UDC 535.361: 620.186 https://doi.org/10.15407/vidbir2021.49.037

MEDIAN BASED ALGORITHM FOR SUB-PIXEL ESTIMATION OF EXTREMA POSITIONS OF DIFFUSE LIGHT REFLECTION SIGNAL

I. B. Ivasiv

H. V. Karpenko Physico-Mechanical Institute of the NAS of Ukraine, Lviv

E-mail: igreg@ukr.net

It has been proposed to utilize the median algorithm for determination of the extrema positions of diffuse light reflectance intensity distribution by a discrete signal of a photodiode linear array. The algorithm formula has been deduced on the base of piecewise-linear interpolation for signal representation by cumulative function. It has been shown that this formula is much simpler for implementation than known centroid algorithm and the noise immune Blais and Rioux detector algorithm. Also, the methodical systematic errors for zero noise as well as the random errors for full common mode additive noises and uncorrelated noises have been estimated and compared for mentioned algorithms. In these terms, the proposed median algorithm is proportional to Blais and Rioux algorithm and considerably better then centroid algorithm.

Keywords: *diffuse light reflectance, discrete signal, extremum position, centroid algorithm, Blais and Rioux algorithm, median algorithm, noise immunity, systematic error, random error.*

АЛГОРИТМ ОЦІНЮВАННЯ СУБПІКСЕЛЬНИХ ПОЛОЖЕНЬ ЕКСТРЕМУМІВ СИГНАЛУ ДИФУЗНОГО ВІДБИВАННЯ СВІТЛА НА ОСНОВІ МЕДІАНИ

І. Б. Івасів

Фізико-механічний інститут ім. Г. В. Карпенка НАН України, Львів

Показано, що важливою проблемою оцінки субпіксельних положень екстремумів розподілу інтенсивності дифузного відбивання світла за результатами вимірювань сигналу сенсора, сформованого на дискретних світлочутливих елементах фотодіодної лінійки, є сильний вплив шумів на периферійних елементах. Вказаний чинник суттєво знижує ефективність відомих алгоритмів оцінки положень екстремумів неперервних сигналів на основі дискретних вимірювань, зокрема, алгоритму центроїда. З іншого боку, стійкий до шумів алгоритм детектора Блейза–Ріо відзначається складністю імплементації. Запропоновано використовувати алгоритм медіани для оцінки субпіксельних положень екстремумів сигналу, який, з одного боку, є достатньо стійкий до шумів, а з іншого – значно простіший в імплементації, навіть порівняно з алгоритмом центроїда. Пояснено основну ідею алгоритму та виведено відповідну формулу оцінки положення екстремуму сигналу на основі подання 5-елементного дискретного сигналу фотодіодної лінійки у вигляді кусково-лінійної кумулятивної функції. Обчислено та порівняно максимальні похибки оцінок положень екстремумів модельованого розподілу інтенсивності дифузного розсіювання світла зі застосуванням до відповідних модельованих дискретних сигналів запропонованого алгоритму, алгоритму центроїда та алгоритму Блейза–Ріо в умовах як відсутності шумів, так і за синфазного та некорельованого шуму. За відсутності шумів максимальна методична систематична похибка становила: для 5-елементного алгоритму центроїда 0,06 мм, 5-елементного алгоритму Блейза–Ріо 0,06 мм та 5-елементного алгоритму медіани 0,04 мм за кроку 1 мм світлочутливих елементів фотодіодної лінійки. Для синфазного шуму в діапазоні співвідношень сигнал/шум від 20 до 100 алгоритм медіани дає дещо більші випадкові похибки (максимальна – 0,08 мм), ніж алгоритм Блейза–Ріо, але значно менші, ніж алгоритм центроїда. Для некорельованого шуму в діапазоні співвідношень сигнал/шум від 10 до 100 запропонований алгоритм медіани забезпечив найменші значення максимальних випадкових похибок (не перевищували 0,14 мм) серед усіх трьох алгоритмів.

