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ЯДЕРНА ФІЗИКА

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CAPTURE CROSS SECTIONS FOR HEAVY-ION REACTIONS PRODUCING COMPOUND SYSTEM WITH Z = 120

The fusion cross sections for reactions $^{50}Ti + ^{249}Cf$, $^{54}Cr + ^{248}Cm$, $^{58}Fe + ^{244}Pu$ and $^{64}Ni + ^{238}U$ are evaluated in the framework of simple barrier-penetration model, which takes into account quadrupole and hexadecapole surface deformations of nuclei.

Keywords: capture cross sections, synthesis of superheavy elements, transuranium elements, subbarrier tunneling, deformed nucleus, hexadecapole deformation.

Introduction

Super-heavy elements with Z = 112 - 118 have been synthesized in Dubna and Darmstadt by using hot fusion reactions $^{48}Ca + X$, where X is the heavy transuranium element. The heaviest element with proton number 118 has been synthesized in reaction $^{48}Ca + ^{249}Cf$. However, it is impossible to use ^{48}Ca beam for the synthesis of more heavy elements, because the elements with Z > 98 are not available for experiments. Therefore it is necessary to search other reactions for the synthesis of elements with proton number more than 118.

Taking into account the long time and complexity of experiment on the synthesis of super-heavy elements it is very useful to estimate the capture cross sections for various collision systems leading to superheavy elements with Z > 118. We have proposed a relatively simple and accurate method of calculating the potential interaction between deformed nuclei and their capture cross sections in previous papers [1, 2].

Note that the range of barrier height variation induced by mutual orientation of heavy well-deformed heavy nuclei is approximately $15 \div 20$ MeV. The range of barrier distance changing is around 2.5 fm [1]. Due to this deformation of nuclei and their mutual orientation during approaching are extremely important for subbarrier and near barrier heavy-ion fusion studies.

Fusion of deformed nuclei

The interaction potential $V(R, l, \Theta_1, \Theta_2, \Phi)$ of two deformed nuclei at distance R between mass centers and mutual orientation described by angles Θ_1, Θ_2 and Φ (Fig. 1) consists of Coulomb $V_C(R, \Theta_1, \Theta_2, \Phi)$, nuclear $V_N(R, \Theta_1, \Theta_2, \Phi)$ and rotational $V_l(R) = \hbar^2 l(l+1)/(2\mu R^2)$ parts

$$V(R, l, \Theta_1, \Theta_2, \Phi) = V_C(R, \Theta_1, \Theta_2, \Phi) + \\ + \eta V_N(R, \Theta_1, \Theta_2, \Phi) + V_l(R). \tag{1}$$

Here η is an adjustable coefficient that determines the contribution of the nuclear potential component $V_N(R, \Theta_1, \Theta_2, \Phi)$ to the total potential. This parameter is used for adjusting the potential at energies higher than barrier height.

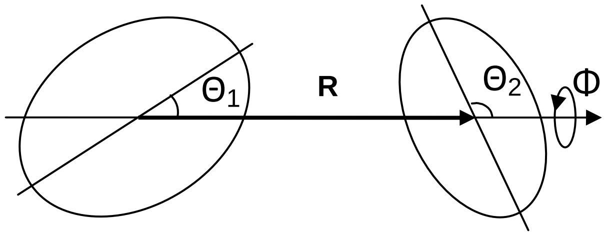

Fig. 1. The angles Θ_1, Θ_2 and Φ, and distance between of mass-centers R describing the arbitrary orientation of colliding nuclei.

The Coulomb interaction of two deformed nuclei is approximated as [1]

$$V_C(R, \Theta_1, \Theta_2, \Phi) = \frac{Z_1 Z_2 e^2}{R}\{1 + \sum_{l \ge 2}[f_{1l}(R, \Theta_1, R_{10})\beta_{1l} + f_{1l}(R, \Theta_2, R_{20})\beta_{2l}] + \\ + f_2(R, \Theta_1, R_{10})\beta_{12}^2 + f_2(R, \Theta_2, R_{20})\beta_{22}^2 + f_3(R, \Theta_1, \Theta_2, R_{10}, R_{20})\beta_{12}\beta_{22} + \\ + f_4(R, \Theta_1, \Theta_2, \Phi, R_{10}, R_{20})\beta_{12}\beta_{22}\}, \tag{2}$$

where Z_1 and Z_2 are the number of protons in corresponding nuclei, β_{il} is the parameter of l-pole deformation of nucleus i (i = 1, 2), $f_{1l}(R, \Theta_i, R_{i0})$, $f_2(R, \Theta_i, R_{i0})$, $f_3(R, \Theta_1, \Theta_2, R_{10}, R_{20})$ and $f_4(R, \Theta_1, \Theta_2, \Phi, R_{10}, R_{20})$ are simple functions [1]. Here R_{i0} is the radius of nucleus i in the case of spherical form.

