ЯДЕРНА ФІЗИКА

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THE INTERNAL CONVERSION COEFFICIENT FOR THE *K*-FORBIDDEN *E*1-TRANSITION WITH THE ENERGY OF 55 keV IN ^{177}Hf

The precise γ-ray intensities of the transitions following the decay of 160-day isomeric state in ^{177}Lu have been measured by using two different types of HPGe-detectors. The values of the internal conversion coefficient and penetration parameter λ for *E*1-transition with the energy of 55 keV were determined from intensity balance of $21/2^-$ 1260 keV level in ^{177}Hf.

Keywords: radioactivity, ^{177m}Lu, γ-spectra, HPGe-detectors, measurements I(γ), internal conversion, penetration parameter.

Introduction

This article continues the series of works on intranuclear conversion in the *K*-forbidden electric multipole transitions excited by ^{177m}Lu decay. Three of these transitions accompany the decay of the 160-day isomeric state in ^{177}Lu: γ55, γ116, and γ228 keV (Fig. 1). All of them are hindered as compared to single-particle estimates. Some anomalies in γ-ray internal-conversion coefficients (ICC), caused by penetration effect, are possible for such transitions.

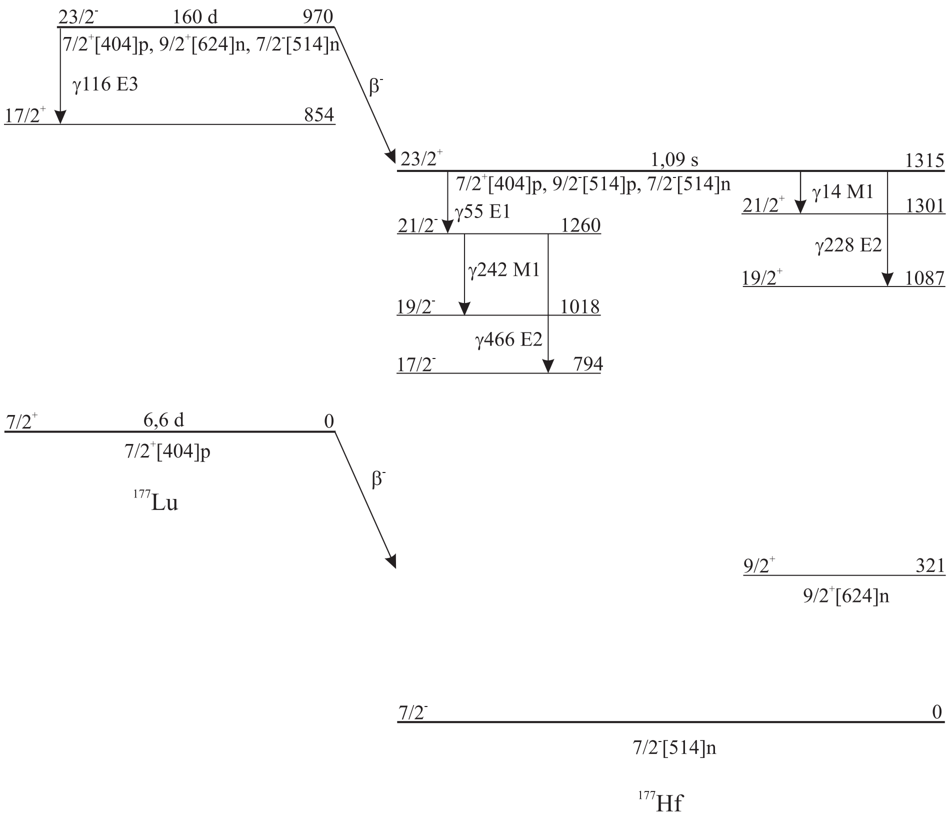

Fig. 1. The partial decay scheme of ^{177m}Lu.

