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ELECTROMAGNETIC FIELD OF THE CIRCULAR MAGNETIC SOURCE IN BICONICAL SECTION

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The problem of axially-symmetric electromagnetic wave diffraction from the perfectly conducting biconical scatterer formed by the finite cone placed in the semi-infinite conical region is solved rigorously using the mode-matching and analytical regularization techniques. The problem is reduced to the infinite systems of linear algebraic equations (ISLAE) of the second kind. The obtained equations admit the reduction procedure and can be solved with a given accuracy for any geometrical parameter and frequency. The numerical examples of the solution are presented. The analysis of the source location influence on the far-field pattern for different geometrical parameters of the bicone is carried out.

Keywords: *finite cone; bicone; rigorous solution; analytical regularization.*

ЕЛЕКТРОМАГНІТНЕ ПОЛЕ ВИТКА МАГНІТНОГО СТРУМУ У БІКОНІЧНОМУ СЕКТОРІ

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Розв’язано осесиметричну задачу дифракції електромагнітної хвилі на ідеально провідному біконусі, сформованому з напівнескінченної та скінченної конічних поверхонь зі спільною віссю симетрії. Задачу сформульовано у сферичній системі координат і зведено до мішаної крайової задачі для рівняння Гельмгольца. Джерелом збудження є круговий виток магнітного струму, поміщений у біконічну область розсіювача, біля його вершини. За цієї умови на формування поля в основному впливає поперечна електромагнітна хвиля. З використанням методу розділення змінних розв’язок крайової задачі подано у вигляді рядів власних мод у кожній з підобластей, утвореній біконусом. Для знаходження невідомих коефіцієнтів розкладу використано метод спряження тангенціальних компонент поля на межах підобластей. Тоді задачу зведено до розв’язання парних рівнянь, поданих у вигляді функціональних рядів. Ці рівняння зведено до нескінченної системи лінійних алгебричних рівнянь першого роду з використанням умови ортогональності власних мод розсіювальної структури. Цю систему трансформовано у систему другого роду методом аналітичної регуляризації, яка полягає у точному аналітичному оберненні сингулярної частини матричного оператора дифракційної задачі. Система рівнянь другого роду допускає розв’язання методом редукції за довільних значень частоти та геометричних параметрів, а отримані розв’язки забезпечують виконання усіх необхідних умов, включаючи і умову на краю. Систему використано для знаходження невідомих коефіцієнтів розкладу розсіяних полів у ряди за власними модами конічних та біконічної областей, а також для розрахунку характеристик дифракційного поля в зоні випромінювання. Наведено кутовий розподіл модуля магнітної компоненти поля в зоні випромінювання для різних значень кутових параметрів біконуса, а також встановлено вплив на цей розподіл азимутальної координати магнітного джерела поля випромінювання.

Ключові слова: *скінченний конус, біконус, строгий розв’язок, аналітична регуляризація*.

Introduction. The rigorous solution of the diffraction problem from biconical structures has attracted attention of scientists due to their application in radio science, optics and nanotechnologies. The numerous scientific papers were devoted to the explanation of the phenomena of electromagnetic waves diffraction from the structures with biconical elements [1–6]. Convenient theoretical models of a biconical antenna analyzed by the mode-matching techniques were first introduced in [1, 2]. This problem was solved in a spherical coordinate system by separation of variables. The solution was represented as a series of sub-regions normal modes, each of which satisfied the boundary condition, as well as the limited energy and radiation conditions. The infinite systems of linear algebraic equations (ISLAE) were obtained using the mode-matching techniques and were solved approximately. Some wave diffraction problems from the finite bicones as well as from the single cones were analyzed rigorously using the Wiener–Hopf technique in [6].

Here, we apply the analytical regularization technique, developed in our previous papers [7–14] to rigorously analyze the far field scattered from the bicone and to study the source location influence on the far field pattern of the biconical scatterer. The time dependence is $e^{-i\omega t}$, and it is omitted throughout the paper.

Formulation of the problem. Let us consider the perfectly conducting biconical surface $Q = Q_1 \bigcup Q_2$ in spherical coordinate system (r, θ, φ), where

$$Q_1 : \{r \in [0, \infty),\ \theta = \gamma_1;\ \varphi \in [0, 2\pi)\}, \quad Q_2 : \{r \in (0, a_1),\ \theta = \gamma_2;\ \varphi \in [0, 2\pi)\},$$

$$\gamma_2 > \gamma_1, \quad \gamma_{1(2)} \neq \pi/2 \text{ (see Fig. 1).}$$

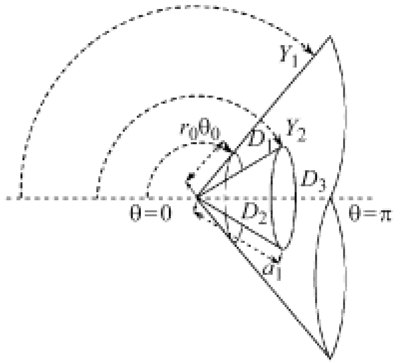

Fig. 1. Geometrical scheme of the bicone.