Ключові слова: *дифузне відбивання світла, дискретний сигнал, положення екстремуму, алгоритм центроїда, алгоритм Блейза–Ріо, алгоритм медіани, завадостійкість, систематична похибка, випадкова похибка.*

Introduction. The problem of estimating the characteristics of corrosion damage of metal surface in the form of submillimetre corrosion spots formed by agglomeration of corrosion products on the elements of the surface microstructure [1–4] continues to be considered. In particular, works [5, 6] proposed to size the surface corrosion microdefects by the determination of the extrema positions for diffuse light reflection intensity distribution (Fig. 1) as long as such signal metrics provided the significant advantage in terms of tolerance to fluctuations of microdefects location.

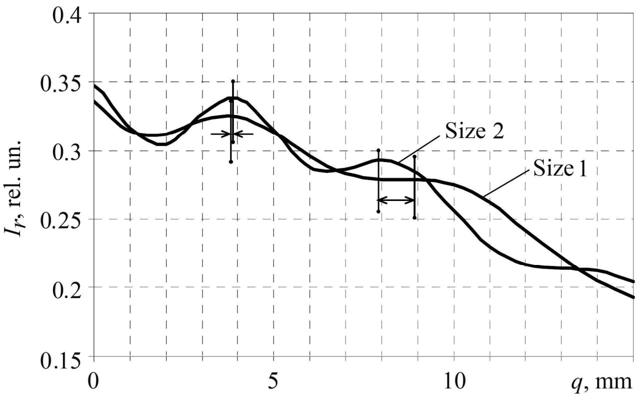

Fig. 1. Extrema positions of the diffuse light reflection signals for different sizes of corrosion grains (I_r – relative intensity measured by photodiode linear array).

At the same time, an effective implementation of proposed metrics requires the determination of extrema positions with sub-pixel accuracy, namely within linear size (hereafter, pitch) of a photodiode linear array (PhDLA) cell. The main drawback of the most popular algorithms for extremum position determination of continuous characteristic by the discrete signal (the centroid algorithm [7–9], wavelet-based algorithms [10], as well as the Gaussian approximation algorithm, the linear interpolation, the parabolic estimator algorithm, etc. [11]) is the significant influence of the noises [9, 11]. From this point of view, the Blais and Rioux (BR) detectors [12] are the most tolerant for the noise [11]. Another algorithm providing the noise immunity is the median algorithm for peak detection [13]. However, it is necessary to describe this algorithm concerning the discrete signal of diffuse light reflection and compare it with BR algorithm and centroid algorithm in terms of the noise immunity as well as complexity of implementation.

Both mentioned above centroid and BR algorithms have some drawbacks restricting their application.

Thus, based on the calculation of the first moments of the diffuse light reflection signal pattern within extremum neighbourhood, the 5-element centroid algorithm

$$X_c = \frac{-2s_{-2} - s_{-1} + s_1 + 2s_2}{s_{-2} + s_{-1} + s_0 + s_1 + s_2} \cdot Pitch\,, \qquad (1)$$

where s_0 is the maximal signal on PhDLA cells, s_{-1}, s_1 are the signals on the left and right (respectively) cells closest to the cell with maximal signal (central cell), s_{-2}, s_2 are the signals on the left and right cells one pitch farther from the central cell, is grounded on the incorrect assumption concerning signal concentration in the centers of the sensor cells. As a result, especially for narrow signal peaks, the centroid algorithm has the considerable methodical systematic error. In addition, double weights for the peripheral pattern elements cause the significant influence of noise on the random error of position determination.

The BR algorithm is more tolerant to noises but, on the other hand, is more difficult to calculate:

$$X_{BR} = \frac{-s_{-2} - s_{-1} + s_1 + s_2}{s_0 + s_{-1} - s_2} \cdot Pitch \text{ if } (s_{-2} + s_{-1}) > (s_1 + s_2) \quad (2.1)$$

and $$X_{BR} = \frac{-s_{-2} - s_{-1} + s_1 + s_2}{s_0 + s_1 - s_{-2}} \cdot Pitch \text{ if } (s_{-2} + s_{-1}) < (s_1 + s_2) \quad (2.2)$$

Therefore, the actual problem is to develop such signal extremum sub-pixel position determining algorithm that is simple and quite tolerant to noises.