In internal conversion theory, by a penetration effect or intranuclear conversion is implied a correction to ICC arising in passing from transition electromagnetic potentials calculated for point-like nucleus to the potentials calculated for finite-size nucleus. Generally, such corrections do not exceed 2 % and have only a slight effect on ICC value. A completely different type of situation occurs in the case of strongly hindered γ-transitions. In such case a contribution from internal conversion may become a crucial factor governing the ICC value. Of course, selection rules, which are responsible for a decrease in probability of γ-radiation, should have essentially smaller effect on the probability of internal conversion.

Appearance of anomalies in ICC of *K*-forbidden transitions is due to admixtures with respect to quantum number *K* in wave functions of initial and final states. There are admixtures that allow conversion transition according to the selection rules with respect to asymptotic quantum numbers, while γ-transition is forbidden. In this case anomalies in ICC caused by the penetration effect are observed. If the selection rules for conversion transition and γ-transition are identical, there are no anomalies. At present, it is very difficult to quantitatively estimate these admixtures. By this reason, it is not feasible to make a prediction of anomalies in ICC for a given *K*-forbidden transition.

Earlier, in Ref. [1, 2] there was found minor variance between experimental and theoretical values of ICC for γ228 and γ116 keV transitions, which cannot be explained by admixtures of different multipolarities with the same parity. Such deviation can be eliminated by assuming the presence of intranuclear conversion.

The total ICC of the γ55 keV *E*1-transiton can be estimated from the balance of intensities of the $21/2^-$ 1260 keV level in ^{177}Hf. Following from the ^{177m}Hf decay scheme (see Fig. 1), this level is powered by the γ55 keV transition and deexcited by two intraband γ242 and γ466 keV transitions having the *M*1- and *E*2-multipolarity respectively. The intensities of the strong γ-rays are known with accuracy of (2 ÷ 5) %, but there is disagreement in evaluation of the intensities of some of the weaker lines, such as γ242 keV. Our current research was to clarify all controversial questions in this area and to provide a more accurate estimate of the total ICC of the γ55 keV *E*1-transiton.

Experimental technique

The relative intensities of γ-rays following the decay of ^{177m}Lu were measured with a gamma-spectrometer that comprises two horizontal coaxial HPGe-detectors: GMX-30190 and GEM-40195, having the resolution of 1,89 and 1,73 keV for the γ1332-line of ^{60}Co and efficiency of 33 and 43 %, respectively.

The radioactive ^{177m}Lu sources were obtained in the (n, γ) reaction as a result of enriched to 27,1 % in 176 mass number lutetium target irradiation with neutrons at the research nuclear reactor WWR-M. The measurements of gamma-ray spectra started two months after the end of irradiation so that ^{177}Lu ($T_{1/2}$ = 6,6 days), having much larger activation cross-section, must have decayed en masse.

The standard ^{60}Co, ^{133}Ba, ^{137}Cs, ^{152}Eu, ^{228}Th, and ^{241}Am γ-sources were used for accurate calibration of detectors for the energy range from 26 to 1620 keV. The shape of the efficiency curve is well described by the Campbell function [3]

$$\varepsilon(E) = \sum_{i=1}^{3} p_{2i-1} e^{-p_{2i}E} + p_7 E^{-p_8} . \qquad (1)$$

Calibration parameters p_i were found by the least-square method. The uncertainty in the efficiency curve of both detectors does not exceed 2 % throughout the energy range.

To minimize possible systematic errors a series of measurements were performed – using different types of HPGe-detectors, at different geometries, at different gains and channel widths of an amplitude-to-digital converter (8192 and 16384 quantization levels of the input signal) – 20 series of measurements in all.

Results and discussion

The γ-ray spectra were analysed using WinSpectrum [4], a computer program which allows determining with high precision the energy and intensity of components that have an asymmetric line shape and the ones that are overlapping. The usage of different types of detectors allowed us to determine the relative intensities of γ-rays for the energy range above 100 keV more precisely. Our data agrees to a great extent with the data of other researchers while having higher precision.