Let bicone Q be excited by the ring source with magnetic current density as

$$J(r, \theta) = I_\varphi^{(m)} \delta(r - r_0) \delta(\theta - \theta_0)/(r_0 \sin\theta_0), \tag{1}$$

where $I_\varphi^{(m)}$ is the magnetic current, $\delta(...)$ is Dirac delta function; r_0, θ_0 are source coordinates, $0 < r_0 < a_1$, $\gamma_1 \le \theta_0 \le \gamma_2$. Nonzero electrical field components E_r, E_θ, excited by source (1), are expressed in terms of H_φ component as

$$\begin{aligned} E_r &= -\frac{Z}{ik} \frac{1}{r \sin\theta} \frac{\partial}{\partial\theta} (\sin\theta H_\varphi), \\ E_\theta &= \frac{Z}{ik} \frac{1}{r} \frac{\partial}{\partial r} (r H_\varphi). \end{aligned} \tag{2}$$

Here, k is the wave number, $k = k' + ik'' = \omega\sqrt{\varepsilon\mu}$, $k', k'' > 0$; ε and μ are dielectric permittivity and magnetic permeability of the medium, respectively; $Z = \sqrt{\mu/\varepsilon}$

is the impedance of the medium.

The problem is formulated in terms of H_φ – diffracted field component and reduced to the solution of the mixed boundary value problem for the Helmholtz equation [11], which satisfies the boundary condition $E_r^t = 0$ on Q, radiation condition and condition of electromagnetic energy limitation, where $E_r^t = E_r^i + E_r$; E_r and E_r^i are the diffracted field and the incident field produced by source (1), respectively.

Solution of the problem. Let us represent the unknown total magnetic field as

$$H_\varphi^t(\rho,\theta) = \begin{cases} H_\varphi^i(\rho,\theta) + \dfrac{i\omega\varepsilon}{\sqrt{\rho}} \sum\limits_{n=1}^{\infty} x_n^{(1)} \Psi_{\nu_n-1/2}(\cos\theta) \dfrac{I_{\nu_n}(\rho)}{I_{\nu_n}(\rho_1)}, & (r,\theta)\in D_1, \\ \dfrac{i\omega\varepsilon}{\sqrt{\rho}} \sum\limits_{n=1}^{\infty} x_n^{(2)} \dfrac{\partial}{\partial\theta} P_{\mu_n-1/2}(-\cos\theta) \dfrac{I_{\mu_n}(\rho)}{I_{\mu_n}(\rho_1)}, & (r,\theta)\in D_2, \\ \dfrac{i\omega\varepsilon}{\sqrt{\rho}} \sum\limits_{n=1}^{\infty} x_n^{(3)} \dfrac{\partial}{\partial\theta} P_{z_n-1/2}(-\cos\theta) \dfrac{K_{z_n}(\rho)}{K_{z_n}(\rho_1)}, & (r,\theta)\in D_3. \end{cases} \quad (3)$$

Here, $D_1 : \{ r\in[0,a_1), \theta\in[\gamma_1,\gamma_2); \varphi\in[0,2\pi) \}$, $D_2 : \{ r\in[0,a_1), \theta\in(\gamma_2,\pi]; \varphi\in[0,2\pi) \}$, $D_3 : \{ r\in(a_1,\infty), \theta\in(\gamma_1,\pi]; \varphi\in[0,2\pi) \}$; $x_n^{(1)}$, $x_n^{(2)}$, $x_n^{(3)}$ are unknown expansion coefficients; $I_\nu(\rho)$, $K_\nu(\rho)$ are modified Bessel and Macdonald functions; $\rho = sr$, $\rho_1 = sa_1$; $s = -ik$; $P_{\chi-1/2}(-\cos\theta)$ is the Legendre function; $\{z_n\}_{n=1}^{\infty}$, $\{\mu_n\}_{n=1}^{\infty}$ are growing sequences of real positive roots of transcendental equations

$$P_{z_n-1/2}(-\cos\gamma_1) = 0,$$

$$P_{\mu_n-1/2}(-\cos\gamma_2) = 0$$

and $\{\nu_n\}_{n=1}^{\infty}$ is the growing sequence of real positive roots of transcendental equation

$$R_{\nu_n-1/2}(\cos\gamma_2) = 0,$$

where

$$R_{\nu-1/2}(\cos\theta) = P_{\nu-1/2}(\cos\theta) P_{\nu-1/2}(-\cos\gamma_1) - P_{\nu-1/2}(-\cos\theta) P_{\nu-1/2}(\cos\gamma_1).$$