Principles of position determination based on median of signals spatial distribution. If we have some signal $s(x)$ distributed continuously on the interval $[X_{\min}, X_{\max}]$ (Fig. 2), we can determine the center of mass X_0 of this signal by two ways. The first way consists in calculation of the signal first moment normalized by signal "mass":

$$X_0 = \frac{\int_{X_{\min}}^{X_{\max}} xs(x)dx}{\int_{X_{\min}}^{X_{\max}} s(x)dx} \quad (3)$$

that is known as the centroid algorithm. For the 5-element discrete signal we obtain formula (1) for X_0 calculation.

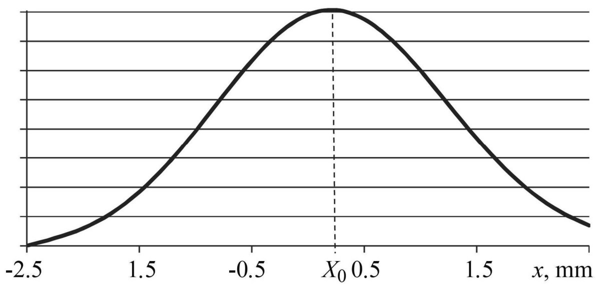

Fig. 2. Continuous signal on interval [$X_{\min}$, $X_{\max}$].

It is quite obviously that the sensitivity of moment functions $\int_a^b x^p s(x)dx$ to signal disturbances is proportional to the moment order p. Consequently, the zero-order moment $\int_a^b s(x)dx$ should be the more tolerant to disturbances. Therefore, another way of the center of "mass" determination that can be more stable is solving the next equation:

$$\int_{X_{\min}}^{X_m} s(x)dx = \int_{X_m}^{X_{\max}} s(x)dx \quad (4)$$

Equation (4) is the equation for signal median (see Fig. 3).

Analytical solution of equation (4) is not a trivial problem. However, we can try to solve it by rewriting integrals in equation (4) into the Stieltjes form and doing the following manipulations:

$$\int_{X_{\min}}^{X_m} dS(x) = \int_{X_m}^{X_{\max}} dS(x) \quad (5)$$

where $S(x) = \int_0^x s(z)dz$ is the cumulative function, and, consequently, equation (5) can be written and solved in the following manner:

$$S(X_m) - S(X_{\min}) = S(X_{\max}) - S(X_m)\,, \tag{6}$$

$$S(X_m) = \frac{S(X_{\min}) + S(X_{\max})}{2}\,. \tag{7}$$

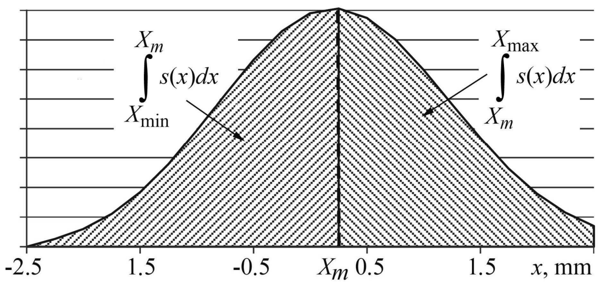

Fig. 3. Definition of signal median.

Now, if we have the 5-element discrete signal s_{-2}, s_{-1}, s_0, s_1, s_2, we can build the piecewise constant cumulative function $S(x)$ with the nodal values $S_{-3} = 0$, $S_{-2} = s_{-2}$, $S_{-1} = s_{-2} + s_{-1}$, $S_0 = s_{-2} + s_{-1} + s_0$, $S_1 = s_{-2} + s_{-1} + s_0 + s_1$ and $S_2 = s_{-2} + s_{-1} + s_0 + s_1 + s_2$ (see Fig. 4). But, piecewise constant function does not allow us to determine the center of "mass" with sufficient accuracy. Therefore, we must apply some interpolation to the obtained set of nodal values. A spline interpolation seems to be quite good for determining the center of "mass" (see Fig. 5). However, on the other hand, derivation of algorithm formula with spline interpolation can be some cumbersome.