The intensity balance at the $21/2^-$ 1260 keV level in ^{177}Hf can be written as

$$(1+\alpha(55))I_\gamma(55) = (1+\alpha(242))I_\gamma(242) +$$

$$+(1+\alpha(466))I_\gamma(466) , \qquad (2)$$

where α(55), α(242), α(466) and $I_\gamma(55)$, $I_\gamma(242)$, $I_\gamma(466)$ are the total ICCs and transition intensities with the energy of 55, 242, and 466 keV, respectively.

Using our data on the intensities of the γ242 and γ466 keV transitions, bringing the experimental value $I_\gamma(55)$ from Ref. [5] and theoretical values of ICC for γ242 and γ466 keV transitions from Ref. [6], we have calculated the total ICC of the γ55 keV *E*1-transiton to be $\alpha(55)_{exp} = 1,08 \pm 0,23$. The theoretical value of ICC in the hafnium for the γ55 keV *E*1-transiton is much lower, $\alpha(55)_{th} = 0,337$. To coordinate them the existence of the admixture of *M*2-multipolarity or the existence of the intranuclear conversion should be assumed.

The value of the admixture of *M*2-multipolarity can be calculated using the expression

$$\alpha(55)_{\exp} = \alpha(E1)\frac{1}{1+\delta^2(M2/E1)} +$$

$$+\alpha(M2)\frac{\delta^2(M2/E1)}{1+\delta^2(M2/E1)}, \qquad (3)$$

where $\delta(M2/E1)$ is $M2/E1$ multipole mixing ratio for γ55 keV transition in ^{177}Hf, $\alpha(E1)$ and $\alpha(M2)$ are the theoretical values of ICC for this transition assuming $E1$- and $M2$-multipolarity respectively.

The obtained value of $\delta^2(M2/E1) = (5{,}2 \pm 1{,}6)\cdot 10^{-3}$ leads to the Weisskopf hindrance factor for the $M2$-component $F_W(\gamma 55\ M2) = (5 \div 9)\cdot 10^6$, while the factors are much higher for other K-forbidden transitions in ^{177}Lu and ^{177}Hf (Table 1).

Table 1. **Weisskopf hindrance factors for *K*-forbidden transitions in ^{177}Lu and ^{177}Hf**

E_γ, keV	Multipolarity, L	$\Delta K = K_i - K_f$	$\nu = \Delta K - L$	F_w	$f_\nu = (F_w)^{1/\nu}$
14	$M1$	7	6	$7{,}0 \cdot 10^{10}$	64,2
55	$E1$	8	7	$3{,}7 \cdot 10^{13}$	86,8
55	$M2$	8	6	$(5 \div 9) \cdot 10^6$	13,1 ÷ 14,4
116	$E3$	8	5	$9{,}1 \cdot 10^8$	61,9
228	$E2$	7	5	$1{,}5 \cdot 10^8$	43,2

Table 1 shows that the $M2$-component for γ-transition with the energy of 55 keV has a 3,0 to 6,6 times smaller hindrance factor per K-forbidenness unit f_ν than other transitions. It means that the $M2$-admixture value is likely to be exaggerated 10^3 to 10^5 times.

Analyses of the cases of anomalous conversion can be made with the inclusion of penetration corrections developed by Church and Weneser [7]. Using the parameterization of Hager and Seltzer [8] the electric ICC's can be written as

$$ICC = \alpha(EL)(1 + A_1\lambda_1 + A_2\lambda_1^2 + A_3\lambda_2 + A_4\lambda_2^2 + A_5\lambda_1\lambda_2), \qquad (4)$$

where $\alpha(EL)$ are the normal (no penetration) ICC's tabulated in Ref. [6], A_i are coefficients calculated in Ref. [8] from electron wave functions for the mulipolarity of interest, and λ_i are the electric penetration parameters. The penetration parameters depend on nuclear structure and are determined from an analysis of the experimental quantities.