The set of positive roots of function $R_{\nu_n-1/2}(\cos\gamma_2)$ starts from $v_1 = 1/2$ and $v_n \neq n - 1/2$, if $n = 2,3,4,...$. Function $\Psi_{\nu_n-1/2}(\cos\theta)$ is determined as

$$\Psi_{\nu_n-1/2}(\cos\theta) = \begin{cases} \dfrac{1}{\sin\theta}, & n = 1, \\ \dfrac{\partial}{\partial\theta}[R_{\nu_n-1/2}(\cos\theta)], & n > 1; \end{cases}$$

$H_\varphi^i(r,\theta)$ is the known field produced by source (1) in the semi-infinite biconical region,

$$H_\varphi^i(r,\theta) = \frac{i\omega\varepsilon}{\sqrt{\rho\rho_0}} \sum_{j=1}^{\infty} B_j \Psi_{\nu_j-1/2}(\cos\theta) \begin{cases} K_{\nu_j}(\rho) I_{\nu_j}(\rho_0), r \ge r_0, \\ I_{\nu_j}(\rho) K_{\nu_j}(\rho_0), r \le r_0, \end{cases} \quad (r,\theta)\in D_1.$$

Here, $B_j = -\rho_0 b_j I_\varphi^{(m)} \Psi_{\nu_j-1/2}(\cos\theta_0)$, $\rho_0 = sr_0$,

$$b_j = \begin{cases} \left[\ln(\mathrm{ctg}\frac{\gamma_1}{2} \mathrm{tg}\frac{\gamma_2}{2}) \right]^{-1}, & j = 1, \\ \frac{1}{\sin\gamma_2} \frac{2\nu_j}{\nu_j^2 - 0,25} \left[\frac{\partial}{\partial\nu} R_{\nu_j-1/2}(\cos\gamma_2) \frac{\partial}{\partial\gamma} R_{\nu_j-1/2}(\cos\gamma_2) \right]^{-1}, & j > 1. \end{cases}$$

Using the mode matching for $E_\theta^t(a_1 \pm 0, \theta)$ and $H_\varphi^t(a_1 \pm 0, \theta)$ and the ortogonality properties of the Legendre functions in the conical and biconical regions we reduce the problem to the solution of the infinite system of linear algebraic equations of the second kind by involving the analytical regularization technique [11, 12]:

$$X - A^{-1}(A - A_{11})X = A^{-1}F. \tag{4}$$

Here, $X = \{x_n\}_{n=1}^{\infty}$ is the unknown vector, $x_n = x_n^{(3)}(z_n^2 - 0.25)P_{z_n-1/2}(-\cos\gamma_2)$; A_{11} is the infinite matrix,

$$A_{11} : \left\{ a_{jn} = \frac{\rho_1 W[K_{z_n} I_{\xi_j}]_{\rho_1}}{\Delta_{jn} I_{\xi_j}(\rho_1) K_{z_n}(\rho_1)} \right\}_{j,n=1}^{\infty},$$

$\{\xi_j\}_{j=1}^{\infty} = \{\nu_j\}_{j=1}^{\infty} \cup \{\mu_j\}_{j=1}^{\infty}$ is the growing sequence; $\Delta_{jn} = \xi_j^2 - z_n^2$; $F = \{f_j\}_{j=1}^{\infty}$ is the known vector with

$$f_j = \begin{cases} \sqrt{\rho_0} I_\varphi^{(m)} \bar{b}_j \Psi_{\xi_j-1/2}(\cos\theta_0) \frac{I_{\xi_j}(\rho_0)}{I_{\xi_j}(\rho_1)}, & \xi_j \in \{\nu_p\}_{p=1}^{\infty}, \\ 0, \ \xi_j \notin \{\nu_p\}_{p=1}^{\infty} \end{cases}$$

and

$$\bar{b}_j = \begin{cases} 1, & j = 1, \\ \left[\sin\gamma_2 \frac{\partial}{\partial\gamma_2} R_{\nu_j-1/2}(\cos\gamma_2) \right]^{-1}, & j > 1; \end{cases}$$