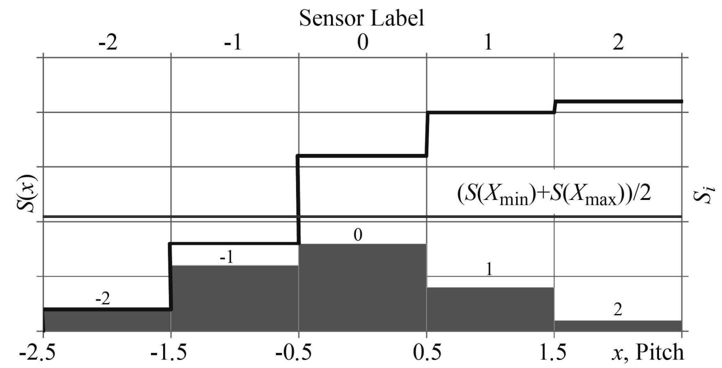

Fig. 4. Cumulative function $S(x)$ for discrete signal.

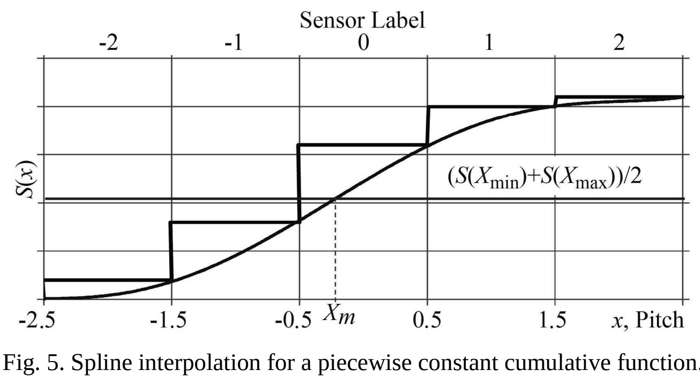

Fig. 5. Spline interpolation for a piecewise constant cumulative function.

Algorithm derivation based on the median of piecewise linear cumulative function. The more simple way of algorithm formula derivation is application of the piecewise-linear interpolation of cumulative function (see Fig. 6).

In this case, we obtain solution in the central (with maximal signal) sensor cell at the cross point of interpolated cumulative function $\tilde{S}(x)$ with $\frac{\tilde{S}(X_{\min})+\tilde{S}(X_{\max})}{2}$ line where $X_{min} = -2.5$, $X_{max} = 2.5$. The equation for the mentioned segment of interpolated cumulative function is derived in the following way:

$$\tilde{S}(x)-\tilde{S}(-0.5)=\frac{\tilde{S}(0.5)-\tilde{S}(-0.5)}{0.5-(-0.5)}\cdot(x-(-0.5)) \qquad (8)$$

or

$$\tilde{S}(x)=\left(\tilde{S}(0.5)-\tilde{S}(-0.5)\right)\cdot(x+0.5)+\tilde{S}(-0.5) \qquad (9)$$

where $\tilde{S}(-0.5)=S_{-1}=s_{-2}+s_{-1}$ and $\tilde{S}(0.5)=S_{0}=s_{-2}+s_{-1}+s_{0}$.

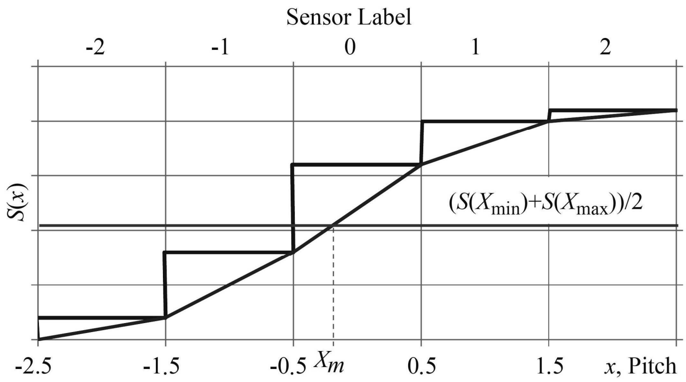

Fig. 6. Piecewise-linear interpolation for the piecewise constant cumulative function.