Applying the proximity theorem [3] we can obtain a simple parametrization of the nuclear part of interaction potential between two deformed nuclei [1]

$$V_N(R, \Theta_1, \Theta_2, \Phi) \approx \frac{C_{10} + C_{20}}{[(C_1^{\mathrm{II}} + C_2^{\mathrm{II}})(C_1^{\perp} + C_2^{\perp})]^{1/2}} \times$$

$$\times V_N^0(d(R, \Theta_1, \Theta_2, \Phi)), \quad (3)$$

where C_i^{II} and $C_i^{\perp}$ are the main curvatures of deformed surface of nucleus i at the point closest to the surface of another nucleus, $C_{10} = 1/R_{10}$ and $C_{20} = 1/R_{20}$ the curvatures of spherical nuclei, $d(R, \Theta_1, \Theta_2, \Phi)$ is the closest distance between surfaces of interacting nuclei, $V_N^0(d)$ is the nuclear part of the interaction potential between spherical nuclei at $d = R - R_{10} - R_{20}$. The nuclear part of potential depends strongly on the value of the closest distance between surfaces of interacting nuclei, therefore we evaluate $d(R, \Theta_1, \Theta_2, \Phi)$ numerically.

The nuclear part of the interaction potential between spherical nuclei $V_N^0(d)$ is described by expression in Ref. [4]. This expression for nuclear part of potential is obtained using the semi-microscopic energy density approach for evaluation of nucleus-nucleus interaction energy [4, 5]. The barrier height and barrier radius of the potential between spherical nuclei evaluated with the help of this expression well agree with corresponding empirical values [4].

The surface curvatures $C_2^{\mathrm{II}\perp}$ depend on corresponding orientation angle(s) and deformation parameters. Useful expressions for surface curvatures are given in [1].

Note that effects of surface deformations on the nuclear part of the interaction between nuclei are considered with the same accuracy as the one for the Coulomb part.

Various orientations of deformed nuclei occur during collisions, therefore the fusion reaction cross section induced by two deformed nuclei should be averaged over all possible mutual orientations of colliding nuclei

$$\sigma(E) = \frac{\pi\hbar^2}{2\mu E} \sum_l (2l+1) < T_l(E, \Theta_1, \Theta_2, \Phi) > . \quad (4)$$

Here μ is the reduced mass of colliding nuclei, E is the collision energy, $< T_l(E, \Theta_1, \Theta_2, \Phi) >$ is the transmission coefficient evaluated at orientation of colliding nuclei specified by angles Θ_1, Θ_2, Φ (see Fig. 1).

We use the WKB approximation for evaluation of the transmission coefficient for subbarrier energies

$$T_l(E, \Theta_1, \Theta_2, \Phi) =$$

$$= \{1 + \exp[\frac{2}{\hbar} \int_{\mathrm{a}}^{\mathrm{b}} \sqrt{2\mu(V(R, l, \Theta_1, \Theta_2, \Phi)\text{-}E)} \mathrm{d}R]\}^{-1} \quad (5)$$

and the Hill - Wheeler approach [6] for over-barrier collision energies.

The inner a($E, l, \Theta_1, \Theta_2, \Phi$) and outer b($E, l, \Theta_1, \Theta_2, \Phi$) turning points in Eq. (5) are determined from corresponding equations

$$V(\mathrm{a}\,(E, l, \Theta_1, \Theta_2, \Phi), l, \Theta_1, \Theta_2, \Phi) = E,$$

$$V(\mathrm{b}\,(E, l, \Theta_1, \Theta_2, \Phi), l, \Theta_1, \Theta_2, \Phi) = E.$$

Discussion and conclusion

Using Eqs. (1) - (5) we evaluate the fusion cross section $\sigma(E)$ values for reaction ^{50}Ti + ^{249}Cf, ^{54}Cr + ^{248}Cm, ^{58}Fe + ^{244}Pu and ^{64}Ni + ^{238}U. These reactions are considered as potential candidates for the synthesis of element Z = 120 of the periodic table.

We used reaction ^{48}Ca + ^{244}Pu determine of adjustable coefficient η. The experimental data for this reaction take from Ref. [7]. The cross sections calculated with allowance for the second-order terms in the quadrupole and hexadecapole deformations of the ^{244}Pu and $\eta = 0.896$ agree well with experimental data, see solid line in Fig. 2.