If, as it is in our case, independent experimental data are insufficient for finding both penetration parameters λ_1 and λ_2, the calculations are limited to one nuclear current parameter λ_1, which, in general, the anomalies in the EL-transitions depend on; the nuclear charge parameter λ_2 is considered to be zero. Because of the fact that in Ref. [8] the penetration coefficients are tabulated only for K-, L-, and M-subshells the following expression was used for the data set analysis

$$\alpha(55)_{\exp} = \alpha_L(1 + A_1^L\lambda_1 + A_2^L\lambda_1^2) +$$

$$+\alpha_M(1 + A_1^M\lambda_1 + A_2^M\lambda_1^2) + \alpha_{N+O}, \qquad (5)$$

where α_L, α_M, α_{N+O} are the theoretical values of ICC, A_i^L, A_i^M are the coefficients for penetration effect analysis in ICC for L-, M-, and N + O-subshells of hafnium respectively.

Theoretical values for the conversion coefficients and penetration coefficients were interpolated from the tables by Hager and Seltzer [6, 8]. The results of calculation are listed in Table 2. Known experimental values of nuclear penetration parameter λ_1 for other K-forbidden $E1$-transitions from the Ref. [9] are also given in the Table.

For quantitative estimates of nuclear penetration parameter λ_1 depending on the Weisskopf hindrance factor F_W on the basis of empirical data it is convenient to draw a graph of such relation using the experimental data given in Table 2 (Fig. 2).

Table 2. **Experimental values of the nuclear penetration parameter λ_1 for the *K*-forbidden *E*1-transitions**

Nucleus	E_γ, keV	$\nu = \Delta K - L$	F_w	λ_1	Reference
^{169}Tm	240,3	2	$2{,}9 \cdot 10^9$	4,5 ± 0,6	10
^{171}Tm	295,9	2	$9{,}3 \cdot 10^8$	2,7 ± 0,6 2,8	11 12
^{171}Tm	308,3	2	$5{,}3 \cdot 10^8$	1,2 ± 0,4 1,2	11 12
^{171}Yb	19,39	2	$1{,}2 \cdot 10^9$	-(1,5 ± 0,5)	13
^{177}Hf	55,15	7	$3{,}7 \cdot 10^{13}$	12 ± 3 or -(17 ± 3)	present work

Continuation of Table 2

Nucleus	E_γ, keV	$\nu = \Delta K - L$	F_w	λ_1	Reference
^{180}Hf	57,6	7	$3,6 \cdot 10^{16}$	$7,8 \pm 1,0$	14
				6,9	15
				$7,0 \pm 0,3$	16
				$6,0 \pm 0,5$*	17
				$6,4 \pm 0,3$*	17
				$7,7 \pm 1,0$*	17
				$7,6 \pm 0,5$*	17
				$7,0 \pm 0,7$	18
				$6,8 \pm 0,2$**	

* Using δ^2 from different references.
** Weighted mean.

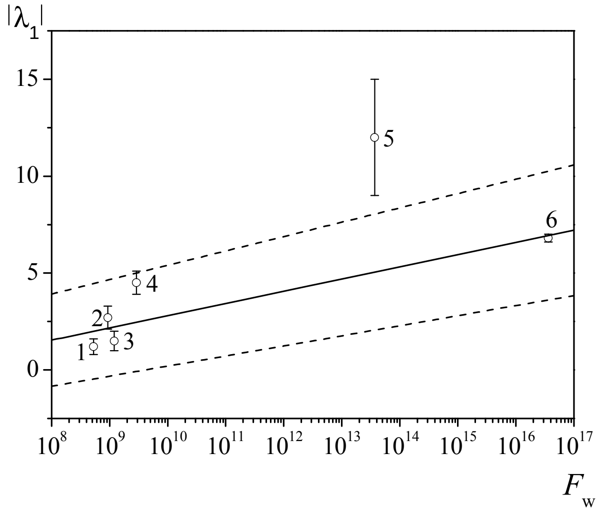

Fig. 2. Relation between the nuclear penetration parameter $|\lambda_1|$ and the Weisskopf hindrance factor F_w for the K-forbidden $E1$-transitions. *1* – ^{171}Tm (308,3); *2* – ^{171}Tm (295,9); *3* – ^{171}Yb (19,39); *4* – ^{169}Tm (240,3); *5* – ^{177}Hf (55,15); *6* – ^{180}Hf (57,6); the number in parentheses is the transition energy in keV. Smaller by absolute value, $|\lambda_1| = 12 \pm 3$ is shown for ^{177}Hf.