Then, taking into account that $\tilde{S}(-2.5)=S_{-3}=0$ and $\tilde{S}(2.5)=S_{2}=s_{-2}+s_{-1}+s_{0}+s_{1}+s_{2}$ the solution of equation

$$\tilde{S}(X_m)=\frac{S(X_{\min})+S(X_{\max})}{2} \qquad (10)$$

can be written as:

$$\left(\tilde{S}(0.5)-\tilde{S}(-0.5)\right)\cdot(X_m+0.5)+\tilde{S}(-0.5)=\frac{\tilde{S}(-2.5)+\tilde{S}(2.5)}{2}, \qquad (11)$$

$$\left(s_{-2}+s_{-1}+s_0-s_{-2}-s_{-1}\right)\cdot(X_m+0.5)+s_{-2}+s_{-1}=\frac{s_{-2}+s_{-1}+s_0+s_1+s_2}{2}, \qquad (12)$$

$$s_0\cdot(X_m+0.5)=\frac{s_{-2}+s_{-1}+s_0+s_1+s_2}{2}-s_{-2}-s_{-1}, \qquad (13)$$

$$X_m=\frac{s_{-2}+s_{-1}+s_0+s_1+s_2-2s_{-2}-2s_{-1}}{2\cdot s_0}-0.5, \qquad (14)$$

$$X_m=\frac{s_{-2}+s_{-1}+s_0+s_1+s_2-2s_{-2}-2s_{-1}-s_0}{2\cdot s_0}. \qquad (15)$$

And finally,

$$X_m = \frac{-s_{-2} - s_{-1} + s_1 + s_2}{2s_0}. \quad (16)$$

Algorithm validation and comparison with centroid and BR algorithms. The simulation based on the modelled signal (see Fig. 7) according to [6] was applied to validate the new algorithm and compare it with the Centroid and BR algorithms. For this simulation, the signal of diffuse light reflection was calculated for a corroded surface with the total area of corrosion spots of $\pi \cdot (0.15)^2 \approx 0.07$ mm^2 and their concentration of 326 cm^{-2}. Corrosion grains were modelled as spherical copper particles with a base radius of 1 μm (or a base diameter d_0 = 2 μm).

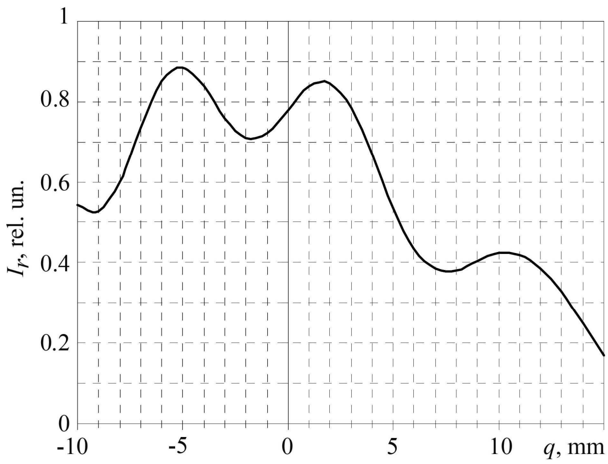

Fig. 7. Diffuse light reflection signals for 2 μm corrosion grains
(I_r – relative intensity measured by photodiode linear array).

The methodical systematic errors for zero noise as well as random errors for different types of noises were calculated for all positions of signal extrema. Then the maximal errors of these position determination were taken as the error estimates.

Thus, the maximal methodical systematic error for the 5-element centroid algorithm, 5-element BR algorithm and 5-element median algorithm has made 0.06 mm, 0.06 mm and 0.04 mm respectively. That is, the median algorithm provides the least methodical error.