The capture cross section for heavy systems is strongly depends on hexadecapole deformation at sub-barrier energies [2]. Deformations of nuclei presented in the Table, and were taken from [8] for quadrupole and [9] for hexadecapole deformations.

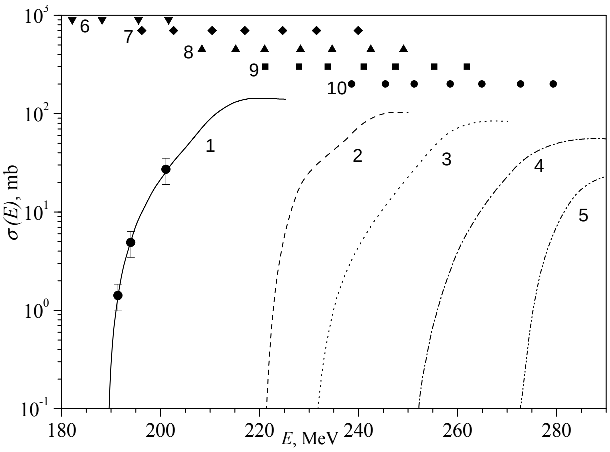

Fig. 2. Fusion cross-section (lines), Q reaction and evaporation of *1*, *2*, *3*, *4*, *5*, *6* neutrons (dots) for reactions: $^{48}Ca + ^{244}Pu$ (line *1* and triangles *6*); $^{50}Ti + ^{249}Cf$ (line *2* and diamonds *7*); $^{54}Cr + ^{248}Cm$ (line *3* and triangles *8*); $^{58}Fe + ^{244}Pu$ (line *4* and squares *9*); $^{64}Ni + ^{238}U$ (line *5* and dots *10*), correspondingly.

During formation the superheavy nuclei the neutrons are evaporated from formed excited compound-nucleus in competition to the fission of the compound nucleus. If the capture cross-section values are reasonably high for the small excitation energies of the compound nucleus, or corresponding collision energies, than the formation of the superheavy nuclei is probable. Therefore the value of capture cross-section should be relatively high around the thresholds of 3 - 5 neutrons for expectable formation of superheavy elements.

Value of deformation parameters of nuclei

X	β_{i2}	β_{i4}
^{48}Ca	0	0
^{50}Ti	0	0
^{54}Cr	0.25	0.045
^{58}Fe	0.2587	-0.019
^{64}Ni	0.179	-0.005
^{238}U	0.2863	0.093
^{244}Pu	0.2931	0.062
^{248}Cm	0.2972	0.04
^{249}Cf	0.55	0

Comparing the values of capture cross-section around thresholds of 3 - 5 neutrons in Fig. 2 we conclude that the most promising reactions for synthesis element with Z = 120 are $^{50}Ti + ^{249}Cf$ and $^{54}Cr + ^{248}Cm$. These reactions are similar to reaction $^{48}Ca + ^{244}Pu$ and they have reasonably high capture cross section at energies close to emissions 3 - 5 neutrons from the compound nucleus, see Fig. 2.

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ПЕРЕРІЗ РЕАКЦІЇ ЗАХОПЛЕННЯ СИСТЕМ ВАЖКИХ ЯДЕР, ЩО ПРИВОДЯТЬ ДО УТВОРЕННЯ КОМПАУНД-ЯДРА З Z = 120

Розраховано переріз захвату для реакцій $^{50}Ti + ^{249}Cf$, $^{54}Cr + ^{248}Cm$, $^{58}Fe + ^{244}Pu$ і $^{64}Ni + ^{238}U$ в наближенні простого підбар’єрного тунелювання, яке враховує квадрупольну та гексадекапольну деформацію поверхні ядер.

Ключові слова: переріз захоплення, синтез надважких елементів, трансуранові елементи, підбар’єрне тунелювання, деформоване ядро, гексадекапольна деформація.

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СЕЧЕНИЕ РЕАКЦИИ ЗАХВАТА СИСТЕМ ТЯЖЕЛЫХ ЯДЕР, ПРИВОДЯЩИХ К ОБРАЗОВАНИЮ КОМПАУНД-ЯДРА С Z = 120

Произведен расчет сечения захвата для реакций $^{50}Ti + ^{249}Cf$, $^{54}Cr + ^{248}Cm$, $^{58}Fe + ^{244}Pu$ и $^{64}Ni + ^{238}U$ в приближении простого подбарьерного туннелирования, которое учитывает квадрупольную и гексадекапольную деформацию поверхности ядер.

Ключевые слова: сечение захвата, синтез сверхтяжелых элементов, трансурановые элементы, подбарьерное туннелирование, деформированное ядро, гексадекапольная деформация.

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