$A : \{a_{jm}\}_{j,m=1}^{\infty}$, $A^{-1} : \{\tau_{nj}\}_{n,j=1}^{\infty}$ are the regularizing operators, $A^{-1}A = I$, I is the identity matrix;

$$\begin{gathered} a_{jm} = (\xi_j - z_m)^{-1}, \\ \tau_{nj} = \left\{ \{[M_-(\xi_j)]^{-1}\}' [M_-(z_n)]' (z_n - \xi_j) \right\}^{-1}. \end{gathered} \tag{5}$$

Here, $M_-(v)$ is the known function determined as

$$M(v) = M_+(v) M_-(v),$$

where

$$M(v) = \frac{P_{v-1/2}(-\cos\gamma_1)\cos(\pi\nu)}{(v^2 - 1/4)P_{v-1/2}(-\cos\gamma_2)R_{v-1/2}(\cos\gamma_2)}$$

and $M_+(\nu) = M_-(-\nu)$; $M_+(v)$, $M_-(v)$ are regular functions in half-planes $Re(v) > -1/2$, $Re(v) < 1/2$, respectively; $M_\pm(v) = O(v^{-1/2})$, if $|v| \to \infty$ in the regularity regions. The unique solution of equation (4) exists in a class of sequences $x_n = O(n^{-1/2})$, if $n \to \infty$, and satisfies the Meixner condition at the conical edge.

Numerical examples. All characteristics of the scattered field are calculated using the reduction method for the solution of the ISLAE (4). The numerical examples of the normalized total magnetic field distribution $|H_\varphi^t(\theta)|$ for different position of the source coordinate θ_0 is presented in Fig. 2. As follows from the behavior of curves presented in this figure the radiation maxima of the biconical scatterer Q are formed in the angle sector $\gamma_1 < \theta < \gamma_2$, if the source is located in the biconical region close to the tip of the bicone.

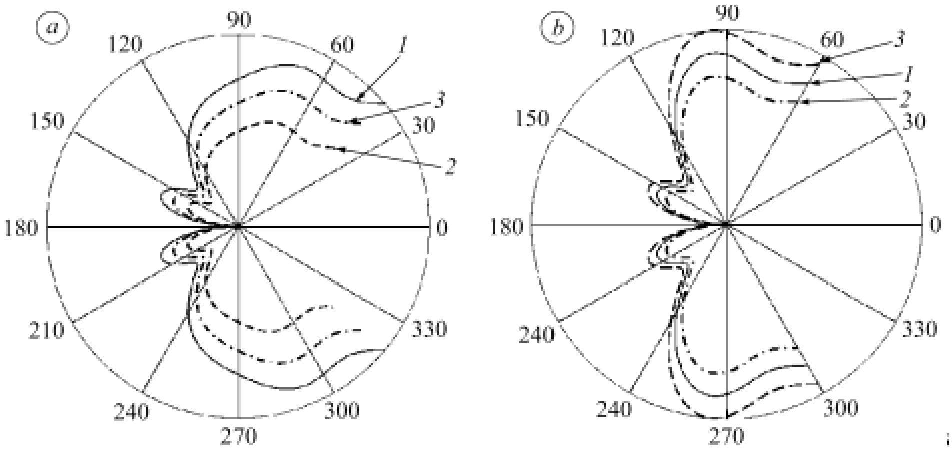

Fig. 2. Influence of the source coordinate θ_0 on the far-field patterns of the bicone Q: $kr_0 = 0.02$, $kr_1 = 6.28$; $\gamma_2 = 130°$; *a* – $\gamma_1 = 40°$, *b* – $\gamma_1 = 60$; *1* – $\theta_0 = \gamma_1$; *2* – $\theta_0 = (\gamma_1 + \gamma_2)/2$; *3* – $\theta_0 = \gamma_2$.

In spite of the small radius of the magnetic ring we observe the effective influence of the source coordinate θ_0 on the far field patterns, if the ring of the source moves towards the semi-infinite conical surface. Therefore, the source parameter θ_0 of the magnetic ring source which is located near the biconical tip can be used for modelling the intensity of radiation of the biconical scatterer.

CONCLUSIONS

The mode-matching and analytical regularization techniques are developed for the solution of axially-symmetric diffraction problem from the bicone Q, formed by the finite and semi-infinite conical shoulders. The key equation (4) which solution satisfies all the necessary conditions is obtained. It is shown that the source parameter θ_0 of the magnetic ring source which is located near the biconical tip can be used for modelling the intensity of radiation of the biconical scatterer.

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