Also, two types of noise were utilized for algorithms comparison:

1) Full common mode (CM) additive noise (noises for each of 5 sensor cell change synchronously)

$$N[5] = \sigma_{noise} \cdot \xi \cdot \mathbf{G} \quad (17)$$

where σ_{noise} is the noise RMS, ξ is centralized random value normally distributed with the standard deviation σ_ξ = 1, **G** is the 5-element unit gain vector for sensor cells

$$\mathbf{G} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}. \quad (18)$$

2) Uncorrelated additive noise

$$N[5] = \sigma_{noise} \cdot [\xi_i], \quad i = \overline{1,5}, \quad (19)$$

where $[\xi_i]$ is the 5-element vector of random values ξ.

A signal-to-noise ratio (SNR) for mentioned noises was determined as $s_{\max} / \sigma_{noise}$, where $s_{\max}$ was the maximal signal for the given PhDLA cells.

Fig. 8 and Fig. 9 show the result of calculated maximal error of extrema positions in case of full CM noise and uncorrelated noise, respectively.

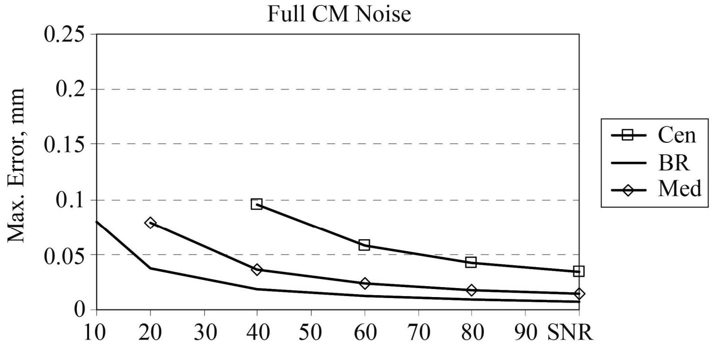

Fig. 8. Maximal random errors of signal extrema positions vs. SNR for centroid, BR and median algorithms (full CM additive noise).

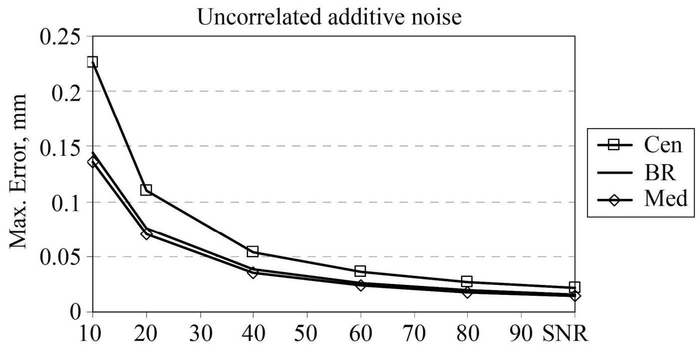

Fig. 9. Maximal random errors of signal extrema positions vs. SNR for centroid, BR and median algorithms (uncorrelated additive noise).

As it can be seen from Fig. 8, the median algorithm provides the maximal random error of extremum position determination that is somewhat greater than BR algorithm, but much less than the centroid algorithm in case of full CM additive noise. On the other hand, in case of uncorrelated additive noise for PhDLA cells (Fig. 9), the median algorithm shows less maximal error than the centroid algorithm and even a little bit less error than BR algorithm.

CONCLUSIONS

The proposed median algorithm uses a much simpler formula in comparison with other algorithms so long as it contains fewer operations in the denominator (neither total signal summation nor comparison). This algorithm has a less methodical systematic error (0.04 mm) than the centroid and BR algorithms. Also, it demonstrates much better noise immunity (for maximal errors in 100% tolerance interval) than the centroid algorithm for full common mode noise. In particular, the maximal random error of the median algorithm did not exceed 0.08 mm for SNR = 20 vs. 0.04 mm for BR algorithm and 0.1 mm for the centroid algorithm. For the uncorrelated noises, the proposed algorithm provides better noise immunity than the centroid algorithm as well as BR algorithm in SNR range from 10 to 100 (e.g. 0.14 mm vs. 0.23 mm and 0.15 mm respectively for SNR = 10).

Therefore, the signals median algorithm could be recommended for the determination of the extrema positions for diffuse light reflection intensity distribution.