For Fig. 2 empirical relation (solid line) between the nuclear penetration parameter $|\lambda_1|$ and the Weisskopf hindrance factor F_w for the K-forbidden $E1$-transitions was determined without considering the γ55 keV transition in ^{177}Hf. It is described by the equation $\lambda_1 = a + b\log(F_w)$. The following values were found by the least-square method: $a = -(3,5 \pm 1,5)$; $b = 0,63 \pm 0,11$. The dashed lines show a 68% confidence interval. As Fig. 2 shows, the obtained experimental value of λ_1 for the γ55 keV transition in ^{177}Hf appear to be higher than expected from the empirical relationship.

Despite that, the explanation of anomalies in the internal conversion coefficients for the $E1$-transition with the energy of 55 keV with occurrence of intranuclear conversion, from our standpoint, is more grounded. As for possible aspects of further research, it would be very interesting to obtain experimental data on the relative intensities of internal-conversion electron lines on L-subshells of ^{177}Hf for this transition, or to attempt to determine more precisely the intensity of the γ55 keV photon in γ-spectrum using high-resolution detectors.

Conclusions

The penetration parameter λ is defined in Ref. [8] as the ratio of the reduced nuclear penetration matrix element to the reduced matrix element for gamma emission. Therefore, if the partial half-life for the radiative transition has been measured, and the experimental value of λ from ICC has been received, one can find the experimental value of the nuclear penetration matrix element and then compare it with theoretical estimates. The calculations of penetration matrix elements for several K-forbidden ($\Delta K = 3$) $E1$ transitions in ^{169}Tm and ^{171}Tm were performed by Sergeenkov and Kharitonov [11] on the basis of Nilsson's model as a result of Coriolis interaction. Theoretical values agree within the error limits with experimental data. But there is still no explanation of the origin of anomalies in the ICC of seven times K-forbidden 57 keV $E1$-transition in ^{180}Hf. It very likely that for such strong forbiddance factors that have not been studied enough matter. Our measurements of the penetration parameter λ for the second in magnitude K-forbidden $E1$-transition in ^{177}Hf could have a positive impact on understanding of underlying processes.

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КОЕФІЦІЄНТ ВНУТРІШНЬОЇ КОНВЕРСІЇ *К*-ЗАБОРОНЕНОГО *Е*1-ПЕРЕХОДУ З ЕНЕРГІЄЮ 55 кеВ У ^{177}Hf

На γ-спектрометрі, що містить два різних типи HPGe-детекторів, з високою точністю поміряно відносні інтенсивності γ-променів, які супроводжують розпад 160-добового ізомерного стану в ^{177}Lu. З балансу інтенсивностей для рівня 21/2$^-$ 1260 кеВ ^{177}Hf визначено коефіцієнт внутрішньої конверсії *E*1-переходу γ55 кеВ та величину параметра проникнення λ.

Ключові слова: радіоактивність, ^{177m}Lu, γ-спектри, HPGe-детектори, інтенсивності γ-променів, внутрішня конверсія, параметр проникнення.

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КОЭФФИЦИЕНТ ВНУТРЕННЕЙ КОНВЕРСИИ *К*-ЗАПРЕЩЕННОГО *Е*1-ПЕРЕХОДА С ЭНЕРГИЕЙ 55 кэВ В ^{177}Hf

На γ-спектрометре, включающем в себя два различных типа HPGe-детекторов, с высокой точностью измерены относительные интенсивности γ-лучей, сопровождающих распад 160-суточного изомерного состояния в ^{177}Lu. Из баланса интенсивностей для уровня 21/2$^-$ 1260 кэВ ^{177}Hf определены коэффициент внутренней конверсии *E*1-перехода γ55 кэВ и величина параметра проникновения λ.

Ключевые слова: радиоактивность, ^{177m}Lu, γ-спектры, HPGe-детекторы, интенсивности γ-лучей, внутренняя конверсия, параметр проникновения.

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