}(0)\right]=-2\left[I_c(\theta)+I_s(\theta)\right],$$

$$\varepsilon\left[\hat{B}_{22}^{c}(0)\right]=-2\left[I_c(\theta)-I_s(\theta)+2\left[I_c^{(1)}(\theta)+I_s^{(1)}(\theta)\right]\right],$$

$$\varepsilon\left[\hat{B}_{22}^{s}(0)\right]=4\left[I_{cs}(\theta)+2I_{cs}^{(1)}(\theta)\right].$$

Let us express bias (12) through the covariance functions of quadrature components of the random process (5). Since

$$2B_{00}(u)\cos\lambda_{11}u+B_{22}^{c}(u)\cos\lambda_{11}u-B_{22}^{s}(u)\sin\lambda_{11}u=$$
$$=R_c(u)+\frac{1}{2}\left[R_c(u)+R_s(u)\right]\cos\lambda_{22}u,\qquad(13)$$

$$2B_{00}(u)\cos\lambda_{11}u-B_{22}^{c}(u)\cos\lambda_{11}u+B_{22}^{s}(u)\sin\lambda_{11}u=$$
$$=R_s(u)+\frac{1}{2}\left[R_c(u)+R_s(u)\right]\cos\lambda_{22}u,\qquad(14)$$

$$B_{22}^{c}(u)\sin\lambda_{11}u+B_{22}^{s}(u)\cos\lambda_{11}u=R_{cs}^{+}(u),\qquad(15)$$

then

$$I_c(\theta)=\frac{1}{2\theta}\int_0^{\theta}\left(1-\frac{u}{\theta}\right)\left[R_c(u)+\frac{1}{2}\left[R_c(u)+R_s(u)\right]\cos\lambda_{22}u\right]du,$$

$$I_s(\theta)=\frac{1}{2\theta}\int_0^{\theta}\left(1-\frac{u}{\theta}\right)\left[R_s(u)+\frac{1}{2}\left[R_c(u)+R_s(u)\right]\cos\lambda_{22}u\right]du,$$

$$I_{cs}(\theta)=\frac{1}{2\theta}\int_{-\theta}^{\theta}\left(1-\frac{u}{\theta}\right)R_{cs}^{+}(u)\,du.$$

Hence

$$B_{22}^{c}(u)\cos\lambda_{11}u+B_{22}^{s}(u)\sin\lambda_{11}u=\frac{1}{2}\left[R_c(u)-R_s(u)\right]\cos\lambda_{22}u+R_{cs}^{+}(u)\sin\lambda_{11}u,\qquad(16)$$

$$B_{22}^{s}(u)\cos\lambda_{11}u - B_{22}^{c}(u)\sin\lambda_{11}u = R_{cs}^{+}(u)\cos\lambda_{22}u + \frac{1}{2}\left[R_s(u) - R_c(u)\right]\sin\lambda_{22}u\,. \quad (17)$$

Taking into account that the symmetric integral of the odd function is equal to zero we obtain:

$$I_c^{(1)}(\theta) = \frac{1}{4\theta}\int_0^{\theta}\left(1-\frac{u}{\theta}\right)\left[R_c(u) - R_s(u)\right]\cos\lambda_{22}u\,du\,,$$

$$I_{cs}^{(1)}(0) = \frac{1}{2\theta}\int_0^{\theta}\left(1-\frac{u}{\theta}\right)R_{cs}^{+}(u)\cos\lambda_{22}u\,du\,.$$

Let us assume that

$$R_c(u) = D_1 e^{-\alpha_1|u|}, \quad R_s(u) = D_2 e^{-\alpha_2|u|}, \quad R_{cs}(u) = D_3 e^{-\alpha_3|u|}, \quad (18)$$

and denote

$$\tau_k(\alpha_i,\theta) = \frac{1}{\theta}\int_0^{\theta}\left(1-\frac{u}{\theta}\right)e^{-\alpha_i u}\cos\lambda_{kk}u\,du\,.$$

Then we have:

$$I_c(\theta) = \frac{1}{2}\left[D_1 r_0(\alpha_1,\theta) + \frac{1}{2}\left[D_1 r_2(\alpha_1,\theta) + D_2 r_2(\alpha_2,\theta)\right]\right],$$

$$I_s(\theta) = \frac{1}{2}\left[D_2 r_0(\alpha_1,\theta) + \frac{1}{2}\left[D_1 r_2(\alpha_1,\theta) + D_2 r_2(\alpha_2,\theta)\right]\right],$$

$$I_{cs}(\theta) = I_{cs}(\theta) = \frac{D_3}{2} r_0(\alpha_3,\theta)\,,$$

$$I_c^{(1)}(\theta) = \frac{1}{4}\left[D_1 r_2(\alpha_1,\theta) - D_2 r_2(\alpha_2,\theta)\right], \quad I_{cs}^{(1)}(\theta) = \frac{D_3}{2} r_2(\alpha_3,\theta)\,.$$

On the basis of the obtained formulae it is possible to calculate the values of bias of LSM-estimator of the covariance component, depending on the length of realization and parameters which describe covariance structure of BPCRP and also to compare the values of biases of the LSM and component estimators.

1. Javorskyj, I. M. Mathematical models and analysis of stochastic oscillations. Nazarchuk, Z. T., Ed.; Physico-mechanical institute of NAS of Ukraine: Lviv, 2013. (in Ukrainian).
2. Javorskyj, I. M.; Kravets, I. B.; Matsko, I. Y.; Yuzefovych, R. M. Periodically correlated random processes: Application in early diagnostics of mechanical systems. *Mechanical Systems and Signal Processing*. **2017**, *83*, 406-438.
3. Javorskyj, I. M.; Yuzefovych, R. M.; Dzeryn, O. Yu. LSM-harmonic analysis of biperiodic non-stationary vibration signals. *Information extraction and processing*. **2017**, *121 (45)*, 14-25.
4. Matsko, I. Y.; Javorskyj, I. M.; Yuzefovych, R. M.; Zakrzewski, Z. Forced oscillations of cracked beam under the stochastic cyclic loading. *Mechanical Systems and Signal Processing*. **2018**, *104*, 242-263.
5. Javorskyj, I. M.; Yuzefovych, R. M.; Dzeryn, O. Yu. Covariance LSM-analysis of biperiodic nonstationary vibration signals. *Information extraction and processing*. **2018**, *122* (*46)*, 19-28.
6. Javorskyj, I. M.; Dzeryn, O. Yu.; Yuzefovych, R. M. Analysis of mean function discrete LSM-estimator for biperiodically nonstationary random signal. *Mathematical Modeling and Computing*. **2019**, *6 (1)*, 44-57.

Received 13.09.2019