The reasonable next step is investigation of applying the spline approximation for the signal 5-element cumulative function to improve the proposed algorithm.

1. Verkhovlyuk, A.M.; Bezpalyi, A.A.; Naymenko, M.I.; Starodub, N.P.; Noga, A.P. Surface defects of products from argentum. *Ukrainian Chemistry Journal*, **2009**, 75, 112–115. (In Ukrainian)
2. Ostash, O.P.; Kondyr, A.I.; Vol'demarov, O.V.; Gladish P.V.; Kurechko M.V. Structural micro-damageability of steels of the steam pipelines of thermal power plants. *Mater. Sci.*, **2009**, 45. https://doi.org/10.1007/s11003-009-9189-0
3. Baltes, H.; Brand, O.; Fedder, G. K.; Hierold, Ch.; Korvink, J.G.; Tabata, O.; Lohe, D.; Haußelt, J. Microengineering of Metals and Ceramics. Part II: *Special Replication Techniques, Automation, and Properties*, Vol. 4; Wiley, 2005.
4. Jordan, A. Microstructure Characterization and Corrosion Properties of Two Recycled Aluminium Alloys AA5050 and AA6011, *Thesis for the degree of Doctor of Philosophy; University of Manchester*, **2016**.
5. Ivasiv, I.B.; Dzhala, R.M. Peaks' Positions Estimation in Diffuse Light Reflection Sensor's Signal for Pitting Corrosion Detection. *Measuring and Computing Techniques in Technological Processes: Proceedings of the XVI International scientific and technical conference* (June 10–15, 2016, Odessa). Odessa–Khmelnytsky, KhNU, **2016**, P. 50.
6. Ivasiv, I.B. Estimation of errors in determining corrosion grain sizes by analysis of diffuse light reflection signal. *Information Extraction and Process*, **2020**, 48(124), 25–34. https://doi.org/10.15407/vidbir2020.48.025
7. Ferguson, J.A.; Sawyers, W.G.; Waddell, K.A.; Ferrige, A.G.; Alecio, R.; Ray, S. Improved centroid peak detection and mass accuracy using a novel, fast data reconstruction method. *Proc. of the 50th ASMS Conf. on Mass Spectrometry and Allied Topics*; 2002 June 2–6; Orlando, Florida; ASMS, **2002**.
8. Arines, J.; Ares, J. Minimum variance centroid thresholding. *Optics Letters*, **2002**, 27(7), 497–499. https://doi.org/10.1364/OL.27.000497
9. Thomas, S. Optimized centroid computing in a Shack-Hartmann sensor. *Proc. SPIE*, **2004**, 5490, 3, pp. 1238–1246. https://doi.org/10.1117/12.550055
10. Lange, E.; Gropl, C.; Reinert, K.; Kohlbacher, O.; Hildebrandt, A. High-Accuracy Peak Picking of Proteomics Data Using Wavelet Techniques. *Proc. Pacific Symp. on Biocomputing*, 2006 (January 3–7, 2006, Maui, Hawaii, USA). – **2006**. – P. 243–254. https://doi.org/10.1142/9789812701626_0023
11. Naidu, D.K.; Fisher, R.B. A Comparison of Algorithms for Subpixel Peak Detection. Image Technology. *Advances in Image Processing, Multimedia and Machine Vision*, Jorge L. C. Sanz, Ed.; *Springer: Berlin, Heidelberg*, **1996**.
12. Blais, F.; Rioux, M. Real-Time Numerical Peak Detector. *Signal Processing*, **1986**, 11, 145–155. https://doi.org/10.1016/0165-1684(86)90033-2
13. Nagaraj, K.; Lewis, S.H.; Walden, R.W.; Offord, G.E.; Shariatdoust, R.S.; Sabnis, J.A.; Peruzzi, R.O.; Barner, J.R.; Plany, J.; Mento, R.P.; Rakshani, V.A.; Hull, R.W. A median peak detecting analog signal processor for hard disk drive servo, *IEEE J. of Solid-State Circuits*, **1995**, 30(4), 461–470. https://doi.org/10.1109/4.375967

Received .09.2021